Part V
Survey Teams

Curriculum and the Role of Research

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Abstract The survey team collected information on the development and use of curriculum from 11 diverse countries around the world. The data show that a common set of mathematics learning goals are established in almost all countries. However, only a few countries report a substantial role for research in designing and monitoring the development of their curriculum. The data also suggest great variation among countries at the implementation level.

Keywords Standards · Curriculum · Technology · Tracking · Textbooks · Research · Teacher support

Introduction

This report is based on an analysis of responses to survey questions on curriculum standards and goals from 11 countries: Australia, Brazil, Egypt, England, China, Honduras, Indonesia, Japan, Namibia, Peru, and six states in the United States.[1] The paper is organized in five sections: standards/curricular goals; relation of standards to the status quo, the role of textbooks in enacting the curriculum, the role of technology in classrooms, and teacher support related to standards/curricular goals.[2]

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The intent of the report is to allow others to examine their standards/curriculum goals relative to those of other countries across the world.

Standards/Curricular Goals

Who Is Responsible for the Development of Standards/ Curricular Goals?

In most countries the ministry of education establishes curricular standards. In the United States, however, control of education is a state's right, and in many states, for example, Montana, state constitutions give control of education to local districts. The federal government influences education through funding initiatives, such as the No Child Left Behind Act in 2001. The 2010 Common Core State Standards (CCSS) initiative is not a federal program but has been adopted and is being implemented by 45 of the 50 states and the District of Columbia. China also does not have a mandated national curriculum. China Mainland, including Shanghai, has common standards; Hong Kong, Taiwan and Macau create their own standards/ curriculum goals.

In many countries, standards/curricular goals are set by historical tradition or cultural norms. For example, Namibia used the Cambridge curriculum when they became independent in 1990 and only recently has begun to develop their its own standards. Brazil 's standards are attributed to the history of the discipline, the prescribed curricula, and the comparative analysis among national documents from different historical periods and national and international documents. Some countries base their standards and guidelines on those of countries with high achievement scores on recent international exams. For example, both England and the United States cite countries such those from the Pacific Rim and Finland as resources for their new standards. Peru noted that an analysis of documents from other countries in South American and from TIMSS, Programme for International Student Assessment (PISA), and National Council of Teachers of Mathematics (NCTM) contributed to the development of their Diseño Curricular Nacional (CND) (National Curricular Design) (2009).

Why Standards?

Over time, many countries have changed from local standards to national standards. For example, Brazil found that the lack of national standards contributed to unequal opportunity for education. For much the same reason, the documented difference in the rigor and quality of individual state standards, the state governors in the United

States supported the development and adoption of the CCSS. The new US standards are intended to be substantially more focused and coherent.

Standards are viewed as political: i.e., Brazil suggests that mathematics curricular goals depend more on political timing, election campaigns and government administrations, where "the logic of an education agenda that transcends governments and politicians' mandates, set as a goal for a democratic and developed society, is not the rule" (Response to ICME 12 Curriculum Survey 2011, p. 6). In the United States the two major political parties have different views on education, its funding and its goals. This has recently given rise to the creation of publicly funded schools governed by a group or organization with a legislative contract or charter from a state or jurisdiction that exempts the school from selected state or local regulations in keeping with its charter. Hong Kong also reported that writing standards seems to be more politically based than research based. Many of the changes in England's National Curriculum (NC) are the result of criticism from the current government that the NC is over-prescriptive, includes non-essential material, and specifies teaching method rather than content. In Peru each new curricular proposal is viewed as an adjustment to the prior curriculum. In this process, radical changes do occur, such as changing the curriculum by capabilities (CND 2005) to the curriculum by competencies (CND 2008) in the secondary education level. These decisions are often the result of a policy change with each new government.

In most countries surveyed, a diverse team, including mathematics education researchers, ministry of education staff, curriculum supervisors, and representatives of boards of education are responsible for developing the standards/goals. In some countries (Japan, Australia) teachers are involved, but in others the design teams are primarily experts from universities, teaching universities or the ministry of education (Indonesia, Egypt). The design of the framework for the National Curriculum in England is carried out by a panel of four, not necessarily mathematics educators, charged to reflect the view of the broader mathematics education community including teachers.

What Is the Role of Research?

Research has different interpretations and meanings in relation to the development and implementation of standards or curricula guidelines. One common response in the surveys was to cite as research the resources used in preparing standards (for example, other countries' standards). In addition, the degree to which research is used in compiling the standards often depends on the vision, perspectives and beliefs of the team responsible for the development.

The use of research related to student learning in developing standards/curricular goals is not common among the countries surveyed. A typical description of the process was given by Hong Kong, where the development team might do a literature review and refer to documents of other countries, but the process is not necessarily well structured and often depends on the expertise of the team members.

England, however, noted that the first version of their National Curriculum (NC) was largely based on the Concepts in Secondary Mathematics and Science project, (Hart 1981) that sought to formulate hierarchies of understanding in 10 mathematical topics normally taught in British secondary schools based on the results of testing 10,000 children in 1976 and 1977. The NC was also based on the ILEA Checkpoints (1979) and the Graded Assessment in Mathematics (1988–1990) projects. The original research-based design of the NC had many unintended consequences. Although the attainment targets were intended to measure learning outcomes on particular tasks, the levels were used to define the order in which topics should be taught, rather than paying attention to the development of concepts over time. The processes of mathematics, originally called "Using and applying mathematics" were defined in a general way related to progressions and levels that made interpretation difficult. As a consequence, the NC was revised several times and as of summer 2012 was again in the process of revision.

After a 1996 survey showed that social segmentation in Brazil seemed to be an obstacle to access to a quality education, research led to the development of the National Curricular Parameters in Brazil (1997). The Board of National Standards for Education (*Badan Standar Nasional Pendidikan*) in Indonesia examined the national needs for education, the vision of the country, societal demands, challenges for the future, and used their findings in developing the curriculum (Ministry of National Education 2006).

What Is the Nature of Standards?

In Brazil, Indonesia, Namibia and Peru, the standards/curricular frameworks are general and provide overarching guidelines for the development of discipline specific content. In the United States, Australia, and Japan, the mathematical standards essentially stand alone, although supporting documents may illustrate how the maths standards fit into the larger national education philosophy and perspective. Some standards include process goals. For example, Australia includes standards for four proficiencies (understanding, fluency, problem solving and reasoning) based on those described in *Adding It Up* (Kilpatrick et al. 2001). The new Australian standards want students to see that mathematics is about creating connections, developing strategies, and effective communication, as well as following rules and procedures. The United States CCSS has mathematical practice standards specifying eight "habits of mind" students should have when doing mathematics. In Brazil ideas such as "learn to learn", "promote independence", "learn to solve problems" are being incorporated into new curricula. In Peru and Indonesia the emphasis is primarily on the processes of problem solving, reasoning and proof, and mathematical communication.

In some cases standards reinforce the role of education in responding to the needs of the country. For example, the Curriculum for Basic Education (1st–9th grade) in Honduras (Department of Education 2003) was developed under three axes: personal, national and cultural identity, and democracy and work. The four pillars of lifelong learning defined by Delors (1996) (personal fulfilment, active citizenship, social inclusion and employability/adaptability) were used to define the mathematical content and methodological guides with problem solving as the central umbrella. Namibia's National Curriculum for a Basic Education outlines the aims of a basic education for the society of the future and specifies a few very general learning outcomes for each educational level (Namibia MoE 2008).

Standards span different sets of school grades or levels and differ in generality. Some countries have grade specific standards for what students should know throughout their primary and secondary schooling (i.e., US, Japan). Australia specifies a common curriculum for grades 1–10 and course options for students in upper secondary. Egypt and Honduras have curricular goals for students in grades 1–9 (age 14). At the high school level, Honduras focuses on post high school preparation with more than 53 career- focused schools for students.

The development of fractions in Australia by the Australian Curriculum and Assessment Reporting Authority (ACARA 2011), the Japanese Ministry of Education, Culture, Sports, Science and Technology (MEXT 2008), the Ministry of Education in Namibia (MoE 2005, 2006), and the US (CCSS 2010) illustrates the difference in standards across countries In grade 1, the standards/goals in the US, Namibia and Australia introduce words such as half, quarter and whole; this happens in grade 2 in Japan. Both US and Japan treat fraction as a number on the number line beginning in grade 3, emphasize equal partitioning of a unit and consider a fraction as composed of unit fractions: 4/3 = 4 units of 1/3. Australia suggests relating fractions to a number line only for unit fractions in grade 3, while Namibia does not mention fractions in relation to the number line. Equivalent fractions are taught in grade 4 in US, Japan, and Australia and in grade 6 in Namibia. Addition and subtraction of fractions with like denominators occurs in grade 4 in Japan, with unlike denominators in grade 5 in the US and Japan, and grade 7 in Namibia and Australia. Australia and Namibia have fractions as parts of collections in grade 2 and again in grade 4 in Namibia, but fractions as subsets of a collection are not mentioned in the standards/goals in the US and Japan. Students are expected to multiply and divide fractions in grade 5 in the US (with the exception of division of a fraction by a fraction, which happens in grade 6), in grade 6 in Japan, and in grade 7 in Australia and Namibia.

The next section describes what is taught in classrooms and how this relates to the standards/curricular goals of the country.

Examining the Status Quo

How Are Standards/Goals Related to the Implemented Curriculum?

Standards play different roles in shaping curriculum. For example, as described above, Peru does not have National Standards, but the mathematics learning goals for students are set out in the Curriculum National Design. With this as a guide, each of the country's regions develops a regional curriculum that considers the diversity of cultures and languages. Similarly, since 2005 Indonesia has National Standards for Education, which include standards for content in each subject area and curriculum structure. Based on these and competency standards, every school develops their own curriculum considering the vision of the school, local culture and students' background. In many of the US states, for example Massachusetts, standards provide a framework with the details of the curriculum, including the materials used for teaching and learning established at the district and school level. Japanese schools base their curricula on the national Course of Study (CS), a "Teaching Guide," resources and guidelines developed by local boards of education in the prefecture, and planning guides from textbook companies. Adaptions are sometimes made based on the situation of the school and its students. When the prefectural or the municipal boards of education develop their own model plans, such as the "nine year schooling system" (ShoChu-Ikkan-Kyoiku), the school in the prefecture or the municipality follows those plans and makes revisions to the CS accordingly.

In some instances, countries turn to other countries with more resources for support in implementing the standards. For example, the Japan International Cooperation Agency supported Honduras in developing curriculum and resources for teachers. Macau uses resources from China Mainland, Hong Kong and Canada.

What Drives the Implemented Curriculum?

Standards, textbooks, or high-stakes examinations seem to drive what happens in classrooms in the countries surveyed. While Hong Kong indicated that standards play that role, teachers in Brazil, Taiwan, Egypt, Honduras, and Japan rely on textbooks, and China mainland cited both textbooks and practice books.

In several countries high stakes examinations are significant in determining what teachers actually teach. In the United States, with the exception of Montana, the states surveyed indicated they followed the curriculum based on the state standards, but in reality most teachers teach only to what they know from experience will be tested (Au 2007). The implemented curriculum in England also seems to be shaped by what is assessed, which determines the nature of the tasks students meet in classrooms. The curriculum in Indonesia is determined both by textbooks and the national examination. Entrance examinations of leading universities impact the curriculum in Brazil and Macau (95 % of the students in Macau attend private schools to prepare for university).

How Do Countries Monitor Implementation of the Curriculum?

Countries use several strategies for monitoring and evaluating the enacted curriculum: large scale research studies conducted by the government or a private agency, small focused research studies on what is being taught and learned, student achievement on high stakes assessments, and approval of textbooks teachers use to deliver the curriculum. Relatively large-scale research studies on students' achievement are carried out in Honduras under the auspices of the Inter-American Development Bank and USAID. The Ministry of Education in Brazil investigated the incorporation of the National Curricular Parameters (PCN) into textbooks and other materials supporting teachers' work, but little research has been dedicated to any of the various stages in the process of curriculum development including the curriculum enacted in classrooms.

Japan administers national assessments on a regular basis in mathematics and Japanese for students in the sixth year of elementary school and the third year of lower secondary school. The results often reveal challenges in knowledge and skill utilization, which lead to revisions in educational policies and classroom lesson plans. These assessments are viewed as invaluable in monitoring and revising the curriculum.

In the United States, perhaps the most significant change in the last decade has been the increasing role of high stakes assessments measuring student achievement in elementary/secondary education. Every year each state assesses each student in grades 3–8 and assesses students once in grades 9–12 using a common state assessment, typically consisting of multiple-choice procedural questions. The results are used to evaluate teachers, administrators, and the curriculum. Little or no evidence exists correlating success on these tests with curriculum (or any other factor). This has not deterred federal and state levels policy makers from making use of the assessment results in these ways. The emphasis on high stakes assessment and accountability are seen in England as well, although it is not clear that the results have contributed to changes in the curriculum or standards.

How Are Changes Made to the Standards/Curricular Goals?

Change occurs in different ways. In the US, the most recent change was brought about by entities outside of the government and teachers. Japan bases changes in goals/standards on research examining student learning. Standards teams summarize, examine, and investigate the results of research studies on what has been achieved though the current Course of Study (CS) and the results of pilot trials of new goals/standards in designated "research schools" (Kenkyu-Kaihatsu-Gakko). They monitor emerging trends, societal needs and international assessments. For example, the most recent revisions to the CS in Japan for elementary and lower secondary schools were in March 2008 and for upper secondary and special needs education in March 2009. In this CS, the aim of mathematics education stresses the student's abilities to express their thinking and utilize mathematics in daily social life. In the CS for lower secondary schools, a new curricular strand "Use of Data" was added to enrich the content of statistics in the compulsory education. International mathematics assessments have helped statistics became a requirement in upper secondary schools. Taiwan and Hong Kong use some research supported by the government to construct and modify the curriculum as well as to inform teacher professional development and resource materials.

The Role of Textbooks

Survey responses indicated commercial publishers, private organizations, and government related organizations were involved in textbook development and distribution but to different degrees. The use of supplementary materials or teacher created worksheets was common in many of the countries. Many countries mentioned national standards/curricular guidelines as tools used in textbook development.

What is the approval or vetting process for textbooks?

In most of the countries with the exception of England and some of the states in the United States, some formal approval is necessary before texts can be used. For example, in Japan, textbooks are edited for adherence to the national curriculum and must be examined and authorized by MEXT. However, each textbook company can design and develop a textbook series with a final draft submitted to MEXT for examination and subsequent revision. During the development process, professionals (such as university researchers and teachers) play a large role in textbook design and development.

Many countries (China, Indonesia, Australia) have multiple textbook options for each grade level. Textbook adoption procedures vary, with decisions made at the national level (Brazil), state level (North Carolina), district level (Japan for elementary and lower secondary), school level (Japan for upper secondary) or even at an individual level (Taiwan). For the most part, the content would be the same across textbook options for each grade level since standards were the main drivers of the textbook development. Textbooks differ in the extent to which the contents are ordered and compiled but often have a similar style. Teachers in England make less use of textbooks than many other countries, and there is no uniform adoption procedure (Askew et al. 2010). In addition, public examination bodies produce textbooks that contain exercises from compilations of past examination questions that are popular with British teachers who see them as preparation for high-stakes assessment.

What Is the Role of Research in the Development of Textbooks?

Most countries mentioned an indirect or no use of research in textbook development. In the United States and England textbooks that are developed through large projects typically involve some research. In the United States, some curriculum materials (such as CMP 2012) are research based and developed with government or other sources of funding. Designers study trialling in classrooms, identify issues that emerge, what is working and not working to inform the next iteration of materials. The cycle may have several iterations, depending on funding and on commercial sales. (If the materials market poorly, the development is quickly terminated.)

Textbooks authored by individual teachers or commercial publishers did not seem to be noticeably influenced by pilot studies, research or research related to learning. In organizing textbook content, Japan makes use of research on high stakes assessment (the National Assessment of Academic Ability and other assessments implemented by local governments), the content and sequence of the old textbooks, and information obtained from teachers on the usability of the textbook and on the students' responses to the textbook problems during the lesson. In Brazil, some authors of mathematics textbooks use research, or rely on research results, to develop books.

Focused research projects on aspects of the curriculum, supplements to illustrate the standards, pilot studies of initiatives, action research and/or small seed projects are common in Hong Kong and Japan. In the United States, research studies on student learning typically focus on specific content areas or the development of a single concept, such as understanding cardinality (i.e., Clements 2012) and have little direct connection to the curriculum. Graduate students carry out many such projects in the United States and in other countries such as Brazil, England and Australia.

The Role of Technology in the Curriculum

What Is the Relationship Between Standards/Curricular Goals and Technology?

From a broad perspective, interacting with technology is seen in most countries as a critical life skill. In Peru, for example, the aim is to develop students' "skills and attitudes that will enable them to use and benefit from ICT … thus enhancing the autonomous learning throughout life" (MoE 2009, p. 17). The National Curricular

Parameters (1997) in Brazil cite the value of technology as important for preparing students for their work outside of school. Australia defines Information and Communication Technology (ICT) as one of seven basic capabilities, i.e., the “skills, behaviours and dispositions that, together with curriculum content in each learning area and the cross curriculum priorities, will assist students to live and work successfully in the twenty-first century” (ACARA 2012, p. 10) Namibia has much the same statement in their National Curriculum for Basic Education emphasizing creating and learning to use software such as Word or Excel. Hong Kong’s Technology Learning Targets calls for technology to enhance learning and teaching; provide platforms for discussions; help students construct knowledge; and engage students in an active role in the learning process, understanding, visualizing and exploring math, experiencing the excitement and joy of learning maths.

Some countries such as Namibia and Peru do not outline how technology should be used in the mathematics curriculum. Others describe the use of technology in mathematics classrooms in very general terms. Indonesia, for example, calls for the use of technology to develop understanding of abstract ideas by simulation and animation. In mainland China, the Nine Year Compulsory Education Mathematics Curriculum Standards emphasized the use of technology to benefit student understanding of the nature of mathematics. In Macau the standards call for educators to consider the impact of computers and calculators on the content and approaches in mathematics teaching and learning. In Taiwan, technology should support understanding, facilitate instruction, and enhance connections to the real world. England’s curriculum documents are more specific, consistently encouraging the use of appropriate ICT tools to solve numerical and graphical problems, to represent and manipulate geometrical configurations and to present and analyse data.

The standards/curricular goals of some countries provide general goals for incorporating technology into the curriculum and then describe specific instances. For example, the United States Common Core State Standards (2010) for mathematical practices call for students to visualize the results of varying assumptions, exploring consequences, and comparing predictions; engage students in activities that deepen understanding of concepts; create opportunities for and learning—comparing and contrasting solutions and strategies, creating patterns, generating simulations of problem situations. These generalizations are followed by statements throughout, such as in grade 7, “Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions” (p. 50) or in algebra, “find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations” (p. 66). The new Australian Mathematics Curriculum specifically calls for the use of calculators to check solutions beginning in grade 3 and, by year 10 includes general statements about the use of technology, “Digital technologies, such as spreadsheets, dynamic geometry software and computer algebra software, can engage students and promote understanding of key concepts (p. 11)”. The curriculum provides specific examples: i.e., students should “Solve linear simultaneous equations, using algebraic and graphical techniques including using digital technology (p. 61).”

Japan has explicit learning goals for the use of technology and its Course of Study provides a guide for teachers that describes how calculators and computers can be used, with specific grade level examples under three headings; (1) as tools for calculation, (2) as teaching materials, and (3) as information/communication networks.

How Is Technology Used in Classrooms?

Respondents cited general issues related to the use of ICT. In England, for example, inspection reports based on evidence from 192 schools between 2005 and 2007 criticized schools' use of ICT, finding effective usage was decreasing and the potential of ICT to enhance the learning of mathematics rarely realized. In Brazil, the number of schools equipped with technological resources is increasing; however, programs using the technology are still restricted to pilot projects.

In Japan a 2010 survey on ICT facilities found that computers (98.7 %), digital cameras (98.1 %), and CD players (95.2 %) were used almost daily or at least two to three times a week (MEXT 2011). Yet, results from international studies such as TIMSS indicate little actual computer use in Japanese mathematics classrooms. At least one computer is typically available in classrooms in Egypt, Peru, China mainland and Macau but rarely used for mathematics instruction. Honduras has a one laptop per child program, but the lack of suitable mathematics related activities limits the use of laptops in classrooms. This was also identified as a problem in England. Brazil reported that a preliminary analysis of research conducted in the country suggests that technologies are used very little. Teachers are uncomfortable with laptops and have few resources for using them.

The availability of technological tools for students varied among countries and within countries. Some have class sets of calculators available; others expect students to provide their own (China Mainland, Macau, Hong Kong). Some schools have computer labs; some have class sets of laptops, while others use a single computer with overheard display (common in China Mainland). Many schools in England have a separate computer suite, where pupils learn to use ICT as a mathematical tool, for example using spreadsheets to generate number patterns or present statistical information but their use to enhance mathematics learning is limited.

Some use computers to provide practice procedures and skills (England, Macau, North Carolina). Some (China mainland, Taiwan, North Carolina) use technology as a way to differentiate instruction. North Caroline describes using interactive sites that allow the learner to manipulate data and objects and then provide immediate feedback; video, games, and other learning activities for struggling students, and providing advanced students with online activities that challenge and invite further learning; real world math practice using tools like Google Earth for measurement, stock market simulations, digital cameras for capturing real-life examples of geometric figures, Skype or other conferencing tools to interact with scientists and mathematicians. Formative and summative assessment was also indicated as a way of bringing technology into the classroom.

Interactive whiteboards are becoming increasingly common, although their role in learning mathematics is not well documented. They are heavily used in Great Britain (in about 75 % of schools) (Schachter 2010), and usage is growing in Japan from 16,403 in 2009 to 60,474 in 2011 (MEXT 2011) and the United States with 51 % of classrooms (Gray 2010). According to England an advantages of interactive white boards include high-quality, diagrams and relevant software to support learning through, for example, construction of graphs or visualization of transformations. A negative effect of interactive whiteboards seemed to be a reduction in pupils' use of concrete manipulatives.

Teacher Support

What Support Is Provided to Teachers to Help Them Know the Curriculum?

The survey results from Brazil and Egypt indicated minimum support is provided to teachers to help them learn about the curriculum. Brazil noted the materials are distributed to teachers usually without any actions involving the teachers. The other countries surveyed provide some form of support for teachers although the amount and form as well as who was in charge of providing support differs. Some countries (i.e., England, China, Japan) have ministry driven efforts to help teachers learn about the curriculum. For example, in Japan, once a new course of study (CS) is determined, the Ministry of Education, using a "trainer of trainers" process, conducts "transmission lectures" (Dentatsu-Koshu) on the principles and content of the new CS to superintendents on the prefectural boards of education who in turn give lectures to the superintendents on the municipal boards of education. The local superintendents then give lectures to all schoolteachers within a period of three years. The Ministry makes information available to teachers by showing concrete teaching examples, especially for large changes from an old to a new course of study. A variety of research meetings and conferences as well as lectures and symposiums are offered to educate teachers on the new CS.

A similar trainer of trainers process organized by the Ministry is also used in Honduras and Peru, although in Peru, some question the effectiveness of the process, given the results of five evaluations available on the web page of the Ministry of Education. Since 2010 the Ministry of Education in Mainland China has invested considerable resources to help teachers (over 1.1 million teachers at the primary level) understand the basic ideas of the curriculum standards and main content of the curriculum. The work is organized and financed by the Ministry but carried out at the local level. In Hong Kong, the Ministry of Education organized a professional development series, "Understanding the Curriculum", to explain the breadth and width of the curriculum. Exemplars, usually a product of collaborative research with schools, are used for illustration.

Other countries have a blend of ministry designed strategies and local initiatives. In Indonesia, the local (district and province) as well as central governments facilitate in-service training for teachers helping them to understand more about the curriculum. District school supervisors, advisors and/or experts from universities do the training and aim to improve the understanding of the Standards of Content, Process and Evaluation. Workshops and sessions on the standards are often organized and provided at the local level by university educators, school districts, curriculum consortia, and non-profit partners for all educators in a region of a state. Web based resources are provided in several countries (Honduras, China Mainland, Hong Kong, Japan). North Carolina provides webinars on the structure, organization, and content of the state standards, and Ohio provides online resources and disseminates curriculum models and other support documents to districts.

What Support Is Provided to Teachers to Help Them Enact the Curriculum?

In some countries support for instruction related to curriculum comes from the ministry of education (China Mainland, Hong King, England, Peru, some states in the United States) and in others it is provided through a combination of ministry of education and local initiatives or at the local level. Support primarily takes three forms: resources, professional development and mentoring.

1. Resources: Supplemental resources, materials created by outside research-based projects, and documents based on the state/national curriculum or standards are often designed and delivered through university programs. In some areas in Brazil, teachers are given written supporting material, videos, and learning resources, and technical pedagogical teams often help teachers in the implementation of the curriculum.
2. Professional Development: A variety of forms of professional development were also cited as ways to help teachers enact the curriculum. In Taiwan the curriculum development council provides lectures at the school level, instruction counselling groups and in-service workshops. Teacher training in Indonesia helps teachers develop teaching plans and provides strategies, methods, and approaches that have been adopted from the current research and theory. Honduras uses a "learn by doing" model for in-service, and many districts in the United States support mathematics "learning communities". Some form of collaborative lesson planning is typical in several of the countries (Japan, Macau, some states in the United States). In many countries (i.e., Hong Kong, United States) universities offer a variety of programs for in-service teacher education; graduate programs are sites for teachers' professional development.

Publishers also organize and deliver professional development workshops (China Mainland, United States).

Japan has a structured system of support. Local boards of education provide training for beginning teachers and for those with five, 10 and 20 years of teaching experiences as well as a variety of professional, non-mandatory training courses to enhance teaching ability and skills; for example, the Tochigi prefectural board of education offers 50 courses a year. Recently, a new teacher training/licensing system has been employed. Ordinary and special licenses are valid for 10 years; teachers need to renew their licenses by attending training courses every 10 years, given by general universities and teacher-training universities. These training courses are required to offer information based on the most recent research.

3. Mentoring: A third form of support in some countries is individualized, such as the Strategic Program for Learning Achievements in Peru where, since 2010, classroom teachers working with children up through the first two years of Basic Education (grades 6–8) receive advice from a specialist teacher. In the United States, many local districts have mathematics coaches who work with teachers, particularly at the elementary level. Hong Kong has dedicated "research schools" that mentor other schools in the implementation of the curriculum. A slightly different strategy is used in Honduras where teachers travel to Japan to see how the curriculum is enacted in classrooms and to learn about mathematics education.

While some cite a research base for professional development, the connection to research is often very limited (Hong Kong, Massachusetts and North Carolina in the United States). England provided ministry organized teacher support designed with a research perspective and later studies investigated the success of the implementation. The National Strategies (DFE 2011) were, from 1998 until 2011, the main delivery vehicle for supporting teachers to understand and implement government teaching and learning priorities. The programme, originally called the National Numeracy Strategy (NNS), was aimed at primary education but was later expanded to include secondary schools with the National Mathematics Strategy (NMS). The National strategies conducted a massive professional development programme, running courses and providing publications, advice and professional development materials such as videos to schools. These also included guidance on course planning, teaching and learning, assessment, subject leadership, inclusion, intervention and mathematics specific content. Detailed assessment guidance, lesson plans, and intervention programs were all provided (DFE 2011). An annotated bibliography of research evidence claimed to underpin the National Strategies (Reynolds and Muijs 1999). However, the research evidence was described as ambivalent and relatively scarce (Brown et al. 2003).

Evaluations of the implementation of the NNS were carried out and indicated some success, but this was contested by many who asserted the gains on National Tests attributed to the programme may be attributed to a careful choice of statistical baseline and to teachers' increasing tendency to orient their teaching towards the tests. When alternative tests were used, smaller gains were noted. Teaching seemed to have changed mainly in superficial ways, and some evidence suggested that in almost no cases were there 'deep' changes. (Brown et al. 2003, p. 668). In 2008 an inspection service found weaknesses in basic teaching skills and had difficultly assessing which initiatives worked and which did not. The frequent introduction of new initiatives, materials and guidance led to overload and diminished the potential effectiveness of each individual initiative (Ofsted 2010). As of March 2012, the Coalition Government abolished the National Strategies programme, and future professional development is decentralized and in the hands of individual schools.

Concluding Remarks

The survey data shows us that a common set of mathematics learning goals are established in almost all countries with a very minor role for research in designing and monitoring the development of their curriculum. Standards, textbooks, or high-stakes examinations seem to drive what happens in classrooms. Countries vary greatly in the amount of support provided to teachers in learning about and implementing the curriculum specified in their standards/goals.

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Key Mathematical Concepts in the Transition from Secondary School to University

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This report[1] from the ICME12 Survey Team 4 examines issues in the transition from secondary school to university mathematics with a particular focus on mathematical concepts and aspects of mathematical thinking. It comprises a survey of the recent research related to: calculus and analysis; the algebra of generalised arithmetic and abstract algebra; linear algebra; reasoning, argumentation and proof; and modelling, applications and applied mathematics. This revealed a multi-faceted web of cognitive, curricular and pedagogical issues both within and across the mathematical topics above. In addition we conducted an international survey of those engaged in teaching in university mathematics departments. Specifically, we aimed to elicit perspectives on: what topics are taught, and how, in the early parts of university-level mathematical studies; whether the transition should be smooth; student preparedness for university mathematics studies; and, what university departments do to assist those with limited preparedness. We present a summary of the survey results from 79 respondents from 21 countries.

[1] *A fuller version of this report is available from* http://www.math.auckland.ac.nz/∼thomas/ST4.pdf.

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Background

Changing mathematics curricula and their emphases, lower numbers of student enrolments in undergraduate mathematics programmes (Barton and Sheryn 2009; and http://www.mathunion.org/icmi/other-activities/pipeline-project/) and changes due to an enlarged tertiary entrant profile (Hockman 2005; Hoyles et al. 2001), have provoked some international concern about the mathematical ability of students entering university (PCAST 2012; Smith 2004) and the traumatic effect of the transition on some of them (Engelbrecht 2010). Decreasing levels of mathematical competency have been reported with regard to essential technical facility, analytical powers, and perceptions of the place of precision and proof in mathematics (Brandell et al. 2008; Hourigan and O'Donoghue 2007; Kajander and Lovric 2005; Luk 2005; Selden 2005). The shifting profile of students who take service mathematics courses has produced a consequent decline in mathematical standards (Gill et al. 2010; Jennings 2009). However, not all studies agree on the extent of the problem (Engelbrecht and Harding 2008; Engelbrecht et al. 2005) and James et al. (2008) found that standards had been maintained. The recent President's Council of Advisors on Science and Technology (PCAST) (2012) states that in the USA alone there is a need to produce, over the next decade, around 1 million more college graduates in Science, Technology, Engineering, and Mathematics (STEM) fields than currently expected and recommends funding around 200 experiments at an average level of $500,000 each to address mathematics preparation issues. This helps to place the transition situation above in context and emphasises the importance of addressing the issues arising.

We found relatively few papers in the recent literature related directly with our brief to consider the role of mathematical thinking and concepts related to transition. Hence we also reviewed literature analysing the learning of mathematics on one or both sides of the transition boundary. To achieve this we formed the somewhat arbitrary division of this mathematics into: calculus and analysis; abstract algebra; linear algebra; reasoning, argumentation and proof; and modelling, applications and applied mathematics, and report findings related to each of these fields. We were aware that other fields such as geometry and statistics and probability should have been included, but were not able to do so.

The Survey

We considered it important to obtain data on transition from university mathematics departments. We wanted to know what topics are taught and how, if the faculty think the transition should be smooth, or not, their opinions on whether their students are well prepared mathematically, and what university departments do to assist those who are not. Hence, we constructed an anonymous questionnaire on transition using an Adobe Acrobat pdf form and sent it internationally by email to members of mathematics departments. The 79 responses from 21 countries were collected electronically. The sample comprised 56 males and 23 females with a mean of 21.9 years of academic teaching. Of these 45 were at the level of associate professor, reader or full professor, and 30 were assistant professors, lecturers or senior lecturers. There were 5 or more responses from each of South Africa, USA, New Zealand and Brazil.

Clearly the experience for beginning university students varies considerably depending on the country and the university that they attend. For example, while the majority teaches pre-calculus (53, 67.1 %), calculus (76, 96.2 %) and linear algebra (49, 62 %) in their first year, minorities teach complex analysis (1), topology (3), group theory (1), real analysis (5), number theory (9), graph theory (12), logic (15), set theory (17) and geometry (18), among other topics. Further, in response to 'Is the approach in **first** year mathematics at your university: Symbolic, Procedural; Axiomatic, Formal; Either, depending on the course.' 21 (26.6 %) answered that their departments introduce symbolic and procedural approaches in first year mathematics courses, while 6 replied that their departments adapt axiomatic formal approaches. Most of the respondents (50, 63.3 %) replied that their approach depended on the course.

When asked 'Do you think students have any problems in moving from school to university mathematics?' 72 (91.1 %) responded "Yes" and 6 responded "No". One third of those who answered "Yes" described these problems as coming from a lack of preparation in high school, supported by comments such as "They don't have a sufficiently good grasp of the expected school-mathematics skills that they need." Further, two thirds of those who answered "Yes" described the problems as arising from the differences, such as class size and work load, between high school classes and university, with many specifically citing the conceptual nature of university mathematics as being different from the procedural nature of high school mathematics. Comments here included "university is much more theoretical" and "Move from procedural to formal and rigourous [sic], introduction to proof, importance of definitions and conditions of theorems/rules/statements/formulas." There is also a need to "…deal with misconceptions which students developed in secondary school…We also have to review secondary school concepts and procedures from an adequate mathematical point of view." Other responses cited: students' weak algebra skills (12.5 %); that university classes are harder (5 %); personal difficulties in adjusting (10 %); poor placement (3 %); and, poor teaching at university (1 %).

Looking at specific mathematical knowledge, we enquired 'How would you rate first year students' mathematical understanding of each of the following on entry to university?' With a maximum score of 5 for high, the mean scores of the responses were algebra or generalised arithmetic (3.0), functions (2.8), real numbers (2.7), differentiation (2.5), complex numbers (1.9), definitions (1.9), vectors (1.9), sequences and series (1.9), Riemann integration (1.8), matrix algebra (1.7), limits (1.7) and proof (1.6). The mathematicians were specifically asked whether students were well prepared for calculus study. Those whose students did not study calculus at school rated their students' preparation for calculus at 2.1 out of 5. Those whose students did, rated secondary school calculus as preparation to study calculus at university at 2.4, and as preparation to study analysis at university at 1.5. These results suggest that there is some room for improvement in school preparation for university study of calculus and analysis.

Since the view has been expressed (e.g., Clark and Lovric 2009) that, rather than being ‘smooth’, the transition to university should require some measure of struggle by students, we asked ‘Do you think the transition from secondary to university education in mathematics should be smooth?’ Here, 54 (68.4 %) responded “Yes” and 22 (27.8 %) responded “No”. Of those who responded “No”, many of the comments were similar to the following, expressing the belief that change is a necessary part of the transition: “Not necessarily smooth, because it is for most students a huge change to become more independent as learners.” and “To learn mathematics is sometimes hard.” Those who answered yes were then asked ‘what could be done to make the transition from secondary to university education in mathematics smoother?’ The majority of responses mentioned changes that could be made at the high school level, such as: encourage students to think independently and abstractly; change the secondary courses; have better trained secondary teachers; and, have less focus in secondary school on standardised tests and procedures. A few mentioned changes that could be made at the university, such as: better placement of students in classes; increasing the communication between secondary and tertiary teachers; and, addressing student expectations at each level. This lack of communication between the two sectors was highlighted as a major area requiring attention by the two-year study led by Thomas (Hong et al. 2009).

Since one would expect that, seeing students with difficulties in transition, universities would respond in an appropriate manner (see e.g. Hockman 2005), we asked ‘Does your department periodically change the typical content of your first year programme?’ 33 (41.8 %) responded “Yes” and 44 (55.7 %) “No”. The responses to the question ‘How does your department decide on appropriate content for the first year mathematics programme for students?’ by those who answered yes to the previous question showed that departments change the content of the first year programme based on the decision of committees on a university or department level. Some respondents said that they change the course content for the first year students based on a decision by an individual member of faculty who diagnoses student needs and background. 15 of the 35 responded that their universities try to integrate student, industry, and national needs into first year mathematics courses. The follow-up question ‘How has the content of your first year mathematics courses changed in the last 5 years?’ showed that 35 had changed their courses in the last 5 years, but 10 of these said that the change was not significant. 17 out of the 35 respondents reported that their departments changed first year mathematics courses by removing complex topics, or by introducing practical mathematical topics. In some of the courses, students were encouraged to use tools for calculation and visualisation. In contrast, six departments *increased* the complexity and the rigour of their first year mathematics courses.

The survey considered the notion of proof in several questions. In response to ‘How important do you think definitions are in **first** year mathematics?’ 52 (65.8 %)

replied that definitions are important in first year mathematics, while 15 presented their responses as neutral. Only 8 respondents replied that definitions are not important in first year mathematics. Responses to the question 'Do you have a course that explicitly teaches methods of proof construction?' were evenly split with 49.4 % answering each of "Yes" and "No". Of those who responded "Yes", 15 (38.4 %) replied that they teach methods of proof construction during the first year, 23 (58.9 %) during the second year and 5 (12.8 %) in either third or fourth year. While some had separate courses (e.g. proof method and logic course) for teaching methods of proofs, many departments teach methods of proofs traditionally, by introducing examples of proof and exercises in mathematics class. Some respondents replied that they teach methods of proof construction in interactive contexts, citing having the course taught as a seminar, with students constructing proofs, presenting them to the class, and discussing/critiquing them in small size class. One respondent used the modified Moore method in interactive lectures. Looking at some specific methods of introducing students to proof construction was the question 'How useful do you think that a course that includes assistance with the following would be for students?' Four possibilities were listed, with mean levels of agreement out of 5 (high) being: Learning how to read a proof, 3.7; Working on counterexamples, 3.8; Building conjectures, 3.7; Constructing definitions, 3.6. These responses appear to show a good level of agreement with employing the suggested approaches as components of a course on proof construction. It may be that these are ideas that the 49.4 % of universities that currently do not have a course explicitly teaching proof construction could consider implementing as a way to assist transition.

Mathematical modelling in universities was another topic our survey addressed. In response to the questions "Does your university have a mathematical course/activity dedicated to mathematical modeling and applications?" and "Are mathematical modelling and applications contents/activities integrated into other mathematical courses?", 44 replied that their departments offer dedicated courses for modelling, while 41 said they integrate teaching of modelling into mathematics courses such as calculus, differential equations, statistics, etc. and 7 answered that their university does not offer mathematics courses for mathematical modelling and applications. Reasons given for choosing dedicated courses include: the majority of all mathematics students will end up doing something other than mathematics so applications are far more important to them than are detailed theoretical developments; most of the mathematics teaching is service teaching for students not majoring in mathematics so it is appropriate to provide a relevant course of modelling and applications that meets the needs of the target audience; if modelling is treated as an add-on then students may not learn mathematical modelling methods. Those who chose integrated courses did so because students need to be equipped with a wide array of mathematical techniques and solid knowledge base. Hence, it is appropriate for earlier mathematics courses to contain some theory, proofs, concepts and skills, as well as applications.

Considering what happens in upper secondary schools, 26 (33 %) reported that secondary schools in their location have mathematical modelling and applications integrated into other mathematical courses, with only 4 having dedicated courses. 44 (56 %) said that there were no such modelling courses in their area. When asked for their opinion on how modelling should be taught in schools, most of the answers stated that it should be integrated into other mathematical courses. The main reasons presented for this were: the many facets of mathematics; topics too specialised to form dedicated courses; to allow cross flow of ideas, avoid compartmentalization; and students need to see the connection between theory and practice, build meaning, appropriate knowledge. The question ‘What do you see as the key differences between the teaching and learning of modelling and applications in secondary schools and university, if any?’ was answered by 33 (42 %) of respondents. The key differences pointed out by those answering this question were: at school, modelling is poor, too basic and mechanical, often close implementation of simple statistics tests; students have less understanding of application areas; university students are more independent; they have bigger range of mathematical tools, more techniques; they are concerned with rigour and proof. Asked ‘What are the key difficulties for student transition from secondary school to university in the field of mathematical modelling and applications, if any?’ the 35 (44 %) university respondents cited: lack of knowledge (mathematical theory, others subjects such as physics, chemistry, biology, ecology); difficulties in formulating precise mathematical problems/ interpreting word problems/understanding processes, representations, use of parameters; poor mathematical skills, lack of logical thinking; no experience from secondary schools; and lack of support. One message for transition is to construct more realistic modeling applications for students to study in schools.

In order to investigate how universities respond to assist students with transition problems we enquired “Do you have any academic support structures to assist students in the transition from school to university? (e.g., workshops, bridging courses, mentoring, etc.).”, and 56 (71 %) replied ‘Yes’ and 22 ‘No’. Of those saying yes, 34 % have a bridging course, 25 % some form of tutoring arrangement, while 23 % mentioned mentoring, with one describing it as a “Personal academic mentoring program throughout degree for all mathematics students” and another saying “We tried a mentoring system once, but there was almost no uptake by students.” Other support structures mentioned included ‘study skills courses’, ‘maths clinics’, ‘support workshops’, ‘pre-course’, ‘remedial mathematics unit’, and a ‘Mathematics Learning Service (centrally situated), consulting & assignment help room (School of Maths). The MLS has a drop-in help room, and runs a series of seminars on Maths skills. These are also available to students on the web.’ Others talked of small group peer study, assisted study sessions, individual consultations, daily help sessions, orientation programmes and remedial courses. There is some evidence that bridging courses can assist in transition (Varsavsky 2010), by addressing skill deficiencies in basic mathematical topics (Tempelaar et al. 2012) and building student confidence (Carmichael and Taylor 2005). Other successful transition courses (e.g., Leviatan 2008) introduce students to the mathematical “culture” and its typical activities (generalizations, deductions, definitions, proofs, etc.), as well as central concepts and tools.

Overall the survey confirmed that students do have some difficulties in transition and these are occasionally related to a deficit in student preparation or mathematical knowledge. However, there are also a number of areas that universities could address to assist students, such as adjusting the content of first year courses, and instituting a course on proving and proof (where this doesn't already exist) and constructing appropriate bridging courses.

Literature Review

A number of different lenses have been used to analyse the mathematical transition from school to university. Some have been summarised well elsewhere (see e.g., Winsløw 2010) but we preface our discussion with a brief list of the major theoretical perspectives we found in the transition-related literature. One theory that is in common use is the Anthropological Theory of Didactics (ATD) based on the ideas of Chevallard (1985), with its concept of a *praxeology* comprising task, technique, technology, theory. ATD focuses on analysis of the organisation of praxeologies relative to institutions and the diachronic development of didactic systems. A second common perspective is the Theory of Didactical Situations (TDS) of Brousseau (1997), where *didactical situations* are constructed in which the teacher orchestrates elements of the didactical milieu under the constraints of a dynamic didactical contract. Other research uses the action-process-object-schema (APOS) framework of Dubinsky (e.g. Dubinsky and McDonald 2001) for studying learning. This describes how a process can be constructed from actions by reflective abstraction, and subsequently an object is formed by encapsulation of the process. The Three Worlds of Mathematics (TWM) framework of Tall (2008) is also considered useful by some. This describes thinking and learning as taking place in three worlds: the embodied; the symbolic; and the formal. In the embodied world we build mental conceptions using visual and physical attributes of concepts and enactive sensual experiences. In the symbolic world symbolic representations of concepts are acted upon, or manipulated, and the formal world is where properties of objects are formalized as axioms, with learning comprising building and proving of theorems by logical deduction from these axioms. We use the acronyms above to refer to each of these frameworks in the text below.

Calculus and Analysis

A number of epistemological and mathematical obstacles have been identified in the study of the transition from calculus to analysis. These include:

Functions: Students have a limited understanding of the concept of function (Junior 2006) and need to be able to switch between local and global perspectives (Artigue 2009; Rogalski 2008; Vandebrouck 2011). Using a TWM lens Vandebrouck

(2011) suggests a need to reconceptualise the concept of function in terms of its multiple registers and process-object duality. The formal axiomatic world of university mathematics requires students to adopt a local perspective on functions, whereas only pointwise (functions considered as a correspondence between two sets of numbers) and global points of view (representations are tables of variation) are constructed at secondary school. An ATD-based study of the transition from concrete to abstract perspectives in real analysis by Winsløw (2008) suggests that in secondary schools the focus is on practical-theoretical blocks of concrete analysis, while at university level the focus is on more complex praxeologies of concrete analysis and on abstract analysis.

Limits: Students need to work with limits, especially of infinite sequences or series. Two obstacles regarding the concept of infinite sum are the intuitive and natural idea that the sum of infinity of terms should also be infinite, and the conception that an infinite process must go through each step, one after the other and without stopping, which leads to the potential infinity concept (González-Martín 2009; González-Martín et al. 2011). According to Oehrtman (2009), students' reasoning about limit concepts appears to be influenced by metaphorical application of experiential conceptual domains, including collapse, approximation, proximity, infinity as number and physical limitation metaphors. However, only physical limitation metaphors were consistently detrimental to students' understanding. One approach to building thinking about limits, suggested by Mamona-Downs (2010), is the set-oriented characterization of convergence behaviour of sequences of that supports the metaphor of 'arbitrary closeness' to a point. Another, employing a TDS framework (Ghedamsi 2008) developed situations that allowed students to connect productively the intuitive, perceptual and formal dimensions of the limit concept.

Institutional factors: An aspect of transition highlighted by the ATD is that praxeologies exist in relation to institutions. Employing the affordances of ATD, Praslon (2000) showed that by the end of high school in France a substantial institutional relationship with the concept of derivative is already established. Hence, for this concept, he claims that the secondary-tertiary transition is not about intuitive and proceptual perspectives moving towards formal perspectives, as TWM might suggest, but is more complex, involving an accumulation of micro-breaches and changes in balance according several dimensions (tool/object dimensions, particular/general objects, autonomy given in the solving process, role of proofs, etc.). Building on this work Bloch and Ghedamsi (2004) identified nine factors contributing to a discontinuity between high school and university in analysis and Bosch et al. (2004) show the existence of strong discontinuities in the praxeological organization between high school and university, and build specific tools for qualifying and quantifying these. Also employing an institutional approach, Dias et al. (2008; see also Artigue 2008) conducted a comparative ATD study of the secondary-tertiary transition in Brazil and France, using the concept of function as a filter. They conclude that although contextual influences tend to remain invisible there is a need for those inside a given educational system to become aware of them in order to envisage productive collaborative work and evolution of the system.

Other areas: One TDS-based research project examined a succession of situations for introducing the notions of interior and closure of a set and open and closed set (Bridoux 2010), using meta-mathematical discourse and graphical representations to assist students to develop an intuitive insight that allowed the teacher to characterise them in a formal language. Another examined the notion of completeness (Bergé 2008), analysing whether students have an operational or conceptual view, or if it is taken for granted. The conclusion was that many students have a weak understanding of ideas such as the suprema of bounded subsets, convergence of Cauchy sequences and the completeness of **R**.

Some possible ways to assist the calculus-analysis transition have been considered. For example, Gyöngyösi et al. (2011) report an experiment using Maple CAS-based work to ease the transition from calculus to real analysis. A similar use of graphing calculator technology in consideration of the Fundamental Theorem of Calculus by Scucuglia (2006) made it possible for the students to become gradually engaged in deductive mathematical discussions based on results obtained from experiments. In addition, Biehler et al. (2011) propose that blending traditional courses with systematic e-learning can facilitate bridging of school and university mathematics.

Abstract Algebra

Understanding the constructs, principles, and eventually axioms, of the algebra of generalised arithmetic could be a way to assist students in the transition to study of more general algebraic structures. Focusing on students' work on solving a parametric system of simultaneous equations and the difficulties they experience with working with variables, parameters and unknowns, Stadler (2011) describes their experience of the transition from school to university mathematics as an often perplexing re-visiting of content and ways of working. The study showed that constructs of number, symbolic literals, operators, the '=' symbol itself, and the formal equivalence relation, as well as the principles of arithmetic, all contribute to building a deep understanding of equation. This agrees with the observations of Godfrey and Thomas (2008), who, using the TWM framework, provided evidence that many students have a surface structure view of equation and fail to integrate the properties of the object with that surface structure.

Students' encounter with abstract algebra at university marks a significant point in the transition to advanced mathematical formalism and abstraction, with concepts introduced abstractly, defined and presented by their properties, and deduction of facts from these properties alone. The role of verbalisation in this process, as a semantic mediator between symbolic and visual mathematical expression, may require a level of verbalisation skills that Nardi (2008, 2011) notes is often lacking in first year undergraduates.

Studies that focus on the student experience in their first encounters with key concepts in abstract algebra describe a number of difficulties. While some have suggested that an over-reliance on concrete examples of groups leading to a lack of skills in proof production, others, such as Burn (1996), recommend reversing the order of presentation, using examples and applications to stimulate the discovery of definitions and theorems through permutation and symmetry. An example of reducing group theory's high levels of abstraction (Hazzan 2001) is to ask students to construct the operation table for low order groups. This was also implemented by Larsen (2009) as a series of tasks exploring symmetries of an equilateral triangle, constructing low order group multiplication tables and culminating in negotiating preliminary understandings of group structure, the order of a group and isomorphism.

In an analysis of student responses to introductory group theory problem sheets, Nardi (2000) identified student difficulties with the order of an element, group operation, and the notions of coset and isomorphism. The duality underlying the concept of group and its binary operation, were also discussed by Iannone and Nardi (2002). They offer evidence of a student tendency to ignore the binary operation, consider the group axioms as properties of the group elements and omit checking axioms perceived as obvious, such as associativity. In addition, research by Ioannou (see Ioannou and Nardi 2009, 2010; Ioannou and Iannone 2011) considers students' first encounter with abstract algebra, focusing on the Subgroup Test, symmetries of a cube, equivalence relations, and employing the notions of kernel and image in the First Isomorphism Theorem. Provisional conclusions are that students' overall problematic experience of the transition to abstract algebra is characterised by the strong interplay between strictly conceptual matters, affective issues and those germane to first year students' wider study skills and coping strategies.

Linear Algebra

A sizeable amount of research in linear algebra has documented students' transition difficulties, particularly as these relate to students' intuitive or geometric ways of reasoning and the formal mathematics of linear algebra (e.g. Dogan-Dunlap 2010). The theoretical framework of Hillel (2000) for understanding student reasoning in linear algebra that identified geometric, algebraic, and abstract modes of description is valuable. For example, the relationship between linear algebra and geometry were at the core of Gueudet's research programme (2004, 2008; Gueudet-Chartier, 2004) that identified specific views on student difficulties. She claims that the epistemological view leads to a focus on linear algebra as an axiomatic theory, which is very abstract for the students and identifies a need for various forms of flexibility, in particular between dimensions. Further work at the geometry-formalism boundary by Portnoy et al. (2006) and Britton and Henderson (2009) has demonstrated some difficulties. First, pre-service teachers who engaged with transformations as geometric processes still had difficulty writing proofs involving linear transformations, and second, students experienced problems moving between a formal understanding of subspace and algebraic problem statements due to an insufficient understanding of the symbols used in the questions and in the formal definition of subspace.

Employing a framework using APOS theory in conjunction with TWM, Stewart and Thomas (2009, 2010; Thomas and Stewart 2011) analysed student understanding of various concepts in linear algebra, including linear independence, eigenvectors, span and basis. The authors found that generally students do not think of these concepts from an embodied standpoint, but instead rely upon a symbolic, process-oriented matrix manipulation manner of reasoning. However, employing a course that introduced students to embodied, geometric representations in linear algebra, along with the formal and the symbolic, appeared to enrich student understanding of the concepts and allowed them to bridge between them more effectively than with just symbolic processes.

Another aspect that has been investigated is students' intuitive thinking in linear algebra. Working with modelling and APOS frameworks Possani et al. (2010) leveraged students' intuitive ways of thinking through a genetic composition of linear independence and systems of equations. Student use of different modes of representation in making sense of the formal notion of subspace was analysed by Wawro, Sweeney and Rabin (2011a), and their results suggest that in generating explanations for the definition, students rely on their intuitive understandings of subspace, which can be problematic but can also help develop a more comprehensive understanding of subspace.

Some research teams have spearheaded innovations in the teaching and learning of linear algebra. For example, Cooley et al. (2007) developed a linear algebra course combined with learning about APOS theory and found the focus on a theory for how mathematical knowledge is generated enriched understanding of linear algebra. Another group of researchers used a design research approach simultaneously creating instructional sequences and examining students' reasoning about key concepts such as eigenvectors and eigenvalues, linear independence, linear dependence, span, and linear transformation (Henderson et al. 2010; Larson et al. 2008; Sweeney 2011). They argue that knowledge of student thinking prior to formal instruction is essential for developing thoughtful teaching that builds on and extends student thinking. In a study on tasks for developing student reasoning they (Wawro et al. 2011b) report how an innovative instructional sequence beginning with vector equations rather than systems of equations successfully leveraged students' intuitive imagery of vectors as movement to develop formal definitions.

Proof and Proving[2]

The transition to university mathematics includes a requirement for understanding and producing proofs. This requires logical deductive reasoning (Engelbrecht 2010) and rigour (Leviatan 2008). Research highlighting examples of this includes

[2] *At the time of writing the book* Proof and Proving in Mathematics Education: *The 19th ICMI study–Hanna & de Villiers,* 2012, *was still in press.*

conceptualisation related to the use of quantifiers (Chellougui 2004), the relationship between syntax and semantics in the proving process (Barrier 2009; Blossier et al. 2009) and logical competencies (Durand-Guerrier and Njomgang Ngansop 2010).

One recommendation is the need for more explicit teaching of proof, both in school and university (Balacheff 2008; Hanna and de Villiers 2008; Hemmi 2008), with some (e.g., Stylianides and Stylianides 2007; Hanna and Barbeau 2008) arguing for it to be made a central topic in both institutions. A possible introduction to proof, suggested by Harel (2008) and Palla et al. (2012) is proof by mathematical induction. However, they propose that it should be introduced slowly, building on students' own pre-existing epistemological resources (Solomon 2006) valuing both ways of understanding and thinking (Harel 2008), and distinguishing between proof schemes and proofs.

A number of potential difficulties in any attempt to place proving and proof more prominently in the transition years have been identified. These include the role of definitions, and the problem of student met-befores (Tall and Mejia-Ramos 2006). Using definitions as the basis of deductive reasoning in schools is likely to meet serious problems (Harel 2008; Hemmi 2008) since this form of reasoning is generally not available to school students, and Hemmi (2008) advocates the principle of *transparency*, which makes the difference between empirical evidence and deductive argument visible to students. In addition, the influence of student met-before can be strong, with Cartiglia et al. (2004) showing that the most recent met-before for university students, a formal approach, had a strong influence on their reasoning. A further difficulty, highlighted by Iannone and Inglis (2011), is a range of weaknesses in beginning university mathematics students' ability to produce a deductive argument, even when they were aware they should do so.

Some consideration has been given to methods of bridging the gap between the fields of argumentation and proof. One pedagogical strategy that may be an effective way to introduce the learning of proof and proving is student construction and justification of conjectures. The idea of an interconnecting problem was employed by Kondratieva (2011) to get students to construct and justify conjectures. Further, conjectures may also have a role during production of indirect argumentation (Antonini and Mariotti 2008), such as that in contradiction and contraposition, by activating and bridging significant hidden cognitive processes. Another approach discussed by Pedemonte (2007, 2008) employs the construct of *structural distance*, and she argues for an abductive step in the structurant argumentation in order to assist transition by decreasing the gap between argumentation and proof. Another proposition is that pivotal, bridging or counterexamples could assist students with proof ideas (Stylianides and Stylianides 2007; Zazkis and Chernoff 2008). A potential benefit of a counterexample is to produce cognitive conflict in the student, while a pivotal example is designed to create a turning point in the learner's cognitive perception. Counterexamples may also foster deductive reasoning, since deductions are made by building models and looking for counterexamples. For Zazkis and Chernoff (2008) a counterexample is a mathematical concept, while a pivotal example is a pedagogical concept, which is within, but pushing the boundaries of the set of examples students have experienced. The role of examples also arose in research by Weber and Mejia-Ramos (2011) on proof reading by mathematicians. This suggests that students might be taught how to use examples to increase their conviction in, or understanding of, a proof. In order to know what skills to teach students, Alcock and Inglis (2008) maintain that identifying different strategies of proof construction among experts will grow knowledge of what skills to teach students, and how they can be employed.

Mathematical Modelling and Applications

Mathematical modelling and applications continues to be a central theme in mathematics education research (Blum et al. 2002), with a primary focus on practice activities. However, it appears that little or no literature exists explicitly discussing these topics with a focus on the 'transition' from the secondary to the university levels, possibly because there have been no roadmaps to sustained implementation of modelling education at all levels. Hence, recent literature relevant to the secondary-tertiary transition issue is briefly considered here.

One crucial duality, mentioned by Niss et al. (2007), is the difference between 'applications and modelling for the learning of mathematics' and 'learning mathematics for applications and modelling'. This duality is seldom made explicit in lower secondary school, and instead both orientations are simultaneously insisted on. However, at upper secondary or tertiary level the duality is often a significant one. The close relationship between modelling and problem solving is taken up by a number of authors. For example, English and Sriraman (2010) suggest that mathematical modelling is a powerful option for advancing the development of problem solving in the curriculum. In addition, according to Petocz et al. (2007), there are distinct advantages to using real world tasks in problem solving in order to model the way mathematicians work. This is supported by the research of Perrenet and Taconis (2009), who describe significant shifts in the growth of attention to metacognitive aspects in problem solving related to the change from secondary school mathematics problems to authentic mathematics problems at university. One difficulty outlined by Ärlebäck and Frejd (2010) is that upper secondary students have little experience working with real situations and modelling problems, making the incorporation of real problems from industry problematic. A second possible difficulty (Gainsburg 2008) is that teachers tend not to make many real-world connections in teaching. One possible solution is to bring together combinations of students, teachers and mathematicians to work on modelling problems (Kaiser and Schwarz 2006). This opportunity may be created through a "modelling week" (Göttlich 2010; Heilio 2010; Kaland et al. 2010), during which small groups of school or tertiary students work intensely, in a supported environment, on selected, authentic modelling problems.

There is some agreement that the secondary school curriculum could include more modelling activities, although high-stakes assessment at the secondary-tertiary interface is an unresolved problem in any implementation (Stillman 2007). Other initiatives for embedding modelling in the curriculum proposed by Stillman and Ng

(2010) include a system-wide focus emphasising an applications and modelling approach to teaching and assessing mathematical subjects in the last two years of school and interdisciplinary project work from primary through secondary school, with mathematics as the anchor subject.

Conclusion

The literature review presented here reveals a multi-faceted web of cognitive, curricular and pedagogical issues, some spanning across mathematical topics and some intrinsic to certain topics—and certainly exhibiting variation across the institutional contexts of the many countries our survey focused on. For example, most of the research we reviewed discusses the students' limited cognitive preparedness for the requirements of university-level formal mathematical thinking (whether this concerns the abstraction, for example, within Abstract Algebra courses or the formalism of Analysis). Within other areas, such as discrete mathematics, much of the research we reviewed highlighted that students may arrive at university with little or no awareness of certain mathematical fields.

The review presented in this report, as well as the longer version, is certainly not exhaustive. However we believe it is reasonable to claim that the bulk of research on transition is in a limited number of areas (e.g. calculus, proof) and that there is little research in other areas (e.g. discrete mathematics). While this might simply reflect curricular emphases in the various countries that our survey focused on, it also indicates directions that future research may need to pursue. Furthermore across the preceding sections a pattern seems to emerge with regard to *how*, not merely *what*, students experience in their first encounters with advanced mathematical topics, whether at school or at university. Fundamental to addressing issues of transition seems also to be the coordination and dialogue across educational levels—here mostly secondary and tertiary—and our survey revealed that at the moment this appears largely absent.

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Socioeconomic Influence on Mathematical Achievement: What Is Visible and What Is Neglected

Paola Valero, Mellony Graven, Murad Jurdak, Danny Martin, Tamsin Meaney and Miriam Penteado

Abstract The survey team worked in two main areas: Literature review of published papers in international publications, and particular approaches to the topic considering what in the literature seems to be neglected. In this paper we offer a synoptic overview of the main points that the team finds relevant to address concerning what is known and what is neglected in research in this topic.

Keywords Poverty · Early childhood · Intersectionality of positionings · Statistical reifications · Macro-systemic perspective · History of mathematics education practices

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Introduction

It is known that socioeconomic factors have an influence on mathematical achievement. Nowadays such link has become a “fact” that researchers, teachers, administrators and politicians have at hand: “the better off you—and your family—are, the more likely you will do well in school, including mathematics”. Such a statement embodies its opposite: “the worse off you—and your family—are, the more likely you will do poorly in school and mathematics”. Studies defining socioeconomic status (SES) and showing its relation to school performance emerged at the beginning of the 20th century. The specification of the relationship for school mathematics was enunciated as a problem for society and for research in the 1960s. However, it is only in the 1980s that such issue started to be a focus of attention of the mathematics education community. What is known so far—which may be part of a commonsense understanding of the topic—and what seems to be forgotten—which are critical readings challenging the commonsense—were the central questions that have guided the work of the survey team.

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What Is Visible

A global literature review for this topic poses challenges such as the multiple languages in which research reports are made available. We gathered literature that would indicate some trends in what is known about the socioeconomic influences on mathematical achievement in different parts of the world. Most of what was reviewed was published in English.

At a general educational level, the relationship between socioeconomic factors and school achievement is inserted in the history of expansion of mass education systems and differential access to education around the world during the 20th century. Meyer et al. (1992) show that the consolidation of Modern nation states is correlated to the expansion and Modern organization of mass systems of education. Many nation states growingly focused on the socialization of citizens with a vision of progress in which the scientific rationality was an articulating element. The link between personal development and the mastery of the curriculum, and such individual mastery and the progress of the nation were established. With the expansion of mass education, the issue emerged of who has access to education and the goods of society and on the grounds of what. To know who was having effective access to education became important.

The report "Equality of Educational Opportunity" (Coleman et al. 1966) was one of the first large-scale national surveys that formulated a model to determine the extent to which educational opportunities were equally available to all citizens in the USA. It allowed individual students' socio-economic, racial and ethnic characteristics to be connected to school inputs in terms of resources available to run education, and to students' individual performance in achievement tests in different school subjects. Internationally, the International Association for the Evaluation of Educational Achievement (IEA) started providing international comparative information about how different national curricula provide different opportunities to learn, and the existence of a lack of equity between different groups of students. Since then, the measurement of educational quality was moved from an input-output model based on school resources to an individualization of the measurement of educational quality in terms of students' achievement, even in mathematics. This fundamental change in the general reports on educational access is central for connecting socio-economic influences with mathematical achievement.

The discussion on what may be the socioeconomic influences on mathematical achievement emerged from general social science research and educational research. Therefore what has become visible about the topic is found in general reports on educational systems around the world, as much as in mathematics educational research literature. Thus any talk about the topic in the realm of mathematics is bound to general discussions about social and educational disparities for different types of students.

At the level of mathematics education research the concern for this connection emerged as a research topic in the 1980s. The studies that address this issue are mainly quantitative and to some extent large scale. It is important to mention that the amount of literature testing different hypothesis about socio economic influences and achievement has increased with the growing importance given to periodic, international, standardized, comparative studies such as TIMSS and PISA since the 1990s.

In different parts of the world there are results about a society's sense of expected, normal school achievement and how different groups of students are compared to the normal expectation. While in the USA, factors that systematically generates differentiation to the expected norm are socioeconomic status (SES) and race, in other countries it is socioeconomic status as in for example in the UK and Australia, or home language and ethnicity in the case of some European countries such as Germany and Denmark, or rurality as in China or many of the African and Latin American countries. Although other factors are also present, the tendency of countries to focus on one factor has influenced the way discussions operate in these countries. In different countries the independent variables considered to be the socioeconomic influences on mathematical achievement —the dependent variable —change. What may be considered the 'socioeconomic' influences on 'mathematical achievement' depends on the systems of differentiation and stratification of the population. It is not any kind of existing, a priori characteristic of individuals and groups of students or of mathematical achievement per se.

Once the general differentiation is possible, similar statistical indicators are adopted in the studies. Prior to the existence of international comparable, standardized national data sets, the variable of socioeconomic status has been one of the most used in the studies. Since its construction in the 1920s, the measurement has been composed by a series of reliable indicators—parents' educational level, family income, possession of appliances, possession of books, etc.—which have not changed much in almost 100 years. The tendency to simplify the measurement is connected to how difficult it is to collect reliable information on this matter from children. The assignment of a socio-economic level to individual students often takes place on very thin evidence. The effect of the measurement, on the contrary, has the tendency to reify a solid state that follows individual children all through their school life. This reification has been documented in studies that have addressed how the discussion of students' differential results is dealt with in the media and public debates.

Even if many studies have a tendency to establish the relationship between a limited variables indicating differential positioning, many studies conclude that those variables intersect. This means that students whose participation in school mathematics results in low achievement experience differential positioning in schooling because they are attributed simultaneously several categories of disadvantage. For example, low achievement in mathematics in certain regions in China is explained by the intersection of rurality, parents' educational level, mother-at-work, and language (Hu and Du 2009). In other words, existing studies devise sophisticated statistical measurements to trace the factors that correlate to differential access to mathematical achievement. However, the very same statistical rationality on which those studies are based imposes a restriction for understanding how the complexity of the intersectionality of variables of disadvantage effect differential results in mathematics.

There is an over-representation of research reports addressing the socioeconomic influences on mathematical achievement in English speaking countries (USA, UK, Australia and New Zealand), while there is little research on this matter in many other places in the world. Such difference may not only be due to the extent of research in mathematics education in these countries, but also to the fact that differential achievement has not been construed as a problem. In East Asia there is little research in mathematics education investigating those who do not perform highly and why. In Taiwan research discards the focus on socio-economic variables and privileges variables such as student's learning goal orientation (Lin et al. 2009). Researchers argue that it is more meaningful to study what educators can impact positively to improve students' results. In South Korea the differentiated achievement is explained in terms of access to private tuition, which reflects a difference in resources that educational policies cannot compensate for (Kang and Hong 2008). In India, it is argued that differential achievement is due to students' mathematical aptitude, gender and urbanity/rurality, the socio-economic and cultural characteristics of communities, and the impact of child work for the lower castes and poorer communities (World Bank 2009).

Existing research both in general education and in mathematics education has constructed the positive correlation between a lower positioning of groups of students with respect to the valued norm of societies, and the results of the school mathematical experience measured in terms of achievement. Poverty, rurality, ethnicity, gender, language, culture, race, among others, have been defined as the variables that constitute socioeconomic influences on mathematical achievement. The question remains whether it is possible to interpret the meaning of “socio-economic influences” and “mathematical achievement” in ways that allow us to go beyond the facts established in the last 50 years of research. In the following sections each one of the members of the team offers a perspective on this issue.

What Is Neglected

Paola Valero on Historicizing the Emergence of Differential Access to Mathematics Education

I discuss the historical conditions that make it possible to formulate the “socioeconomic influences on mathematical achievement” as a problem of research in mathematics education. How and when the problem has been made thinkable, up to the point that nowadays it is part of the commonsense or taken-for-granted assumptions of researchers and practitioners alike? My strategy of investigation builds on thinking the field of practice of mathematics education as a historical and discursive field. There are at least three important conditions that make the problem possible:

Education, Science and the Social Question. The social sciences and educational research are expert-based technologies for social planning. In the consolidation of Modernity and its cultural project in the 20th century, the new social sciences were seen as the secular rationality that, with its appeal to objective knowledge, should be the foundation for social engineering. Statistical tools in the social sciences allow generating constructs that identify the ills of society that science/education needs to rectify. This is an important element in how educational sciences address the differential access of children to the school system. Constructs, such as students’ “socio-economic status”, later on expanded to school and communities socio-economic status, emerged in the 1920s in a moment where the newly configured social sciences started to address the “problems” of society. Educational sciences made it possible to articulate salvation narratives for facing the social problems for which education was a solution (Tröhler 2011). Measurements of intelligence, achievement and socio-economic status were and still are technologies to provide the best match between individuals and educational and work possibilities. The double gesture of educational sciences of promoting the importance of access to education and reifying difference by constructing them as a fact inserts human beings in the calculations of power.

Mathematics and progress. During the second industrialization the justification for the need for mathematics education was formulated clearly in the first number of

L'Enseignement Mathématique. In the times of the Cold War, the justification was related to keeping the supremacy of the Capitalist West in front of the growing menace of the expansion of the Communist Soviet Union. Nowadays, professional associations and economic organizations argue that the low numbers of people in STEM fields can severely damage the competitiveness of developed nations in international, globalized markets. The narrative that connects progress, economic superiority, and development to citizen's mathematical competence is made intelligible in the 20th century. The consolidation of nation states and the full realization of the project of Modernity required forming particular types of subjects. The mathematics school curriculum in the 20th century embodied and made available cosmopolitan forms of reason, which build on the belief of science-based human reason having a universal, emancipatory capacity for changing the world and people (Popkewitz 2008). In this way, subjects are inserted in a logic of quantification that makes possible the displacement of qualitative forms of knowing into a scientific rationality based on numbers and facts for the planning of society. Thus, from the turn of the 19th century to present day, the mathematics curriculum is an important technology that inserts subjects into the forms of thinking and acting needed for people to become the ideal cosmopolitan citizen.

Mathematics for all. That high achievement in mathematics is a desired and growing demand *for all* citizens is a recent invention of mathematics education research. Between the years of reconstruction after the Second World War and the Cold War, school curricula were modernized with focus on the subject areas for the purpose of securing a qualified elite of college students. In the decade of the 1980s the new challenge of democratization and access was formulated. At the "Mathematics Education and Society" session at ICME 5 it was publicly raised the need to move towards inclusion of the growing diversity of students in school mathematics (Damerow et al. 1984). The systematic lack of success of many students was posed as a problem that mathematics education research needed to pay attention to and take care of. Mathematics education researchers, the experts in charge of understanding the teaching and learning of mathematics as well as of devising strategies to improve them, took the task of providing the technologies to bring school mathematics to the people, and not only to the elite. "Mathematics for all" can be seen as an effect of power that operates on subjects and nations alike to determine who are the individuals/nations who excel, while creating a narrative of inclusion for all those who, by the very same logic, are differentiated.

It is on the grounds of at least these three interconnected conditions that the "socioeconomic influences on mathematical achievement" has been enunciated as a problem of research in the field. I do not intend to say that underachievement is an unimportant "social construction". My intention is to offer a way of entering into the problem that makes visible the network of historical, social and political connections on which differential social and economic positioning is related to differential mathematical achievement.

Mellony Graven on Socio-Economic Status and Mathematics Performance/Learning in South African Research

South Africa's recent history of apartheid and its resultant high levels of poverty and extreme social and economic distance between rich and poor continue to manifest in the education of its learners in complex ways. The country provides a somewhat different context for exploring the relationship between SES and education than other countries. The apartheid era only ended in 1994 with our first democratic elections. Education became the vehicle for transforming South African society and a political rhetoric of equity and quality education for all emerged. Thus educational deliberations focused on redressing the inequalities of the past and major curriculum introductions and revisions were attempted. Engagement with SES and mathematics education became *foregrounded* in policy, political discourses and a range of literature since 1994 although in must be remembered that transformation of education was a priority of the eighties period of resistance and the people's education campaign (although heavily suppressed at the time). Yet for all the political will and prioritization little has been achieved in redressing the inequalities in education.

Much of the recent data available on the relationship between SES and mathematics performance can be 'mined' from large scale general education reviews. These studies provide findings indicating patterns or correlations between school performance and socio-economic context. Several indicate that correlations are exacerbated in mathematics. These reports highlight a range of factors or areas that affect learner performance, such as social disadvantage, teachers' subject knowledge, teaching time, teacher absenteeism, resources, poorly managed schools, poverty effects including malnutrition and HIV/AIDS. In general reports present a consistent picture. In South Africa, since poverty affects more than half of our learners, studies tend to focus on the poorest (but largest) SES group when looking at challenges in education. Many reports point to numeracy scores and mathematics results being consistently below other African neighbour countries with much less wealth. Furthermore, South Africa has the highest levels of between-school performance inequality in mathematics and reading among SACMEQ countries.

What might be somewhat different from other countries exploring SES and mathematics achievement is that South African poverty levels are extreme even while there is relative economic wealth. Fleisch (2008) argues that poverty must be understood in its full complexity and not only in economic terms and argues for "the need to understand the underlying structural dimensions of persistent poverty, which engages the complexities of social relations, agency and culture, and subjectivity" (p. 58). He also notes that "Poor families rather than being just a source of social and cultural deficit, are important supporters of educational success […] poor South Africans share with the middle class an unqualified faith in the power of education. For poor families education is the way out of poverty, and as such many spend a large portion of their disposable income on school fees, uniforms and transport […]" (p. 77)

Mathematics education research conducted in South Africa almost inevitably touches on issues of equity and redress when engaging with the context of studies. One important area is research on language and mathematics education. The overlap between language of learning with SES and mathematics achievement is referred to in almost all of the large quantitative studies above (as a correlating factor) and the data provides for a complex picture that cannot easily be explained in terms of causal relationships. Setati and collaborators (e.g., Barwell et al. 2007) urge that multilingualism needs to be reconceptualised as a resource rather than a disadvantage. In this way the deficit discourse around multilingualism and how it negatively correlates with mathematics performance should be reframed. Most language 'factors' referred to in the literature above position multilingualism as a factor that correlates with low mathematics performance but this should not be read as causal.

Recent research by Hoadley (2007) analyses how learners are given differentiated access to school knowledge in mathematics classrooms. She argues that the post-apartheid curriculum with its emphasis on everyday knowledge has had a disempowering effect in marginal groups who are not exposed to more specialised knowledge of mathematics. The result is that "the lower ability student, paradoxically, is left free to be a local individual but a failed mathematics learner" (Muller and Taylor 2000, p. 68). In its implementation teachers in low SES schools struggled to make sense of these changes resulting in even further mathematics learning gaps between 'advantaged' and 'disadvantaged' learners (Graven 2002). The result has been that "students in different social-class contexts are given access to different forms of knowledge, that context dependent meanings and everyday knowledge are privileged in the working-class context, and context-independent meanings and school knowledge predominate in the middle class schooling contexts" (Hoadley 2007, p. 682).

While studies relate poverty, class, race and access to English to differentiated learning outcomes from a variety of perspectives, most, I would argue, are not sufficiently concerned with the impact of extreme income inequality within a context of widespread and deep absolute poverty. Many poor countries achieve much better educational outcomes compared to South Africa but have lower levels of inequality. A deeper understanding of inequality as a core component of SES, and not just of the nature and impact of poverty might enrich our understanding of the relationship of SES to mathematical educational outcomes.

Murad Jurdak on a Culturally-Sensitive Equity-in-Quality Model for Mathematics Education at the Global Level

Equity, quality, and cultural relevance are independent dimensions in mathematics education. I refer to this 3-dimensional framework as *culturally-sensitive, equity-in-quality in mathematics education*. In the period 1950–2008 the agendas of equity and quality in education, and of mathematics education have moved in different directions. While the provision for universal primary education was paramount between 1950 and 2000, educational quality received low priority during that period. In the first decade of the 21st century, quality education for all has emerged as a top priority. On the other hand, mathematics education literature shows that the evolution of mathematics education was dominated by quality concerns in scholarly discourse between 1950 and 1980. The social and cultural aspects of mathematics education started to emerge as legitimate research in the 1980s. Towards the end of 1980s, equity became a major concern in mathematics education. The first decade of the 21st century witnessed the beginning of convergence towards an increased emphasis on achieving equal access to quality math education (Jurdak 2009).

In the last half of the past century, the decline of colonization was a major reason for the emergence of the two-tiered system of mathematics education. During colonization, many developing countries adopted the mathematics education of their colonial rulers. However, as colonization dismantled, the developing countries invested most of its resources in increasing coverage at the expense of the quality of education, and educational research and development. Thus developing countries did not accumulate enough 'credentials' in mathematics education to fully participate in the international mathematics education community. This situation led to the formation of a two-tiered system of math education at the global level. The upper tier, referred to as the *optimal mode of development*, includes the developed countries that are integrated in the international mathematics education community. The lower tier, referred to as the *separate mode of development*, consists of the marginalized countries which have yet to be integrated in the international activities of mathematics education.

The majority of countries having average or high quality index (measured in terms of national achievement in TIMSS 2003) and low or average inequity index (measured in terms of size of between-school variation) generally fit the optimal mode of development. These countries have high or average mathematics achievement performance, contribute significantly to international research in mathematics education, and assume leadership roles in international mathematics education organizations and conferences. On the other hand, the majority of countries having low quality index in mathematics education, irrespective of its equity index, fit in the separate mode of development. These countries have low mathematics performance, have little contribution to international research in mathematics education, and normally have humble participation in international mathematics education conferences, such as the ICME. In other words, they are marginalized by the international mathematical education community and left to follow their own path in developing their mathematics education. Some of these countries use the preservation of cultural values as an argument to rationalize the lack of their integration in the international mathematics education community. Other countries do not have the resources to participate and contribute to the international math education community.

A country classified as fitting in the separate mode of development of mathematics education is likely to be relatively poor, low in the spread and level of education among its population, and belongs to a socioeconomically developing region (Arab states, Latin America, and Sub-Saharan Africa). On the other hand, a country classified as following the optimal mode of development of mathematics education is likely to be relatively rich, high in the spread and level of education among its population, and is part of a developed region (North America, Western and Eastern Europe, East Asia and the Pacific). There seems to be a divide between developing and developed countries in mathematics education, and some of the significant factors that contribute to that divide (socioeconomic status of a country, its educational capital, and its culture) seem to be beyond the sphere of influence of local or international mathematics education communities, whereas the other factors are not. For example, policies that govern international organizations and conferences may be addressed by the international mathematics education community.

The international mathematics education community has a responsibility to find ways to encourage and enable mathematics educators to be integrated in the international mathematics education community. The participation in and contribution to international mathematics education conferences and international mathematics education journals are critical for such integration. One measure in this regard would be to favour the participation of mathematics educators from developing countries. Writing and presenting in English is a major barrier to the participation of many mathematics educators in international conferences. Some form of volunteered mentoring by their colleagues who can provide their support in reviewing and editing manuscripts could be a desirable strategy. Providing opportunities for presentations in international conferences in languages other than English would broaden access to such conferences. All these measures may hopefully help enhance the integration of more mathematics educators in the international community.

Danny Martin on Politicizing Socioeconomic Status and Mathematics Achievement

In the United States discussions about the relationships between SES and schooling processes and outcomes—persistence, achievement, success, failure, opportunity to learn, access to resources, and so on—are long and enduring. These discussions have surrounded mathematics education—more so than being generated and sustained by mathematics educators—as much of the research and policy generated to support various positions about socioeconomic status has been produced in fields like sociology, economics, critical studies, and public policy.

In many of these studies there is often a deficit-oriented narrative that is generated and reified about "poor" children and families, while normalizing certain middle- and upper-class children and families. SES is often used as a proxy for "race" but the discussions are often unwilling to explore the impact of racism in generating socioeconomic and achievement differences. The dialectic between race and social class is important. In fact, a number of dialectics are important with respect to SES as one considers its racialized, gendered, and contextual nature. The processes undergirding its formation and strata in a given historical and political context may help to explain outcomes like school achievement in ways that are more insightful than just placing human bodies into various socioeconomic strata and characterizing their achievement in relation to human bodies in other strata.

There have been recent reports that consider race, class, gender, ethnicity, and language proficiency in relation to mathematics education (e.g., Strutchens and Silver 2000; Tate 1997). They support the intuitive finding that higher socioeconomic status is associated with increased course-taking and higher achievement on various measures of mathematics achievement. However, the story is less clear when one considers that many "Asian" students from the lowest socioeconomic levels in the U.S. outscore White and other students at the highest socioeconomic levels. Moreover, many of these reports leave unexplained high achievement among African American, Latino, and Native American students, who are disproportionately represented among the lower socioeconomic levels in the U.S.

I would argue that while SES is positively correlated with achievement, mathematics education research in the U.S. context still has far to go in addressing the complexity of these issues. Tate (1997), for example, noted that in defining and operationalizing socioeconomic status, "Typically the mathematics-achievement literature is organized according to a hierarchy of classes—working class, lower-middle class, middle class, and so on. This hierarchy often objectifies high, middle, and low positions on some metric, such as socioeconomic status (SES)" (p. 663). This objectification presents SES as static and uncontested and not influenced by larger political and ideological forces.

There is complexity that goes unexplored even within the socioeconomic strata that are used. In the U.S. it is generally true that even among poor and working class "Whites" and "Blacks", within-class racism often mitigates the opportunities of Blacks. Across economic strata, the sociology and economics of schooling suggest that "Whites" often enjoy the capital associated with their "Whiteness" even in a supposed meritocracy that many claim and wish for in our society (e.g., Jensen 2006). I would argue that such considerations extend to mathematics education to affect the conditions under which students learn and in which opportunities unfold or are denied.

My particular orientation is to move "race" to a more central position in the conversation on SES within the U.S. context (Martin 2009). It might be argued that "race" is not a central concern in other national and global regional contexts. I would disagree based on the histories of nationalism, colonialism, xenophobia, anti-Muslim sentiments, and anti-multiculturalism throughout Europe, South America, and other locations. Every context, without exception, experiences a historically contingent "racial" ordering of its society that also structures its socioeconomic ordering. Research on the global contexts of racism(s), in all its forms, makes this point clear for the U.S., Europe, Brazil, Asia, and so on. So, while it may not be an issue of "White" and "Black" in a particular location, there are likely to be some other forms of "race" and "racism" that are at play (including differences that result from "lighter" and "darker" skin), whether they be manifested in the lives of Indians living in Singapore, the ideologies of the Danish People's Party (DF) in Denmark, or the rise of xenophobic nationalism throughout Europe.

We know that SES does not explain all of the variation in achievement and does not explain why some "poor" or low SES children in a given context succeed academically and why some "rich" or high SES children do less well. Analyses of SES often treat it as a static variable and often do not examine human agency or the manipulation of SES by those in power. SES is intimately linked to other variables that may impact schooling processes and achievement. These other variables include gender; geographic location; language status; immigrant status and the prevailing racial context in given society including nationalism, anti-immigrant sentiment, xenophobia; quality of health care and pre-school systems; history of colonialism; the prevailing political context and ideologies that dominate that context; larger economic system; and so on.

I argue for a more politicized view of SES that takes into account race and racism, political projects, socioeconomic projects and manipulation, among others. SES may be conceptualized differently in different contexts. The common reporting line "the more economic resources one has, the greater their achievement is likely to be" is not an interesting finding even if it gets repeated in research. It does not explain why some have more resources than others. We, in mathematics education, should continue to trouble that imbalance.

Tamsin Meaney on Back to the Future? Mathematics Education, Early Childhood Centres[1] and Children from Low Socio-Economic Backgrounds

In the last two decades, early childhood has become the focus for much discussion in regard to overcoming inequalities in educational outcomes between groups. Although there is a perception that such a connection has only been newly recognised, the history of early childhood centres shows otherwise. For example, May (2001) outlined how preschools in New Zealand have changed dramatically from being charitable organisations for the urban poor in the late nineteenth century to now being seen as essential for all children, to the extent that children who do not attend are perceived as likely to be problems for society. The right to determine the appropriate care for young children through education arose during the history of early childhood centres.

[1] Throughout history and across the world, different names have been given to institutions set up outside of homes for the care and education of young children. To overcome this confusion, the term early childhood centres has been adopted.

> An activity such as preschool, like most of the welfare institutions, is marked by its history. There is a clear relationship between a country's traditions in preschool and school system and its administration and integration of new challenges and demands. (Broman 2010, p. 34; own translation)

I suggest that the history of early childhood centres as carers and educators of poor children has produced different sorts of mathematical education programmes. The physical care of young children, who are seen as unable to look after themselves, always has been part of the role of early childhood centres. As well, characteristics of the child, from their character to their imagination, have been perceived as being in need of moral care. Education, including mathematics education programmes, reflected these different perceptions of moral care. Many instigators of early childhood centres have considered that education could overcome faults in children, particularly poor children. Table 1 provides a summary of the main early childhood centres for the last two hundred years and the sorts of moral care and education provided to children.

In recent years, a moral deficiency that early childhood centres are supposed to overcome is a lack of school readiness in regards to mathematics knowledge. An analysis by Greg Duncan and colleagues of six longitudinal studies suggested that early mathematics knowledge is the most powerful predictor of later learning, including the learning of reading (Duncan et al. 2007). The mathematical programmes, now being advocated in early childhood centres, reflect society's wish to

Table 1 Summary of the kind of care and education provided in early childhood centres

	Time	Care	Education	Mathematics
Robert Owen—Infants School	Early 19th century	Care of the character	Broad curriculum	Arithmetic from manipulating objects from nature
Frederick Frobel—Kindergarten	1837 to end of 19th century	Spiritual care could only occur in schools	Playful and based on children's own interests	Geometry and other math learnt through engagement with gifts and occupations
Margaret McMillan—Nursery Schools	Early 20th century	Care of the imagination	Physical and mental development through play	Math learning was incidental to using their imagination to explore the world
Maria Montessori—Children's houses	Early 20th century	Care for children's personalities	Learning though the senses, using children's interests. School preparation	Materials were math in they required comparisons
Diversity of approaches	Middle to late 20th century	Care for psychological well-being	Learning to play with other children	Experiences were valuable for later school math learning
Present day	1990s to present	Care for academic well-being	Content becomes the focus of education	Math concepts have become the focus of preschool programs

care for poor children's academic needs, which are considered to be at risk and which could result in them being non-productive workers in the future (Pence and Hix-Small 2009). If all children could receive a quality early childhood education then the risk of society having citizens with insufficient education and unable to gain jobs would be alleviated.

A consequence of the acceptance of early childhood centres' right to determine the education necessary to appropriately care for young children is leading to the imposition of a homogenised view of young children, including as young mathematics learners. Providing mathematics programs for this homogenised child can result in a lack of recognition and undervaluing of what poor children bring to early childhood centres. Although the jury is still out on the long-term effectiveness of present structured mathematics programmes, an education that does not recognise nor value children's transition back into their home communities (Meaney and Lange 2013) will result in some children becoming failures before they begin school.

Miriam Penteado on Mathematics Education and Possibilities for the Future

The Brazilian educational system is organized as shown in Table 2 below. For both basic school and the higher education there are two parallel systems: the private and the public. Concerning basic schools, in general, private schools have more status and offer better learning and teaching conditions for students. On the other hand, public schools include the majority of the Brazilian population. The teaching and learning conditions in public schools is very poor. Many schools are in bad structural condition and there are cases of no electricity, no potable water, etc. It is known that Brazilian public schools students study less content than those in private schools. Furthermore, in Brazil there is lack of teachers. It is difficult to find people who want to be educated as a teacher, and there is a set of reasons for this: low salary, low social status, and violence. The best teacher students who graduate are hired in private schools with better working conditions than in public schools.

Concerning higher education the situation is the opposite of what happens in basic schools. *Public universities* are those with the highest investment in research and teaching. In fact, in the last years part of the policy of the Brazilian government has been to increase the investment in higher education making available to the system a considerable amount of resources. It is more difficult to gain enrolment as undergraduate student in public universities than in private, especially in more

Table 2 The Brazilian educational system

Basic school	Primary and secondary level (9 years—from 6 to 14 years old)
	High school level (3 years—from 15 to 18 years old)
Higher education	Different length

prestigious courses such as medicine or engineer. For this reason, those who attend private schools are more likely to become a student at a public university. Many students from public school do not even dream of having further education at a public university. The choice (when it is the case) is to work during the day and take a course in the evening at a private faculty.

Considering the situation it is possible to state that a person with high socio-economic background follows the route: from private school to public university. One with low socioeconomic background follows the route: from public school to private faculty. There is financial governmental support for students from public schools to study in private faculties. Only a small percentage of the Brazilian population has further education at the tertiary level (private or public system). According to the OECD[2] the number of Brazilian people within 25–64 years old who has completed tertiary education has increased to 11 %. However it is still low when compared with other countries.

Public universities are trying to facilitate access for students from low SES, however it is not for any career. As an example, one can use a socio economic report of a public university in Sao Paulo State for the year 2010. The distribution of students in relation to their background (basic school in the private or the public system) in university courses such as medicine and mathematics is very different. While students who enter medicine have studied in private institutions (85.9 % of students have attended a private primary school and 94.6 % have attended a private high school), the majority of mathematics students (future mathematics teachers) have studied primary and high school in public institutions (an average of 72.5 % for public primary schools, and 74.6 % for public high schools). Thus one can see that medicine does not function as any social-ladder, while mathematics has the possibility to do so.

That socioeconomic factors influence students' educational life is common sense. Given this, one could think that there is not so much to say about the survey theme. However, this common sense could be challenged. When working with students in so-called disadvantaged context one can consider the question: What possibilities could be constructed together with the students?

It is important for a mathematics education to create new possibilities for students. Creating possibilities for students could mean thinking of the opportunities they might obtain for the future. One could think as students' possibilities for, later on in life, to participate as (critical) citizens in political issues. To consider the conditions for coming to "read and write" the world, to use an expression formulated by Paulo Freire (1972).

There might exist a tendency to consider low achievement related to the students and to their background. And from this perspective one can start discussing strategies for compensating the, say "low cultural capital". One can pay attention to the general living conditions of the students, including their conditions of getting to school. One can consider their learning with reference to their worlds and their foregrounds.

[2] http://www.oecd-ilibrary.org/economics/country-statistical-profile-brazil_csp-bra-table-en.

One can claim that it is an important aim for mathematics education to help to establish possibilities within the horizon of students' foregrounds (Skovsmose 2005). To make them recognise that: This could also be for me!

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