Part IX
Discussion Groups

Current Problems and Challenges in Non-university Tertiary Mathematics Education (NTME)

James Roznowsk and Huei Wuan Low-Ee

Report

The countries represented during the discussion group's meetings included: Australia, Canada, China, Iran, Israel, Philippines, Singapore, the United Arab Emirates, and the United States. The types of institutions varied among the countries and included: two and four-year vocational institutions, community colleges, programs related to retraining adults, and a one-year preparatory program in Israel for individuals coming out of military service and interested in attending university.

The discussion group team suggested five questions that were developed from the proposal. These dealt with:

- Student placement;
- Student learning of mathematics;
- Use of technology in teaching, learning, and assessment of mathematics;
- Classroom research; and
- Faculty development.

The attendees were asked to select the topics they were most interested in discussing. The topics with the most interest were technology, classroom research, and faculty development.

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To facilitate the discussion of issues related to technology, attendees were asked to answer the following question: What types of technology are available to teach, learn and assess mathematics at your institution? The notes from this discussion follow.

Notes on Uses of Technologies at Non-university Tertiary Institutions

Types of technologies:

- Graphing calculator
- Computer software (e.g. Excel™)
- CAS—good for learning math
- Internet resources
- Projectors and presentation devices
- Online learning platforms (homework problems)—there can be differences between the way in which a topic is explained in online tutoring software and how the instructor teaches, especially when the online tutoring comes with the textbook. In Singapore, instructors typically develop their own materials.
- Course management software (e.g. Blackboard™, WebCT™, Angel™)

Graphing calculators are handled with a variety of approaches around the world —either not allowed, allowed, or required. Singapore—graphing calculators are compulsory for mathematics at A-levels, and also adopted by high schools offering integrated programmes, the calculators are reset before the start of the A-levels mathematics papers. US—graphing calculator may be required (may be rented) in community college, but then may not be allowed at universities. Canada—required at high school, not permitted at most post-secondaries. Philippines—had assessments both with and without graphing calculators within a single course. Used among pre-service teachers in a course on technology. Worksheets are given to guide the students to gain further understanding from the work done with the hand-held calculators.

The discussion related to technology also led to a discussion of developmental mathematics and different terminologies used in different countries or regions. At some institutions “intermediate” algebra and “college” algebra are considered separate topics. Some states in the US are not having developmental math taught in lecture format, and students are being placed in front of computers instead. There is some concern about how well weak students will do under these conditions. There are similar issues in the Philippines. Universities are phasing out foundations mathematics. In Singapore, many of the students who opt to join the five polytechnics are weaker in math. At Singapore Polytechnic, weaker students are given customised CD-ROMs and are required to complete exercises before starting at the polytechnic. They are given assessment test upon arrival, and that determines whether or not they are required to attend face-to-face remedial math sessions. Recorded lectures given by selected lecturers to supplement the regular lectures are also provided in Blackboard.

For student teachers in Australia the problem is that when teachers first start, they usually reproduce what they saw when they were in school, so in training teachers, lecturers try to introduce more and different tools that are available.

The second session of DG1 focused on classroom research. Individuals did 5-min presentations on the different aspects of classroom research. These included research projects on student/teacher interaction, assessment of student learning, and potential research topic areas. After the presentations, the group discussed the expectations of instructors at their varied institutions with regard to conducting classroom research. At many institutions in the United States and Canada, research is not an expectation of faculty at community or vocational colleges. They are often not given the time or resources needed to do classroom research. In fact, at some institutions, instructors who conduct classroom research may be thought of as taking time away from their assigned responsibilities.

An attendee from Singapore shared information about a new requirement of faculty at her institution to participate in research projects. Questions by others involved faculty reaction to this expectation and professional development for faculty who are new to such research. Information about classroom research in Singapore was distributed to the discussion group attendees after ICME. It was offered as a model that can be adapted to encourage classroom research by instructors at institutions in a variety of countries.

Before the end of the session, participants reviewed ways to continue the discussion beyond ICME 12. The session closed with agreement among attendees that the discussion was of great value and arrangements were developed to make sure a proposal for the continuation of the discussion group would be submitted for ICME 13.

Creativity in Mathematics Education

Hartwig Meissner

Report

The Discussion Group 2 on Creativity in Mathematics Education at ICME-12 in Seoul, affiliated with MCG, was a great success. Co-chaired by Board members Emily Velikova and Vince Matsko, the group focused on issues relating to creativity in the classroom as well as training both pre-service and in-service teachers regarding effective ways of fostering creativity.

DG 2 faced difficulties in communicating with ICME organizers, and as a result, participants were not able to see the proposals before the conference. As a result, the chairs decided to begin each of the two ninety-minute sessions with brief summaries of the papers delivered by their authors. These were both lively and informative, and gave the 100+ participants in DG 2 a chance to be brought up to speed.

The sessions focused on the following questions, developed by the Board earlier in the year:

- What does creativity mean in the process of teaching and learning mathematics?
- How can we develop or stimulate creative activities in and beyond the mathematics classroom?
- How might we balance mathematical skill training and mathematical creativity?
- What should be done in teacher training programs at the pre-service and in-service levels to foster creativity in the classroom?

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(Note: a link to the DG 2 website, including a full description of the Discussion Group as well as all the submitted papers, is accessible from the MCG website.)

The first session addressed the first two questions, while the second centered around the last two. The chairs subdivided the first two questions into six, so that smaller discussion groups could be formed. DG 2 participants selected the question they wished to discuss, and elected a representative who summarized the discussion at the end of the session. Many participants remarked on quality of the discussions, and all were stimulated by the sharing of ideas.

The second session was rather smaller than the first, occurring on the last full day of ICME-12. As a result participants voted to have a large group discussion rather than breaking into smaller groups. This proved to be effective—and even though participants were tired after a hectic week, it proved difficult to make sure everyone had a chance to speak. Those involved had a real passion for creating engaging activities in the mathematics classroom, and there was no shortage of ideas to share.

Of course, in discussions like these, more questions are raised than are answered. These questions came up as a response to concerned teachers truly wanting to be more creative in the classroom. Among the questions raised by the DG 2 participants were:

- How do we decrease pressure on students so that they are more free to be motivated and involved in mathematics?
- How can we use technology to allow students to demonstrate originality, flexibility, and fluency of thought?
- How can we develop creativity within a pre-service teacher's university experience?
- Given we believe that *all* students can be creative, how can we create opportunities for students to do so?
- How can we deliberately foster creative thinking to encourage innovation?
- How can we provide accessible resources for teachers so that they may more easily bring creative activities into their classrooms?
- How can we change the climate of university education departments so that developing creativity in teachers is valued and addressed in the curriculum?
- How can creativity in mathematics education be made a priority at a regional or national level?

Of course none of these questions has an easy answer. But one or more of them might be suitable for a discussion forum or a special session of a conference on education. We welcome contributions to this newsletter from mathematics educators who have successfully answered one of these questions either in their classroom, or who made an impact regarding one of these questions on a local, regional, or national level.

Issues Surrounding Teaching Linear Algebra

Avi Berman and Kazuyoshi Okubo

Report

Linear Algebra is one of the most important courses in the education of mathematicians, scientists, engineers and economists. DG 3 was organized by The Education Committee of International Linear Algebra Society (ILAS) in order to give mathematicians and mathematics educators the opportunity to discuss several issues on teaching and learning Linear Algebra including motivation, challenging problems, visualization, learning technology, preparation in high school, history of Linear algebra and research topics at different levels. Some of these problems were discussed. Around 50 participants participated in the discussion.

Motivation

The interest in learning linear algebra can be motivated by real life (high tech) applications and by challenging problems. The following examples were mentioned:

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Google's Page Rank; Edge detection methods; Neural networks; Problems in graph theory; Properties of the Fibonacci sequence; Computer games—"Fiver" (see http://www.math.com/students/puzzles/fiver/fiver.html), Another popular game that can be solved by linear algebra (see http://matrix.skku.ac.kr/bljava-v1/Test.html).

Technology

Sang-Gu Lee from Korea described the work on Sage. He talked about what can be done in Mobile learning Environment and mentioned his coming regular lecturer on Mobile LA. He mentioned Bill Barton' words on Monday "We know that Education using ICT will improve the quality of Math Education. But it is clear that we are not THERE yet" (ICT Revolution in Math Education). http://www.sciencetimes.co.kr/article.do?todo=view&atidx=0000063949. Avi Berman from Israel described the use of clickers to promote the students' involvement and the communication between them and the professors.

J. L. Dorrier from France raised a question "Is it a good idea to use computers?" Sang-Gu said "If we do it properly, there is no reason for not using technology for education. I am used to teach and discuss in a traditional way, but I encourage our students to use whatever they can use for better understanding of Linear Algebra." Dorrier said that he prefers to wait with the use of computers to a later stage. Other participants said that they use Maple from the beginning of the course. Ludwig Paditz from Germany pointed out that technology sometimes gives incorrect results. "Modern calculators (CAS) resolve some problems but sometimes incorrect". It is important that students check if the computed results make sense.

Understanding

Megan Wawro from the USA described her research with Chris Rasmussen on how students engage with eigenvalue-eigenvector system making connections with functions.

Sepideh Stewart (New Zealand, USA) described her PhD thesis on teaching and learning Linear Algebra. She made a framework using Tall's embodied symbolic and formal words of mathematical thinking in conjunction with Ed Dubinsky's APOS theory. She found that the majority of the students were comfortable in the symbolic world but struggled with formal definitions and theorems. She also found that embodied (giving body to an abstract idea) thinking helped some students to have a better grasp of Linear Algebra. Her thesis is available on the web.

Teaching Mathematics and Engineering Students

Saeja Kim at U Mass, Dartmouth said she is not in favor of introducing Linear Algebra in an abstract way. She suggested to start with linear equations pointing out that most students are struggling with even the basics ideas. Avi said he also starts with linear equations but quickly moves into more abstract theory. Dorrier said that Linear Algebra is not about solving systems. The main thing is having mental views. He believes that abstraction ability of math majors should be developed. Avi said that at the Technion this is done also for students of electrical engineering and computer science.

Michelle Zandieh (Arizona) and many other participants emphasized the importance of Geometry and Visualization.

Chris Rasmussen (San Diego) asked how the differential equations and the linear algebra courses can be combined. The question was answered by a presentation by Karsten Schmidt from Denmark titled: "Revising the first semester math course for engineering students".

The names of the participants and photos from the two sessions can be found in http://matrix.skku.ac.kr/2014-Album/ICME12-DG3-report-v1.htm.

The Evolution of Mathematics-Teachers' Community-of-Practice

Nitsa Movshovitz-Hadar and Atara Shriki

Aim and Rationale

A successful implementation of educational change depends on teachers' professional development, and their ability to translate innovative ideas into practice. Although teaching, by its very nature, is a complex practice, most teachers work in isolation, making their own planning and decisions, and solve pedagogical problems having limited consultation with and feedback from their colleagues. The past decade has seen increasing demand to improve school mathematics, which, as a result, generated a need for teachers to join forces and share individual knowledge and experience with the community. Thus, the need to nurture mathematics teachers' communities of practice became a primary goal.

Wenger (1998), who coined the term "community of practice" (CoP), maintains that in order for a community to be recognized as a CoP, a combination of three characteristics, cultivated in parallel, is necessary: (i) The domain: A CoP is identified by a common domain of interest; (ii) The community: A CoP consists of members who are engaged in joint activities and discussions, help each other, share information, and build relationships that enable them to learn from one other; (iii) The practice: Members of a CoP are practitioners. They develop a shared repertoire of resources, such as experiences, stories, tools, and ways of addressing recurring problems, thus learn with and from each other. In general, national communities of mathematics teachers conform to Wegner's first two characteristics: they definitely share an interest in mathematics, its teaching and learning, meet in professional

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conferences, read professional journals, and share a professional jargon enabling them to learn from one another. However, the third characteristic, to a large extent, is still missing in many communities of mathematics teachers, as only few develop a shared repertoire of resources. Even those communities of mathematics teachers who do develop such resources usually count on leaders of the community to put them together for the benefit of the entire community.

In light of the above, DG4 focused on issues related to the formation of a mathematics teachers' CoP (MTCoP) and their on-going handling from both theoretical and practical points of view.

Session 1: Triggers and Needs for CoPs to Be Formed—Theory and Practice

Following a short introduction that presented views from three continents (Barbara Clarke, Australia; Jiansheng Bao, China; Diane Resek, USA) participants were asked to share experiences and promising practices, and to consider the following questions in small groups:

- What triggers and needs for CoPs to be formed, can you identify based upon your own experiences/beliefs/research?
- Who are the initiators and what are their drivers?

In as much as possible, please anchor your perceptions in a theoretical framework.

The following are some of the issues and challenges identified during the discussion:

- Arriving at shared goals for the purpose of teaching, defining problems of teaching, and agreeing on problem definitions/boundaries is not a simple process, but no doubt challenging;
- Sometimes groups are dysfunctional and there are some features to be wary of in groups: For example, blaming the student rather than taking personal responsibility;
- It can be challenging to develop a genuine CoP due to norms of privacy being evident in many schooling cultures. For example: reluctance to 'open the classroom door' to other teachers;
- Getting teachers to focus on results of their change of practice versus just doing activities should be at the heart of working with MTCoPs;
- Leadership, trust, sustainability, and quality of relationships are required for an effective community of practice. These issues raise questions regarding who should lead and run a CoP (School teachers? University professors? Researchers? Consultants?), and how the nature of leadership effects the commitment and sustainability of the group;

- The community needs to continue learning, which may require redirection. Systemic support can be effective when it establishes a culture of professional collaboration with appropriate expertise.

Session 2: Forming, Running and Sustaining an Effective MTCoP

Following a short introductory presentation (Atara Shriki and Nitsa Movshowitz-Hadar, Israel), three subgroups were formed focusing on three themes that emerged from the first session:

- Forming and running of MTCoP: Bottom up vesus top-down models;
- Collective efficacy: How do we build mutual trust, sense of belonging and ownership;
- Sustainability of MTCoP.

In relating to these questions, participants were asked to provide concrete examples from their previous experience. Since it turned out that participants observed reciprocal connections between forming and running of MTCoP and its sustainability, we present these concerns together.

The following is a brief summary of the issues discussed:

Forming, running and sustaining MTCoPs. The design of professional development programs is mostly 'top down', done by teacher educators who are not necessarily members of the MTCoP to whom the program is targeted. The designers of such programs hardly ever ask teachers for their urgent needs and spend time responding to them. This might be one of the reasons for the unsustainability of most MTCoPs. Therefore, the question is what should be done in order to nurture these CoPs as independent groups that keep developing professionally without external assistance. It is also assumed that sustainability is dependent on the initial motivation for the group and whether it was internally or externally initiated. Namely, the sustainability of a MTCoP is directly affected by the driving force of the community. There has to be a desire (whether intrinsic or external) to change, to learn, and to transform. Some further related questions are: How to bring teachers to acknowledge the need to change their practice? What would teachers consider as change? How can teachers develop their ability to reflect on their change of practice?

Trust and Efficacy. Tensions exist in a functioning MTCoP. Although these are not bad, they need to be managed productively to move the group forward. One needs conflict to make changes, but also needs to build a rapport. There can be tensions between leadership and the ownership of participants, and tensions between making meetings compulsory versus having voluntary participation. Thus, it is necessary to be aware of these possible tensions, and discuss them openly with teachers.

Conclusion

DG4 provided an opportunity for productive dialogue and sharing of experiences from a range of contexts and countries. There are many positive experiences and experiential knowledge that need to be shared. We hope to continue these conversations into the future.

Reference

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. Cambridge, UK: Cambridge University Press.

Uses of History of Mathematics in School (Pupils Aged 6–13)

Bjørn Smestad

Report

Activities concerning history of mathematics have been a part of ICMEs since ICME2 in Exeter in 1972. They are now regular features of ICMEs, organized by the HPM (International Study Group on Relations between History and Pedagogy of Mathematics). The premise of this discussion group was that research on history of mathematics in education tends to have older pupils and students in mind, and that there is a lack of both research and resources on how to include a historical perspective when teaching younger pupils.

Three key questions were pointed out in the invitation to the discussion group:

- Which ideas from HPM can be used with children (aged 6–13) in such a way that produces good results (e.g. improved student engagement, positively impacted student learning)?
- What would be criteria for finding, developing and selecting materials to be used with children (aged 6–13)?
- How does the HPM community in particular (and mathematics education community more broadly) assure that high-quality material that cover a variety of topic are produced and shared?

Question 1 was discussed in the first session and questions 2 and 3 were discussed in the second session.

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In the first session, Kathy Clark gave a short introduction of ideas from the vast literature on how and why to include history of mathematics in teaching. Thereafter, Narges Assarzadegan gave a short talk on how she has been working with her students in Iran on the topic. Kathy Clark subdivided question 1 into further sub-questions: What *are* the ideas for which HPM contributes meaningfully to the mathematical experience of pupils aged 6–13? What are the forms of *good results* we wish to happen? *How do we know* when good results occur? What are some of the *obstacles* that teachers using HPM with pupils of this age may encounter—and what are ways to address or minimize the obstacles?

Group discussions on the first question brought forward a wealth of ideas: the use of historical instruments, finding good problems from history to engage children of this age range, using concrete materials to visualize mathematics, working with words instead of symbols, exploring cross-curricular themes (for instance historical measuring units), using source material from the middle ages, studying materials from the cultures of children's parents and grandparents, and studying positive/negative numbers through history, to mention a few. More generally, it was pointed out that although "storytelling" was in our introduction described as just one of many ways of working with history of mathematics to kids, storytelling is indeed particularly important at this age level and should not be disparaged. Teachers that are able to fascinate their pupils with great (and meaningful) stories from the history of mathematics have a wonderful gift.

The good results participants wish to happen at this age level mostly has to do with the attitudes of the children: we want them to see mathematics as a fascinating cultural and human activity and make them connect to it in new ways. We will probably never be able to prove beyond doubt that using history of mathematics with children do have positive effects, as history of mathematics will always be just one of several elements a teacher uses simultaneously to engage his students. For the teacher, however, such proofs are not necessary—just seeing the pupils engaged is good enough.

Of course, there are obstacles—both in terms of resources and in teachers' opinion that history of mathematics will take time from mathematics. Moreover, as work on history of mathematics is not mandated in curricula in most countries, there is the ever-present need to justify it to colleagues who are not interested. This can also be lonely work. Some of these issues can partly be remedied by working on what we discussed in session 2, however.

In the second session, as an introduction to discussions on question 2 and 3, Bjørn Smestad and Kathy Clark gave some good examples of use of history of mathematics in teaching, including some from online sources.

The group came up with a long list of criteria for materials, noting that not every resource need to fit every criterion. The resource should:

- Include significant mathematics (and be curriculum-related)
- Include activity/task/problem/something for pupils to "do"
- Fire-up the imagination; inspire pupils to do mathematics
- Tell a story

- Have multiple representations (pictures, text, sound, video, interactivity)
- Show mathematics as a human endeavor (e.g., have a cultural aspect)
- Be doable in a "reasonable amount of time"
- Generate discussion, debate among the pupils
- Be authoritative and accurate

There are lots of materials on the internet, and at first you feel lost as it is difficult to see what is of good quality. After a while, you start being able to determine what "makes sense", but still you need to sort through a lot of bad stuff while looking for the gems. (But to get even there, you will probably need experience in using the materials—and where do you get that?) Thus, there is a need of a "clearing house" for keeping valuable materials in one location. This idea was developed further later in the discussion: what we need is a "Kantor project" (named after Moritz Kantor), mimicking the "Klein Project" in providing high-quality resources to teachers, for instance with comments both from historians of mathematics and from teachers who have used the resources with pupils (including information on how it was used and the perceived outcomes). In addition, the need for History of Mathematics courses and better resources at libraries, were mentioned.

The discussion group consisted of about 25 people from around the world, with a good mix of well-known faces in the HPM community and newcomers. This lead to good discussions where everybody took part. In that respect, we view the discussion group as a successful experience, and hope that the discussions here will inspire further work on teaching with history of mathematics for young pupils. We do hope there will be increased dissemination of ideas for this purpose in the years to come.

This report was written by Bjørn Smestad. He is happy to be contacted at bjorsme@hioa.no for further information on the works of this DG.

Postmodern Mathematics

Paul Ernest

Aim and Rationale

The goal was to elucidate the nature of postmodern mathematics and mathematics education, including the multiplicity of the subject of mathematics; and to explore and share ideas of how postmodern perspectives offer new ways of seeing mathematics, teachers, learners, mathematics education theories, research and practice. The two key themes originally used to organizer the sessions are as follows.

Theme 1 What does postmodern mathematics mean? Part of this means exploring perspectives of mathematics as a plurality, as having multiple forms, identities, locations. Even the name mathematics is plural, although modern(ist) usage overlooks this anomaly in treating it/them as a single entity.

Theme 2 What does postmodern mathematics education mean? This involves considering multiple-self perspectives of the human subject as teacher, learner or researcher.

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Background

Postmodernism rejects a single authoritative way of seeing mathematics, teachers and learners, for each can be seen and interpreted in multiple ways. Mathematics can be seen as axiomatic and logical leading to indubitable conclusions, but it can also be seen as intuitive and playful, open-ended, with surprises and humour, as evidenced in popular mathematical images and cartoons. Additionally it can be seen in its applications in science, information and communication technologies, as well as in everyday life and ethnomathematics. All of these dimensions are part of what makes up mathematics and they all co-exist successfully.

This perspective asserts that there is no such thing as mathematics. There is no unique object named by the term. There is no unique or fixed identity for the various knowledge realms, activities, practices, forms, identities or locations connoted by the term. However, mathematics (in the plural) do exist—evidently there is a multiplicity connoted by this term that varies according to the times, places, and purposes for which they are invoked. These different mathematics bear what Wittgenstein terms a family resemblance. There is no essential defining character shared by them all, but to a greater or lesser extent, they are recognizably related.

It is also important to recognize that all human subjects have multiple selves and that we all (mathematicians, teachers and learners—and we are always all three) have access to different selves: authoritative knowers, researchers, learners, appreciators and consumers of popular and other cultures, as well as having non-academic selves. Thus mathematics teachers can be seen as epistemological authorities in the classroom as well as co-explorers of unfamiliar realms both mathematical and cultural; and as ring-masters in the mathematical circus. Students can be seen as receivers of mathematical knowledge, but also as explorers, interpreters and sometimes even creators of mathematics and cultural realms that can be related to mathematics. All of these perspectives and selves are resources we can use to enhance the teaching and learning of mathematics, but many are currently overlooked or excluded.

Discussion Group aimed to provoke discussion on these and other issues, to raise and discuss these ideas and explore and generate examples relevant to classroom practices.

Papers, resources and discussions were shared on-line before the conference both via the official ICME-12 site and via Allan Tarp's MATHeCADEMY.net website. The purpose was to begin discussions before the conference and to share planned presentations in advance so that participants could prepare themselves and presentations could be kept short and much of the group time devoted to discussion and contributions from participants. As judged by informal feedback, the discussion group was very successful in this respect.

Key Questions

- Is there such a thing as postmodern mathematics?
- What is postmodern thinking in mathematics and mathematics education? What is new or different about it and what are the implications for research in mathematics education?
- Given a postmodern multiple-perspectives view of mathematics what illuminations and surprises can be found for mathematics and its teaching and learning in multidisciplinary sources including: history of mathematics, ethnomathematics, science, information and communication technology, art works, stories, cartoons, films, jokes, songs, puzzles, etc.?
- How might the new emphases and differences foregrounded by postmodern perspectives impact in the primary and secondary mathematics classrooms? What concrete examples serve to illustrate these differences?
- How can a multiple-selves view of the human subject be reflected in the mathematics classroom and in mathematics teacher education? How can a multiple-selves view of the teacher facilitate teacher education?
- To what extent are the theories and presentations offered at the conference and elsewhere in publications actually post-modern?

The discussion group opened with the showing of an animated movie discussion on Postmodern Mathematics Education between avatars for Paul Ernest and Allan Tarp. The movie is accessible at: http://www.youtube.com/watch?v=ArKY2y_ve_U and the script for the animated movie has been published in The Philosophy of Mathematics Education Journal no. 27, retrievable via http://people.exeter.ac.uk/PErnest/.

Among the key distinctions made in this dialogue were those between philosophical and cultural postmodernism. The former is concerned with multiple epistemologies and representations of knowledge, whereas the latter is more frivolous concerning the eclectic conjoining of different styles, with pastiche, bricolage and irony. The second distinction is between Anglo postmodernism (e.g., Rorty) with its focus on knowledge and uncertainty, and continental postmodernism with its emphasis on power (e.g., Foucault).

The remaining presentations in the group were as follows:

- Bill Atweh, Is the Good a Desire or an Obligation? The Possibility of Ethics for Mathematics Education
- Bal Chandra Luitel and Peter Charles Taylor, Fractals of ‘Old’ and ‘New’ Logics: A Post/modern Proposal for Transformative Mathematics Pedagogy
- Peter Collignon and Regina D. Möller, Postmodern Analysis
- Regina D. Möller and Elisabeth Mantel, Postmodern approaches in teacher education
- Allan Tarp, Postmodern Mathematics Education in Practice
- Paul Ernest, The importance of being erroneous in maths: to be wrong or not to be wrong?

Two further papers accepted for the group have been shared online as the presenters were unavoidably detained at the last minute and unable to present in person:

- Mônica Mesquita and Sal Restivo All Human Beings as Mathematical Workers: Sociology of Mathematics as a Voice in Support of the Ethnomathematics Posture and Against Essentialism
- Ilhan M. Izmirli Wittgenstein as a Social Constructivist

An invitation was given to all presenters and participants to comment or publish papers in *The Philosophy of Mathematics Education Journal*. Papers 1, 2, 7 and 8 have been published in *The Philosophy of Mathematics Education Journal* no. 27 (2013), retrievable via http://people.exeter.ac.uk/PErnest/.

The meetings were well attended and there was a lively and controversial set of presentations, questions and responses including extensive audience generated questioning and dialogue. The most frequent question directed to speakers was: "The presentation was interesting, but it could also be given under a cognitivist or constructivist framework! In what way was it postmodern?"

Several presenters answered this from their own perspectives. Paul Ernest's answer is that the social constructivism he has been developing is intended to be a postmodern philosophy of mathematics and mathematics education because:

- It rejects absolutism and accepts multiple perspectives on mathematics and teaching and learning;
- It is grounded in human practices (language games embedded in forms of life—following Wittgenstein)—and this pre-knowable grounding and reality takes precedence over any theorizing (and also installs a deep ethics as the first philosophy of mathematics education);
- It refuses priority to either A the objective or subjective forms of knowledge, B the social or individual forms of being, or C the structural or agentic forms of power—arguing that these are mutually constitutive pairs—two sides of the same coin.

All participants were delighted to be involved with a controversial contemporary topic that is genuinely best approached through discussion.

Improving Teacher Professional Development Through Lesson Study

Toshiakira Fujii and Akihiko Takahashi

Discussion

The purpose of this proposed Discussion Group is to facilitate discussion and initiate collaborative research with colleagues around the world to seek effective ways to improve teacher professional development through Lesson Study.

Each session, Session 1 (Tuesday, July 10) and Session 2 (Saturday, July 14) was began by two of four key questions addressed by the panel. Then the panelists and the participants had fruitful discussion around the key questions.

Session 1 (Tuesday, July 10).

Key Questions

- What are the key elements of Lesson Study that can help teachers gain mathematical knowledge for teaching?
- What are the key elements of Lesson Study that can help teachers develop expertise in teaching mathematics effectively?

Chair: Toshiakira Fujii (Japan); Discussant: Susie Groves (Australia); Panel: Jennifer Lewis (USA), Yoshinori Shimizu (Japan), Akihiko Takahashi (USA),Tad Watanabe (USA), Nobuki Watanabe (Japan); Reporter: Yo-An Lee (Korea).

Session 2 (Saturday, July 14).

Organizers Co-chairs: Toshiakira Fujii (Japan), Akihiko Takahashi (USA); Team Members: Susie Groves (Australia), Yo-An Lee (Korea); Liaison IPC Member: Mercy Kazima (Malawi).

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Key Questions

- How can an established effective professional development model such as Lesson Study be translated for use in different cultures?
- How can a professional development model such as Lesson Study be adapted for use in pre-service teacher education?

Chair: Akihiko Takahashi (USA); Discussant: Lim Chap Sam (Malaysia); Panel: Koichi Nakamura, (Japan), Anika Dreher (Germany), Don Gilmore (USA), Berinderjeet Kaur (Singapore), Thomas E. Ricks (USA); Reporter: Yo-An Lee (Korea).

Results

As the result of two-day discussion the following questions are raised by the discussion group for further discussion:

Question 1: Although we recognize the roles of lesson study facilitators/leaders/outside experts are important, what does expertise mean in conducting lesson study is still not entirely clear.

Outside experts can push LS members in terms of (1) content knowledge (what kinds of knowledge is needed at each particular teaching episode), (2) pedagogical knowledge (how to help students with particular contents), and (3) interactional (whether LS groups are mature enough to take and produce criticism).

In Japan, university in-service training can link to school practices where expertise come into play. In-service training, there are multiple levels. Notable is that LS study groups often have expert among themselves who can handle some aspects of lesson study. In general, it is hard to find a capable expert.

Question 2: Although lesson study is a form of professional development based on collaboration, sometimes teachers feel uncomfortable when criticized by their colleagues.

American teachers are not familiar with or comfortable with criticizing. You want to criticize lesson, not teaching. This could be an effective way of handling the pressure of giver and taker of criticism. One way to handle the pressure is to have groups work on lesson plans and choose one person to teach at the last minute. Lesson study helps teachers work together and to see what kids are doing. Lesson study should try to move their focus from "what I did wrong" to "what the issues were" "what students did (not) learn?"

Question 3: May be good idea to explore more about authentic school based lesson study in Japan.

What students should learn may be clearer in Japan. Developing school research theme might be a good starting point for group of teachers. (Theme means "what do you want to do in the class"). Although novice teachers don't know how to anticipate student responses, they learn how to anticipate student responses through school-based lesson study.

Theory and Perspective of Mathematics Learning and Teaching from the Asian Regions

Chun Chor Litwin Cheng

Report

The DG has prepared a questionnaire to collect data of teachers' practise in China, Taiwan, Hong Kong, and Korea. The results, together with the literature search in theory and practice in mathematics education was prepared into a booklet of 90 pages for discussion during the ICME-12. There were two sessions of discussion during the ICME-12 and the following is a report of work and discussion of the DG during the ICME-12.

The Chinese Framework and Theories in Mathematics Education

Two practices in China dated back to the 13 century. One is the technique of using analogy by Yang Hui (楊輝) in 1275, which work on two problems which shared the same structure and one can apply the method of the first question to solve the second problems. Another technique is the using of more than one solution to tackle the same problem by Li Zhi (李治, 1248) when he investigates cases of circles inscribed in a right angle triangle.

Organizers Co-chairs: Chun Chor Litwin, Cheng (Hong Kong), Hong Zhang (China); Team Members: Eunmi Choi (Korea), Po-Hung, Liu (Taiwan), Lianghuo Fan (UK); Liaison IPC Member: Shiqi Lee (China).

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Approaches in Understanding and Learning Mathematics in Taiwan

Fou-Lai Lin (National Taiwan Normal University) suggested that the components of being a good teacher include (1) Vary methods, (2) "Skillfully waits to be questioned" and "Hear the questions", (3) Teach students "how to learn", and (4) Know the reasons why teaching is successful or failed. And conjecturing approach is the principle of teaching mathematics.

Different Approaches in Understanding and Learning Mathematics in China

We know that the models and approaches developed in Mainland China these years include:

1. "Four Basic" model (structure approach and heuristic approach)
2. Problem solving model (structure approach)
3. Trial Teaching and Learning approach (heuristic approach),
4. GX experiment and model, (through correspondence, induction and deduction)
5. Teaching through variation approach (structure approach),
6. Demonstration, imitation and practise approach (structure approach) and
7. Dialectic approach for abstraction and internalization (structure approach).

The Characteristics of Chinese Mathematics Education and Four Basics Model

Zhang Dian-zhou (Eastern China Normal University) proposed the 5 aspects of characteristics in China Mathematics Education:

1. good lesson introduction of new topics,
2. technique of interaction among teachers and students in large classes,
3. teaching of mathematical thinking with variation method,
4. variation in teaching and exercises, and
5. fluency in practice-for-sophistication.

Zhang also proposed "Four Basics" model in mathematics teaching. The model has three dimensions and these three dimensions intertwined with each other in the process of learning.

Dimension 1: the accumulation of Basic Mathematics Knowledge (relational and procedural).

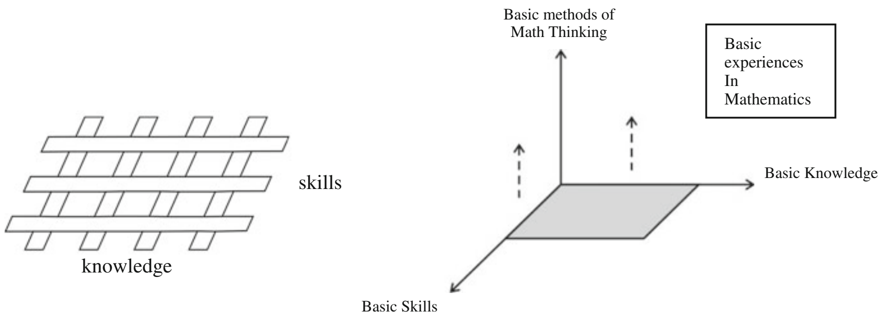

Fig. 1 Dimensional representation of the foundations of mathematics module

Dimension 2: the development of Basic skills (mathematical skill and skills to known procedure).
Dimension 3: the process of Basic Mathematical Thinking (application, formation of method of Mathematical Thinking, and develop new method).

The Basic Experiences in mathematical activities form as glue to connect the three-dimensional module (Fig. 1).

The Korean Framework and Theories in Mathematics Education

A survey conducted by Chung in Korea found that the most important thing that teachers considered in teaching and learning mathematics are (1) understanding ‘concepts’, (2) ‘principles’, and (3) ‘process’. In Korea, teacher’s role is described as “Goon Sa Boo Il Che” (君師父一體), that means King, Teacher and Father are the same one. These circumstances can be explained by culture tradition under Confucian Heritage Culture (CHC) in Korea. Though passive learning in traditional classroom is changing into more active learning in recent reformed classroom. But the zealous of learning under CHC culture still the core of the classroom in Korea.

Kyung Hwa Lee of the Seoul National University indicated that: “Good” Mathematics Teaching and “Good” Teachers usually means typical Korean math teacher have the orchestration of lessons based on the following four areas (a) Systematic instruction, (b) Coherent explanation, (c) Complete practice, and (d) Efficient imprinting.

The Japanese Framework and Theories in Mathematics Education

Masami Isoda (University of Tsukuba) indicated that there are a few traditions in the Japanese teaching of mathematics. The first one is the Japanese Problem Solving Approach for Learning by/for students. The second one is Problematic Situation explained by the Contradiction between Conceptual and Procedural Knowledge Originated from Mathematics Curriculum. And the third one is learning how to develop mathematics.

The aims of the traditions are achieved through the following teaching approaches in classroom:

1. Categorizing students' ideas from Meaning and Procedure.
2. Explaining Contradiction by Meaning/Conceptual and Procedural Framework
3. Procedurization of meaning,

Using Technology to Integrate Geometry and Algebra in the Study of Functions

Scott Steketee

Report

This group discussed the potential value of using technology-supported geometric transformations to introduce and develop function concepts. This approach (referred to here as *Geometric Functions*) and related representations can be used to help students develop intuitive understandings, avoid and overcome misconceptions, and deepen their understanding of variables and functions.

Why is this approach not more widely used? What are the benefits and obstacles? How can this approach be encouraged and facilitated? The session agendas and notes are available on the DG9 wiki, which also contains links to resources (including movies and existing student activities): http://wiki.geometricfunctions.com/index.php/ICME_12_Discussion_Group_9.

Our DG addressed a number of questions. Why are geometric transformations not more widely integrated into the study of function? What are the benefits, and what are the obstacles? What experiences have discussants had in promoting such an approach? How can we best encourage and facilitate such a change in students' experience of function?

Each session began with a whole-group introduction, broke into small-group discussions addressing particular areas, and concluded with a whole-group summary. The bullet points below are based on the reports from the small groups.

Organizers Co-chairs: Scott Steketee (USA), Cheah Ui Hock (Malaysia); Team Members: Ang Keng Cheng (Singapore), Aleksandra Cizmesija (Croatia), Ali Lelice (Turkey); Liaison IPC Member: Hee-chan Lew (Korea).

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What We Know

- Students need to experience a variety of functions to form a robust conception.
- Though other examples may be given, the conventional approach quickly settles down to $\mathcal{R} \rightarrow \mathcal{R}$ functions defined by equations. But many important functions do not merely map real numbers to real numbers.
- The Geometric Functions approach may contribute to a richer concept of function.
- Transforming points versus transforming shapes is an issue; we need to be clear about this distinction.
- We have anecdotal evidence that teachers don't connect algebra and geometry. In most (all?) countries geometric transformations are taught independently of functions. (One participant observed that five mathematics educators from five different countries agreed that geometric transformations are an independent topic from functions in their mathematics curriculum.)
- Students have difficulties with many function-related concepts (variable, function, domain, range, relative rate of change, composition, and inverses).

Research Questions

- How does the Geometric Functions approach differ from current practice? What might it add? Might important elements be lost?
- How can Geometric Functions expand students' understanding of function? How might students' conceptions of variable, function, domain, etc. be strengthened?
- How might this approach help students' concept of function move along the action-process-object (APOS) sequence?
- How does current thinking on embodied cognition support the Geometric Functions approach? How do students experience Geometric Functions as embodied?
- It's important to present functions in a way that does not introduce misconceptions. What impact does this approach have on common misconceptions about function?
- How can physical activities supplement technology-based activities?

Implementation Issues

- When the teacher starts using Geometric Functions activities, how quickly should she go to technology? Can she use some real-world activities before using virtual activities? (See the Function Dance activity, www.geometricfunctions.org/function_dances.html, for one example of this real to virtual transition.)

- Transformations are sometimes taught in the elementary curriculum. How does this affect the use of Geometric Functions in teaching transformations?
- What would teachers need to know about math that may be unfamiliar to them?
- Professional development and support for teachers should be just-in-time.
- Team teaching may be very useful when teaching unfamiliar topics.
- How could teachers get comfortable with the technology? Ideally the technology should be transparent, so that the focus is on the math. Students' experiences should be mathematical rather than magical.
- The goal of experiences with Geometric Functions is to facilitate conversations about what functions really are, and about the connections between Geometric Functions and functions that are normally studied ($\mathcal{R} \rightarrow \mathcal{R}$ functions expressed as equations).
- Assessment is a problem, since students often do not have the opportunity to use technology during tests. How can this situation be corrected?

Conclusion

Discussion Group 9 concluded that teaching geometric transformations as functions has significant potential for improving students' understanding of function concepts and for avoiding common misconceptions, and that dynamic mathematics technology is a promising way for students to experience geometric transformation as a conceptual metaphor on which to ground their conception of function.

DG9 further concluded that more research should be done to establish the benefits of the Geometric Functions approach and to determine effective ways to implement it. Geometric Functions challenge both wide-spread curricular assumptions (that functions belong to algebra, not geometry) and teachers' typical mathematical background and knowledge, and require careful thought and preparation for effective implementation.

Given the poor student understanding of function concepts that results from current practices, and the proven value of incorporating students' sensory-motor systems in the learning process, we encourage mathematics educators and education researchers to take seriously these twin arguments for studying and implementing the Geometric Functions approach.

New Challenges in Developing Dynamic Software for Teaching Mathematics

Zsolt Lavicza and Balazs Koren

Report

The principal aims of the discussion group were to discuss the development of a variety of mathematical software applications developed during the past decades. Among the most often utilised software types in education are Computer Algebra Systems (e.g. Derive, Mathematica, Maple, Maxima), Dynamic Geometry Systems (e.g. Cabri Geometry, Geometer's Sketchpad, Cinderella, GeoNEXT, GeoGebra), Spreadsheet and Statistics Software (e.g. Excel, SPSS, Fathom, R). Most of this software has been designed by keeping in sight primarily their usability for research purposes while others were predominantly aimed for their use in teaching. In the recent years we could observe, among others, three important trends in the development of these software tools: (1) Designers of research oriented software products started to involve features and support for educational purposes; at the same time teaching oriented software have been becoming increasingly more powerful so their use in some research is increasing; (2) The distinction between different types of software has begun to blur as many products integrate features from other types of software; (3) The computer platforms are diversifying; with the appearance of smart phones, tablets, and Interactive Whiteboards (IWB) in recent years, as well as online services such as Wolfram Alpha, challenging the design and development of mathematics software.

Organizers Co-chairs: Zsolt Lavicza (UK), Markus Hohenwarter (Austria); Andrian Oldknow (UK), Tolga Kabaca (Turkey), Kyeong Choi (Korea); Liasion IPC Member: Frederick Leung (Hong Kong).

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The Discussion Group aimed to elaborate some of the outlined issues with a mix of short presentations, questions, and reflections from the audience. DG-10 was divided into two sessions both of them were attended by approximately 60–70 participants.

Session I

In the First session Zsolt Lavicza and Balazs Koren outlined the aims of the session and set a schedule for presentations and discussions. The first presentation was given by Balazs Koren, Hungary introducing the different software available and used in mathematics teaching and research. Balazs wanted to have this presentation to challenge thinking of the audience and cluster software into tentative groups that was supposed to be dissected or rearranged into new categories and groups during the sessions. In the past decades, three kinds of tools were mainly used in schools: desktop software, specialised handheld calculators and more recently tablet and mobile phone apps, as well as web-based applications assisting mathematics teaching and learning. However, this grouping is getting outdated and the community should develop new clusters and characterizations to advance software applications and related theories. It seems that the borders between categories are getting more and more overlapped and we are converging towards more complex and adaptive systems in the near future.

The second presentation was given by Tolga Kabaca from Turkey. Dr Kabaca described his experiences with the mathematics community using a variety of software in Turkey. He emphasized that it is necessary to allow teachers to develop their own teaching applications, but at the same time there should be a structured system as well as training that allow them to bring technology into the classroom. In addition, Dr. Kabaca mentioned the importance of getting feedback for both software and material developers directly from the software tools.

Peter Boon from the Freudenthal Institute, Netherlands described the needs for extended environments around mathematical software. It is getting common to embed mathematical software into Learning Management Systems (LMS) or to a so called Digital Mathematical Environment (DME), which are enabling teachers and students manage learning within and outside classrooms, offer assignments of problems and collect data from their solutions. Developing such LMS and DME systems is difficult and may take years of improvements and modifications as well as needs an interaction with the used mathematics software environments.

Chris Sangwin, UK outlined his experiences in developing assessment tools for mathematics. STACK is an assessment environment developed by Dr. Sangwin including a number of own solutions, but at the same time drawing on the resources of other mathematical software. Assessment is one of the most controversial and difficult issues in today's educational debate so that creating an environment is challenging and risky. However, such environments are necessary for the educational system and ample thoughts have been invested into such developments.

The discussions during the sessions reflected the topics of the presentations. At the end there were suggestions for questions to be further investigated through debates and research:

- How do we or should we classify software applications?
- What does Dynamic means in a software environments?
- Do the use technology to enable mathematics learning, if yes, how?
- Do software need to offer and restructure social dynamics in classrooms and on the web?
- We need to emphasize the pedagogical uses of the software and develop them accordingly to enhance further opportunities for learning.

Session II

We consider this session as a historical event as all widely used software creators or leaders of their teams were represented in the room. Jean-Marie Laborde (Cabri), Ulli Kortenkamp (Cindarella), Zsolt Lavicza and Balazs Koren (GeoGebra), and Nicholas Jackiw (Geometer's Sketchpad).

The session also started with the presentation of Ulli Kortenkamp, Germany, who highlighted the difficulties and processes in the development of mathematical software. Dr Kortenkamp emphasized that there are a number of issues could arise when mathematical theories needed to be implemented in a computer environment. For example, matching Euclidian and Hyperbolic geometry into a single software could be challenging thus the community of mathematicians and software developers need to have forums to discuss possibilities for implementation.

Tatsuyoshi Hamada, Japan talked about a wide range of software developed in Japan and the difficulties of their spread across groups and universities. Professor Hamada created a downloadable live Linux application called Math Libre, which is a collection of freely available software tools for mathematics teaching and research. Trough this collection the authors aims that schools, teachers, and students can choose the best applications fitting their needs in education. The project contributed to the involvement of technologies in the curricula in many schools around the world.

Jean-Marie Laborde, France stressed the importance of quality and the mathematical correctness of software development. Professor Laborde described that software development is a costly process and needs to be done in a complex way to ensure the correct mathematical background of the underlining processes within the software. Thus, it is important that while choosing a software must be made based on quality and rather than economy.

Finally Zsolt Lavicza, UK outlined the development of an open source project and the importance of a community surrounding the software. GeoGebra has become a successful mathematical tool, because teachers and students found them on the Internet and started to contribute to both software and material development.

Due to the large user base and the responsiveness of developers to the requests of the users the software was developing quickly and attempt to correct the problems arising during its use.

The debate after the presentations initiated further ideas and questions:

- How we can deal with infrastructure issues in schools, in particular in lesser-developed regions in the world?
- How can we encourage education to use current rather than outdated technologies?
- Do we need to develop specialised or general software? Do we need to connect development with other fields such as with video game?
- How can we learn from the success of long existing and sustained software packages such as R?
- How can we best support LMS and DMS with mathematical software development?
- How can we deal with the complexity of mathematical software development?
- How can we set some guidelines for assessment with computers?
- Possibly we need flexible and customisable tools in the near future
- We need to produce more books and learning materials with different tools

DG-10 offered an inspiring environment to discuss issues for both developers and users of products. The presentations and reflections were fruitful, but because software use in mathematics education is still around the start line with the exception of some larger projects the session ended with more questions than answers. However, the beginning of such discussion is valuable and offered food for thoughts for participants and we believe already impacted the development of software.

Mathematics Teacher Retention

Axelle Faughn and Barbara Pence

The main question addressed in this Discussion Group is whether professional development can have a positive effect on the retention of mathematics teacher, and if so what is the nature of professional development that leads to teacher retention and how (what are the mechanisms by which) such professional development supports teacher retention.

Background information, literature on Mathematics Teacher Retention, and important guiding questions were listed on the Discussion Group Website at https://sites.google.com/a/cmpso.org/icme2012/ for participants to actively engage in discussions guided through individual contributions by researchers from Israel, New Zealand, Norway, South Africa, India, and the United States. The major theme of *Supporting mathematics teachers*: *Transition into the workplace and professional development* underlined all discussions. Sub-themes were classified under seven major strands: (1) Mathematics Content and Pedagogy, including Technology; (2) Models of Support; (3) Communities of Practice, including online and lesson study; (4) Teacher Leadership; (5) Research; (6) Policy; (7) Mathematics Teacher Identity. Discussion group organizers were also interested in the magnitude of teacher retention issues in various countries, as well as their local and global impact on mathematics education.

Initial discussions included concerns about what happens at the pre-service level. Choosing teaching training is often a last choice for students, which makes producing qualified teachers a challenge. Many participants indicated trends towards an increasingly diverse range of people choosing teaching as a career choice.

Organizers Co-chairs: Axelle Faughn (USA); Barbara Pence (USA); Team Members: Glenda Anthony (NZ), Mellony Graven (South Africa), Claire V. Berg (Norway); Liasion IPC Member: Oh Nam Kwon (Korea).

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In countries such as NZ mathematics teachers recruits included an increasing number of career switchers who did not necessary regard teaching as a life-long career. In other countries such as SA, the critical shortages of mathematics recruits and subsequent high number of out-of-field teachers contributed to high attrition rates. In the United States, 50 % of teachers leave within their first 5 years of teaching, and the mode for teaching expectancy of beginning mathematics teachers is 1 year. Problems contributing to teacher attrition that were highlighted included the inappropriate or challenging placement of new teachers, and the rise in number of unqualified teachers teaching mathematics.

Karsenty described a model of support for new teachers of at-risk students in Israel (SHLAV) based on weekly on-site meetings with a mentor. The support in this model was personalized and included discussions on material/content, teaching strategies, and affective issues of students at risk. Through this support model, mentees felt empowered and gained confidence, but the questions of sustainability of such a model was raised in light of cost-effectiveness and what happens once funds for such programs run out. The idea of fostering deep changes in the school through material sharing and networking to form a community that shares information and resources was put forward as an answer to sustainable change.

Graven noted that support in the form of one day training sessions which negate teacher experiences and communicate a 'fix-it' type of approach based on giving teachers new ways of teaching undermines teacher confidence. In South Africa, 55 % of teachers say they would leave if they could. Emphasizing life-long learning as a continuous professional process and redefining one's identity from hiding shame of not knowing to acknowledging one-selves as life-long learners while knowing where to ask should be embedded in professional development interventions. Other components of successful professional development include providing teacher autonomy, fostering a sense of belonging, empowering through increased confidence, which all bring sustainable changes in practice. The question of sustainability through teacher leadership and promotion was raised, as well as the development of a strong professional identity through leadership. Questions linking teacher retention and identity were also considered.

Common themes across different contexts concerned a lack of qualified mathematics teachers, difficulty with recruitment of quality teachers, and teacher dissatisfaction with the profession. East Asian, where the status of teachers in the society is still high and salaries competitive, were noticeably absent from this discussion. In contrast to interventions that offer 'add on' support, East Asian countries offer systemic opportunities and expectations for teachers to grow and to advance in their professions. China has a well-established and coherent professional development system through the use of teaching researchers who serve as collaborators, facilitators and mentors, and get involved at different levels of teaching research activities by developing research lessons. This is the basis of an intricate ranking and promotion system that includes lesson competitions and ensures that theory is implemented and tested in the classroom. Teachers in East Asia are actively involved at every level of the teaching profession, from training of pre-service teachers, to development of curricular material and delivery of professional development. Madhana Rao described a two-tiered educational system in the district of Warangal of Andhra Pradesh State of India where private institutions are linked to teacher attrition while government schools retain 100 % teachers. Stability in this case is also attributed to a state-led system that provides promotional opportunities and a regular salary to teachers, with professional development interventions.

Participants from other countries reported that teachers who want to become leaders and see their influence increase do not always have the institutional support to do so. When Australian teachers reach the top of the pay scale after 10 years of teaching they can only be promoted as administrators. Anthony examined New Zealand's induction system with time allotted for mentees and mentors to meet regarding concerns pertaining to the teaching profession such as learning about school context, completion of accreditation requirements, and the necessity and tools to become a professional inquirer. The mandated induction program with extra time for planning and support for Year 1 and 2 beginning teachers is a significant factor in low attrition rates. However, the development of a wider community base of support in teacher education programs is necessary to help inform and equip pre-service teachers to proactively counter dissatisfaction and disappointment with the profession and the nature of school culture.

Berg's model is a community of inquiry that aims at replicating material and teaching processes introduced during workshops into the classroom, such as asking good questions during an inquiry activity. Two participating groups qualified as "New Comers" and "Old Timers" showed different levels of appreciation for the professional development, which brings up the question of what is a minimum length for a professional development project to induce sustained change in classroom practices. In this project participants found a community and support outside of their school, a situation echoed by Pence in the California-based Supporting Teachers to Increase Retention project. Pence provided a glimpse on 10 professional development site models aimed at supporting teachers and increase retention. Findings from the project included the emergence of professional communities across all sites and the importance of leadership in keeping teachers motivated and involved. The professional development model for each site was targeted over multiple years, content specific, challenging, and went beyond mathematics content and pedagogy to focus on establishing teaching as a "noble" profession requiring work and preparation, growth that is complex, on-going, and supports the realization that there is a great deal to learn.

Contributions

Contributions to this Discussion group can be found at https://sites.google.com/a/cmpso.org/icme2012/

Mathematics Teacher Educators' Knowledge for Teaching

Kim Beswick and Olive Chapman

Report

The aims of DG12 were to:

- Facilitate discussion of key issues related to the knowledge required by mathematics teacher educators (MTEs).
- Identify different emergent strands in research that can be related to this area.
- Summarise research and research/theoretical perspectives related to knowledge for mathematics teacher education.
- Identify research directions and potential collaborations that will move the field forward.

Four broad areas were suggested to frame discussions. In summary these were:

- To what extent are the various knowledge types for mathematics teachers described by Shulman (1987), Ball et al. (2008) and others applicable/transferable to MTEs? How does the knowledge needed by MTEs differ from that required by mathematics teachers? Is it a kind of meta-knowledge or something as distinct from the knowledge for teaching mathematics as knowledge for teaching science is?

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- Who researches MTEs' knowledge? What are the dilemmas and opportunities associated with researching ourselves? What evidence is there of the knowledge required by MTEs? What measures/criteria are there for successful mathematics teacher education and how are they connected to MTEs' knowledge? What methodologies might be effective in building such an evidence base?
- How is knowledge for mathematics teacher education acquired? How is the transition from mathematics teacher to mathematics teacher educator made and what is gained or lost in the transition? To what extent and in what ways is knowledge for teaching mathematics necessary for MTEs? What theories of learning are useful? What models are/should be used?
- Why might it be important to articulate knowledge for MTEs? What contribution can understanding it make to our work and to mathematics education more broadly? Who wants to know about this knowledge and why?

The first session was attended by more than 45 participants from at least 18 different countries. There was a broad range of experience and expertise in relation to the topic with many participants acknowledging that they had not given MTEs' knowledge serious consideration prior to attending the discussion group. Discussion in session 1 focussed on areas 1, 3 and 4 and ended with participants writing down one or more questions that they had about MTEs' knowledge. These were grouped into five themes, summarised below, that formed the basis of discussion in the second session.

Theme 1: The Nature of the Knowledge Needed by MTEs

What knowledge of mathematics is needed by MTEs? What differences are there between teaching at university level and school? What is the distinction between Mathematics Knowledge for Teaching (MKT) and mathematics knowledge for MTEs? Is MKT the 'curriculum' that MTEs teach? How do MTEs' conceptions of teaching and learning develop? How can we research these? How do these conceptions translate into their teaching? Is there a connection to student learning? What aspects of MTEs' knowledge are important? What knowledge do MTEs for in-service teachers need? How is it different from knowledge needed for pre-service teacher education? How can MTEs for in-service MTEs be educated?

Theme 2: Different Types of Mathematics Teacher Educators and Implications for the Knowledge Needed

Who are the MTEs? How does local context impact on MTEs? What kinds of courses would cater for the differences between MTEs (e.g., mathematicians, former mathematics teachers, mathematics education researchers)? Is the same knowledge needed by all MTEs? Is it possible for one person to have/develop all the knowledge necessary? Is it helpful to consider mathematics education as team work?

Theme 3: Research Methodologies/Approaches

In what ways might teacher collaborative inquiry among MTEs provide a methodological framework for research in this area?

Theme 4: Acquisition of Knowledge for Mathematics Teacher Education

How can programs be developed specifically for MTEs of mathematics teachers at different schooling levels? How can professional development for existing MTEs be provided? What is the importance of role models in the development of MTEs? What knowledge is acquired through apprenticeship models? What are the relationships between MTEs' background and the way they acquire knowledge? How can MTEs develop the capacity for inquiry into their own practice? What is the role of collaboration and mentoring?

Theme 5: The Importance of Research in This Area

How can we ensure that the appropriate resources are allocated towards this work?

Future Directions

Many participants indicated their interest in progressing the work through a book or journal publication. There was also interest in international comparative research on MTE backgrounds and the relationship of this to MTE practice and outcomes.

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The Role of Mathematics Education in Helping to Produce a Data Literate Society

William Finzer

Report

There were 30 participants from 17 countries. The challenge presented to the group was as follows: "The data revolution is everywhere except the classroom. In general, students finish their schooling seriously under-prepared to participate in the emerging data-driven society. This represents an enormous loss of scientific discovery, solutions to social problems, economic advancement, …."

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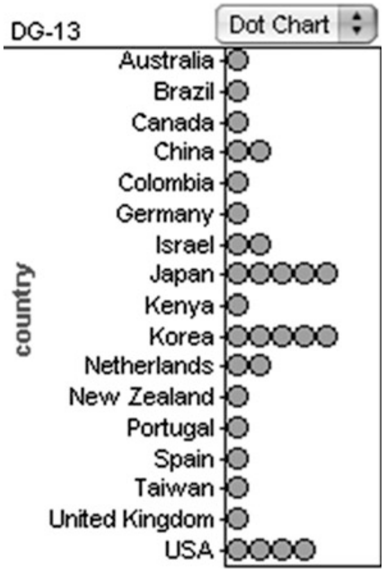
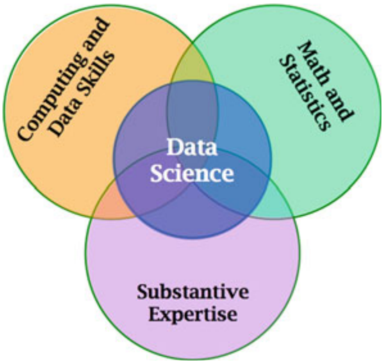


Highlights from Participants' Introductory Comments

- Biggest challenge in Kenya is infrastructure, so there is lack of computers and other technology.
- There is a new program in Portugal to try to emphasize statistical ideas in the early years, however the teachers do not have the knowledge to be able to implement this currently.
- One specific topic in Brazil in general education is data handling, but teachers do not know what it means. There is a new program to introduce data handling to master's students who will be teachers.
- One of the difficulties in Columbia is the different goals among different groups in the country. For example, government versus teacher goals. And the curriculum does not match what is happening in the schools.
- Students are underprepared not just technically, but ideologically. The problem with thinking that if data is on the web, then it's true. The other problem is if you believe in something strongly enough, then you don't need any data.
- Students in Korea explain interpretations very superficially. They are good at computing, but it's difficult for them to do data analysis.
- There is statistical literacy in the curriculum in New Zealand. The students start to talk about their own data, then critique other's data. In high school, it moves to looking at media reports and so on. The challenge is professional development for the teachers because they are not used to having these discussions in the classrooms.

- In China, probability and statistics have been content areas since 1978, but they haven't become the focus area until the new curriculum standards in 2001 and 2003. Teachers will now pay much attention to probability and students might get high scores in examination, but still not be able to solve authentic problems.



Highlights from Discussion of "Regarding the Problem, to What Extent Is What We Are Talking About Something that Goes Beyond 'the Need for More and Better Statistics Education?'"

- The expansion of statistic education should be manipulation of data, raising critical issues about data, and critiquing data.
- We are looking for boundary crossing between different perspectives including statistical education, heuristics (e.g. representativeness, availability, anchoring, etc.), and responsible critical social perspectives.
- It is not enough to teach more and better statistics in terms of computations. We think students need to be able to communicate, tell a story, and explain their interpretation. There needs to be something else in terms of communication.
- What is taught currently in schools in statistics is too narrow. It is just calculations that even when taught well, is still too narrow. (For example, archival of data is something that statisticians do not think about it, but in economics it is important. Also data cleaning. 90 % of the effort is spent in the data cleaning area, with very little time spent in data analysis.)

Highlights from Discussion of "What Strengths Do Mathematics and Statistics Educators Bring to Bear on the Problem? Conversely, with What Aspects of the Problem Are Mathematics and Statistics Educators Unlikely to Bring Expertise?"

- ability to think quantitatively, defining algebraic expressions, handicap—fear of there not being a right answer
- statistics is one part of mathematics in china, the mathematics teacher is the statistics teacher, statisticians are teaching statistics only, don't consider statistics at school level, good statistical knowledge but don't know how to teach at school level, guide the curriculum design, how to teach have no idea (lecturers)
- know the concepts of scientific enquiry (PPDAC), statisticians know the more concrete of the cycle, huge data and how to handle huge data, important and big role for statistics for big data age
- mathematics doesn't use context, statistics brings context, conflict. Is this content mathematics or social studies? PISA had a strong effect on what to teach in mathematics
- basic concepts in statistics no longer have the same status, huge data sets, different types of data, introduce students to different types of data, how to talk about signal of data, how to present the data, need to work at global level, how do we sample our global data, the basic ideas need to be revisited—data analysis, not statistics as this is an old word
- students learn statistics in the mathematics classroom so always want to know the right answer, "so what is the answer", this is a problem, look at decision making in a comparison situation
- society is not necessarily data driven but there is a flood of data.

Highlights from Discussion of "What Should Be the Role of Mathematics Education in Helping to Produce a Data Literate Society?"

- **classrooms**—Active engagement of students in the investigative process (PPDAC), importance of technology in teaching and learning statistics—creating the displays etc. so can focus on interpretation, also to build conceptual development, exploring outliers as an example.
- **concepts**—Variation and prediction, sample as a starting point, where does data come from, what is data, very important to have simple counting procedures and also by deciding what to measure, defining difficult variables and this is crucial for validity, ordinal and other scales.

- **policy**—What are the key ideas that we need to keep and/or build that we will always need; what do we have to change what we are doing in our classrooms? Ethical issues, big data not always available to all citizens, open data sets being available to all; statistics across the curriculum; look for New Zealand example and follow (blindly) cognizant of the local situation—collaboration amongst statisticians, educators and teachers in leading the reform in the curriculum; may need to collaborate with other disciplines, collaborate with internet experts, data people, computer experts and create discipline of data science.

Mathematical Modeling in Connecting Concepts to Real World Application

Zhonghe Wu and Lijun Ye

Aim and Rationale

In recent years, achieving mathematics proficiency has received notable attention [RAND 2003; National Research Council (NRC) 2001] What useful, appropriate, practical, and effective strategies can be developed and used to enhance student proficiency in mathematics is still a puzzle to mathematics educators. This urgent need becomes a challenging task for mathematics educators seeking research-based strategies to support classroom teachers to enhance their teaching leading to student proficiency.

The Mathematical Modeling is a research-based teaching model (Lesh and Zawojewski 2007; Niss et al. 1991) that builds conceptual understanding and problem solving skills. The mathematical modeling also reflects the core components of proficiency defined by research studies (Hill and Ball 2004; NRC 2001; RAND 2003)—conceptual understanding, computational skills, problem solving, mathematical reasoning, and mathematical disposition.

Organizers Co-chairs: Zhonghe Wu (USA), Lijun Ye (China); Team Members: Shuhua An (USA), Zhongxiong Fan (China), Ling Wang (China); Liaison IPC Member: Morten Blomhoej (Denmark).

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Key Questions

The following five broad areas frame the territory of the discussion.

- What is Mathematics Modeling? Why Mathematics Medeling?
- What is the relationship between mathematical modeling and mathematical proficiency? What does role of Mathematics Modeling play in teaching and learning mathematics for K-12 students?
- How is mathematical modeling used in primary school?
- How is mathematical modeling used in secondary school?
- What are the challenges and issues of mathematical modeling in teacher professional development?

Mathematics and Culture in Micronesia: An Exploration of the Mathematical Aspects of Indigenous Practices

A.J. (Sandy) Dawson

Aims

The aims of the discussion group are to (1) discuss the findings of the three indigenous authors (Mamangon, Moses and Velasquez) investigations thus far relative to mathematics and culture in Micronesia, (2) explore the challenges and successess achieved in using elders to uncover and validate indigenous knowledge and practices, (3) explore the pedagogical issues of how to translate the findings into materials and approaches suitable for elementary school children, and (4) consider implications for future research in other indigenous cultures. Indigenous mathematics, enthnomathematics, cultural-based mathematics

Key Questions

The discussion will allow an exchange of ideas, successes, and challenges in supporting indigenous activities, capturing the mathematics contained therein, and preserving those activities and the mathematics for future generations. The key questions addressed in the discussion group are:

- What mathematics has been uncovered by examining indigenous practices and activities of Micronesian peoples?
- How can this mathematics and the associated practices be used to teach mathematics to indigenous children?

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- What are the challenges to conducting such research particularly working with elders and dealing with what, at times, is seen as 'protected' knowledge, and developing approaches to the teaching of mathematics with the focused populations?
- What lessons can be learned from this work with indigenous populations?

Structure of Sessions

Session One: 1.5 h

- 10 min: introductions and setting the scene
- 50 min: discussion of findings of the mathematics found in Micronesian cultural practices
- 25 min: discussion of pedagogical strategies developed for use with indigenous children
- 5 min: summary of session and closing remarks.

Session Two: 1.5 h

- 5 min: overview of previous day's discussion
- 15 min: further discussion of the pedagogical strategies
- 40 min: small group discussions regarding the challenges of conducting the research and devising implementation strategies
- 20 min: large group discussion and consolidation of the major issues, challenge and observations made during the discussion group—lessons learnt, honouring indigenous knowledge and ways of knowing
- 10 min: closing summary and suggestions for furthering the conversation begun here.

Project MACIMISE

Project MACIMISE (Mathematics and Culture in Micronesia: Integrating Societal Experiences) is supported by a National Science Foundation grant (0918309). This material in this paper is based on work supported by that grant. The content does not necessarily reflect the views of the NSF or any other agency of the US government. The Project is a collaborative effort between Pacific Resources for Education and Learning (PREL) and the University of Hawaii-Mānoa (UHM) with PREL as the lead organization.

Can Art Save Mathematics?

Dirk Huylebrouck

Aim and Rationale

"Can art save the world?" is a well-known catchphrase in art circles. As most participants to the ICME are mathematicians, the title of this DG was reformulated more modestly as: "Can art save mathematics?" Indeed, some call mathematics a supreme art form as it enjoys total freedom, unrestricted by material limitations. An art form with the "collateral advantage" of having many real life applications, sure. However, if it can be considered as art, why don't art and mathematics more often collaborate, for their mutual benefit?

In the past, carpenters or painters sometimes helped mathematicians in the construction of mathematical models which sometimes had artistic ambitions (intarsia, for instance), but today's computers allow mathematicians to express themselves in total freedom, without the help of intermediate persons or tools. However, mathematicians aren't necessarily artists and so this technological improvement does not necessarily guarantee better art. Also, while in the past the lack of mathematical knowledge by artists was a burden for the development of mathematical art, today this should no longer be the case: computer developments make mathematics more accessible to artists, despite their usual aversion for the pure sciences.

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Yet how do we bridge the gap between mathematics and art so that mathematical art becomes an equally well “established” art field as, for instance, biological art or kinetic art? It would be beneficial for society because it would help to unite the “two cultures” of J.P. Snow, and because today’s society needs designers interested in scientific developments.

Key Questions

- How much art should “artistic mathematicians” know in order to produce more than embellished mathematical results, so that their artistic mathematics are not mere “kitschy attempts”?
- How much mathematics should “mathematical artists” grasp in order to get really involved in the pure sciences, so that their mathematical art is not mere “baby math”?
- Or else, instead of turning mathematicians into ‘artists’ and artists into ‘mathematicians’, wouldn’t it be better both sides simply cooperate—and if so, what should be the framework for such a collaboration?
- How can mathematics departments take mathematical art achievements into account in their output evaluation? For example, are mathematical art journals included in the journal rankings?
- How should the refereeing process work in this case where “peers” are by definition hard to find since the creative process implies every mathematical artwork should be unique? In the art world, refereeing is seldom done by peers.
- What is the difference between a scientific paper on mathematical art and a poetic artistic portrayal? The objectives of a purely mathematical paper are well known, but what about those of a paper on mathematical art?
- As for its implications in teaching mathematical art to art students, what are their specific needs and aspirations? The scientific “aha-Erlebnis” and “problem solving” are not sufficient, so how do we stimulate the creative mathematical approach?
- Is there a need for teaching mathematical art? The implications could be diverting students’ attention from classical mathematics material (leading to “easy credit” courses). However, it could also raise awareness of the usefulness and the beauty of mathematics, inspiring students to continue taking math courses

Teaching of Problem Solving in School Mathematics Classrooms

Yew Hoong Leong and Rungfa Janjaruporn

Aim and Rationale

The 1980s saw a world-wide push for problem solving to be the central focus of the school mathematics curriculum since the publication of Polya's book about solving mathematics problems in 1954. However, attempts to teach problem solving typically emphasised the learning of heuristics and not the kind of mathematical thinking used by mathematicians. There appears to be a lack of success of any attempt to teach problem solving within school curriculum. Problem solving strategies learned at lower levels tended to be ignored instead of being applied in their mathematical engagements at the higher levels, possibly because of the routine nature of the high-stake national examinations. The era of mathematical problem solving, its research and teaching and learning in schools ended, ambivalent on research findings and imprecise on recommendations for its teaching in schools. Based on the teaching and research experience of the organising team, we think that problem solving should still be the direction for teaching mathematics in schools. As such, this discussion group is proposed to identify the practices in teaching problem solving in school mathematics classrooms across different parts of the world, and how these practices are linked to the success.

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What Is the Place of Problem Solving in the School Mathematics Curriculum?

As a result of the publication of Polya's book about solving mathematics problems in 1954, National Council of Teachers of Mathematics and the worldwide educational reforms in school mathematics have recommended the study of problem solving at all levels of the mathematics curriculum. The reform documents indicate that problem solving should be the central focus of these mathematics curricula. It should not be an isolated part of mathematics instruction, it should be an integrated part of mathematics learning. Moreover, there is an expectation in these reform documents that even young students will improve their mathematical knowledge and procedures with understanding of problem solving. As a result of these reforms, problem solving has occupied a major focus in worldwide school mathematics curricula. Problem solving had been identified as both a major goal of instruction and a principal activity in mathematics teaching and learning.

How Much Curriculum Time Is Spent in Problem Solving in Comparison to the Other Components of Mathematics Curriculum?

By learning problem solving in mathematics, students acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that will serve them well outside the mathematics classroom. In addition, problem solving is one of the basic skills that students must take along with them throughout their lives and use long after they have left school. To improve young student's mathematical knowledge and procedures, it is essential that the teacher should know how to teach problem solving and how to approach problem-solving instruction. They should also identify problem solving as a major goal of instruction and a principal activity in mathematics teaching and learning.

What Is the General Perception on the Importance of Mathematical Problem Solving Among the School Teachers?

Although worldwide educational reforms in school mathematics have recommended the study of problem solving at all levels of the mathematics curriculum, problem solving remains an unfamiliar idea for most school mathematics teachers. Many teachers lack the knowledge and confidence to teach mathematical problem solving. They also do not recognize the importance of mathematical problem solving in their classrooms. As such, the question, "How should we go about helping teachers increase their knowledge and confidence to teach problem solving?" has often been asked, and it is timely to ask whether examine teacher professional development programs that are targeted in this area. Apart from workshops on problem solving for teacher—which is the traditional mode of teacher development—there has been a shift towards models that develop teachers as owners and collaborators of innovations in the teaching of problem solving. One example of the latter is the mathematics problem solving for everyone (MProSE) project based in Singapore. Since three members of the organizing committee are investigators of this project, there was substantial capacity to present the details of MProSE in the DG meetings during the conference, which attracted much interest and lively discussion.

How Is Mathematical Problem Solving Assessed?

In order to have experience on problem solving, students should be expected to solve various types of problems in their own way on a regular basis and over a prolonged period of time. Non-routine problems and open-ended problems that provide students with a wide range of possibilities for choosing and making decisions should be used. Students should be asked to show their solutions in writing. A student's written work on a problem can be used to help evaluate progress in problem solving. A rubric scoring scale, both holistic scoring and analytic scoring, are methods for evaluating a student's written work on a problem.

In MProSE, the team introduced the "mathematics practical" into problem solving lessons using a "Practical" Worksheet. The students were encouraged to treat the problem-solving class as a mathematics "practical" lesson. The worksheet contains sections explicitly guiding the students to use Pólya's stages and problem solving heuristics to solve a mathematics problem. The scoring rubric focuses on the problem solving processes highlighted in the Practical Worksheet. The rubric allows the students to score as high as 70 % of the total 20 marks for a correct solution. However, this falls short of obtaining a distinction (75 %) for the problem. The rest would come from the marks in Checking and Extending. The intention is to push students to check and extend the problem (Stage 4 of Pólya's stages), an area of instruction in problem solving that has not been largely successful so far.

Summary

While the participants of the DG are from a wide range of jurisdictions—and hence different social-educational contexts, there is general agreement that mathematics problems solving of the type advocated originally by Polya and subsequently developed by other researchers remain important but elusive. There are many challenges, not least of which is teacher development. It is heartening to note—from the sharing of participants—that innovative projects were conducted, such as MProSE, to address this challenge.

Erratum to: Congratulatory Remarks: Minister of Education and Science, and Technology

Ju Ho Lee

Erratum to: "Congratulatory Remarks: Minister of Education and Science, and Technology" in: S.J. Cho, *The Proceedings of the 12th International Congress on Mathematical Education*, DOI 10.1007/978-3-319-12688-3_4

In the Opening Chapter 'Congratulatory Remarks: Minister of Education, Science and Technology', the second half of the text is missing. The full text should read as follows:

First of all, congratulations on the opening of the 12th International Congress on Mathematical Education.

I am glad that this important math event is being held in Korea this year.

Also, it is a great pleasure to welcome math education researchers and math teachers from more than 100 countries.

With the aim of transforming Korea into a nation of great science and technology capacity, and a nation of outstanding human talent, the Ministry of Education, Science and Technology of Korea is focusing on three important points in designing and implementing its policies.

The three points are "creativity", "convergence", and "human talent". Creativity enables us to think outside the box, convergence allows us to go beyond the

The online version of the original chapter can be found under
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traditional boundaries between disciplines, and finally human talent builds the very foundation that make all these possible.

Without a doubt, these are the most essential elements in today's knowledge based society. Math is the very subject that can foster much needed creativity and convergence, and is becoming a core factor in raising national competitiveness.

Math is behind everything.

The ICT revolution would have been impossible without the binary system.

The technology behind the CT scans can be traced back to simultaneous equations.

The launching of Korea's first carrier rocket—KSLV-1—will be controlled by a computer program that is based on complicated math equations.

As such, math is behind all technologies we are benefiting from. Anywhere you go in the world, math is considered one of the most important school subjects.

Countries around the world are increasing investment in math education because logical and rational thinking abilities are essential for our students to become creative talents, especially in today's knowledge-based society. Both abilities can be achieved through effective math education. The Korean government is making efforts to improve math education to foster creative talents.

We are planning to revise math textbooks to introduce story-telling methods, so that students can see math principles at work every day in their real lives.

The classroom environment will be changed to help students experience and experiment, rather than simply solving math problems.

The government also stands ready to provide necessary support for math teachers for their professional development.

Once again, congratulations on the opening of the event and I wish all of you a very successful and productive Congress.

Thank you.