Part VI
National Presentations

National Presentation of Korea

Sun-Hwa Park

Introduction

Korea, as represented by the Korea Institute for Curriculum and Evaluation (KICE), is pleased to present a National Presentation at ICME-12. We think this is a precious opportunity where we can introduce the mathematics education in Korea. Following an overview of the mathematics education in Korea, we plan to present the policies and many efforts we devote to improve our mathematics education.

Presentations (Two Sessions)

Session I (80-min Session)

1. Overview of Mathematics education in Korea
2. The National Mathematics Curriculum of Korea
3. The Development and Characteristics of Korean Mathematics Textbooks
4. Teaching and Learning practices of mathematics classroom in Korea
5. The educational practices for the Mathematically-gifted and the underachieving students

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Session II (70 min Session)

6. National Assessment of Educational Achievement
7. College Scholastic Ability Test (CSAT) in Korea
8. Achievement on International Assessment in Korea
9. Mathematics Assessment at school level
10. Mathematics Teacher Education in Korea

Overview of Mathematics Education in Korea

Here we provide an overview of the mathematics education in Korea that consists of five areas: mathematics curriculum, textbooks, teaching and learning, educational evaluation, and teachers' education. First, we introduce the national curriculum of mathematics and the revision that has been made throughout the history of curriculum. Second, we introduce the mathematics textbooks that are used at each school level. Third, various types of elementary and secondary classroom mathematics teaching will be reviewed. Fourth, we discuss the three kinds of mathematics assessment: National Assessment of Educational Achievement (NAEA), College Scholastic Aptitude Test (CSAT), and international student assessments. Korea has been one of the top performing countries in students' mathematics achievement in the international assessment such as PISA and TIMSS. Behind its shining accomplishments, the mathematics education in Korea also has some noticeable afflictions which remain for us to resolve. Here we attempt to obtain constructive advice from other countries on our difficulties and problems both in theoretical and in practical perspectives.

The National Mathematics Curriculum of Korea

Korea has a national curriculum system. After the curriculum is announced at the national level, the writers of textbooks begin to develop textbooks on the basis of the national curriculum. Once the development of textbooks is completed, new curriculum is implemented in the school. It takes about 2 years to apply the newly announced national curriculum to the school. The new curriculum is sequentially applied from the 1st grade.

It was 1954 that the mathematics curriculum at the national level was first announced in Korea. Since then, the national curriculum has been revised 8 times. The 7th curriculum was announced by public notification in 1997. Then in 2007 and 2009 revisions were made to respond to rapidly changing external environment in recent decades in Korea.

The latest mathematics curriculum emphasizes mathematical creativity designed to equip students with capacities on basic learning ability, divergent thinking ability, problem-solving ability, originality, and ability to create new values. It also reinforces mathematical process including problem-solving ability, reasoning ability, and communication ability. To facilitate creative class activities, it was organized to reduce more than 20 % of the existing mathematics contents, and to apply 'the grade cluster system' to enhance connection and cooperation between grades.

The Development and Characteristics of Korean Mathematics Textbooks

Mathematics textbooks in the elementary, middle and high schools are being developed based on the curriculum. Some materials suggested here are still under development, which will give us a general picture about the mathematics textbooks in Korea. In Korea, three different kinds of textbooks are used in schools: (1) Government-copyrighted textbooks, (2) government-authorized textbooks, and (3) government-approved textbooks.

Elementary school mathematics textbooks are government-copyrighted textbooks, which are developed by the institute commissioned by the government. Secondary school mathematics textbooks are government-authorized textbooks and government-approved textbooks, which are published through the authorization procedure to guarantee high quality textbooks. In addition to textbooks, student workbooks and teacher guidebooks are developed. Student workbooks are to help students' self studies and teacher guidebooks are to help teachers apply various teaching methods and guarantee quality teaching.

Teaching and Learning Practices of Mathematics Classroom in Korea

Here we investigate the teaching and learning practices that are implemented in the mathematics classrooms in Korea. Various types of teaching practices in the elementary and secondary classrooms will be reviewed. At the elementary level, we investigate the factors of similarities and differences of the teaching and learning across mathematics classrooms. Also, we explore the general characteristics of mathematics classrooms such as activity-based lessons, emphasis on cooperative learning and communication. At the secondary level, we find the characteristics of mathematics classrooms such as differentiated lessons, subject-based classroom system, preparation for university entrance exam, and the cases of teaching practices. We provide some example cases for a better understanding of the teaching and learning practices at each level.

The Educational Practices for the Mathematically-Gifted and the Underachieving Students

In this section, we investigate mathematics education for the gifted and the underachieving students in Korea. We will describe mathematics educational systems, contents, and plans of actions focused on these particular students. First, for the mathematically-gifted students, we will introduce the development of various policies and explain three types of institutions for the gifted: schools, education centers, and classes. Additionally we introduce several gifted education programs that are implemented at the elementary and secondary level schools. Second, for the mathematically underachieving students, we will explain various new policies implemented after 2009 where the government, local offices of education, and schools actively participating in supporting underachieving students. Additionally, we will introduce the institutions and programs for the underachieving and the Internet website, called Ku-Cu (www.basics.re.kr), which is operated by KICE to support education for the children with underachievement.

National Assessment of Educational Achievement

The major aims of the National Assessment of Educational Achievement (NAEA), targeted for all schools in Korea, are to (1) acquire information and implications on directions to improve curriculum, teaching, and learning methods, (2) review the educational quality, (3) diagnose and remedy each student's performance level, and (4) examine educational accountability of school education. The NAEA is implemented targeting sixth-grade elementary school students, third-grade (9th) middle school students, and second-grade (11th) high school students across the nation. Test items of the NAEA are developed in contents and behaviour areas based on the national curriculum. Here we describe in depth the NAEA such as the structure, testing time, development of assessment tool, domains of assessment, and the scoring and reporting results. We also discuss the recent trend of the assessment results.

College Scholastic Ability Test (CSAT) in Korea

In this section, we discuss the College Scholastic Ability Test (CSAT). The CSAT has been implemented since 1994, and it was adapted with the changes of the national curriculum and college recruitment systems. The current CSAT for mathematics consists of Mathematics 'GA' (Korean) type and Mathematics 'NA' (Korean) type. Students who will major in mathematics and natural science at college should take Mathematics 'GA' (Korean) type and other students should take Mathematics 'NA' (Korean) type. Test items are developed to examine students' competencies on calculation, mathematical understanding, reasoning, and problem-solving.

Starting from the 2014 school year, the CSAT will be improved to reflect the aims of the 2009 revised curriculum and reduce the importance of the CSAT in the college admission to enhance the autonomy of each college. The title, Mathematics 'GA' (Korean) type and 'NA' (Korean) type, will be changed to Mathematics A type and B type.

Achievement on International Assessment in Korea

In the international student assessment part, we will analyze the characteristics of Korean students' mathematics achievements revealed in two representative international assessments: PISA and TIMSS. Korea has continuously been ranked among the top performing countries in PISA and TIMSS, which has been the result of more students with a high level of proficiency and less students with the lowest levels of proficiency compared with other countries. More than 2/3 of Korean students have performed at the excellent level, and 98 % of them have performed above the basic level. The proportions of Korean students with the highest level of proficiency in PISA and TIMSS, however, have been decreased, which requires policy measures to deal with the situation. We also find further implications from the test results.

Mathematics Assessment at School Level

The evaluation at the school level is administered according to the curriculum. Mathematics assessment at the school level is distinguished by the student's grade. We start with introducing the principles of assessment, the types of assessments and schedule, and the assessment methods. Usually, there are diagnostic assessments at the beginning of school year, scheduled examinations such as mid-terms and final exams at the single school level and performance assessment and quizzes at the class level. Even though the national curriculum strongly recommends various assessment methods, selection type focusing on multiple choice items and constructed-response type problems focusing on short-answer types are in the majority. However, constructed-response items that require the students to create their own answer have also been treated in fair proportion and are applied to not only scheduled examinations, but also performance assessments and diagnostic assessments. We further provide information about the analysing, reporting, and application of the assessment results.

Mathematics Teacher Education in Korea

Here we will discuss about pre-service teacher education, the teacher employment test and professional development of teachers. First, we will review the curriculum of various teacher education programs for the elementary and secondary level prospective teachers in Korea, which features a strong zeal for education. We will also examine the teacher employment test including the procedure, structure, and test areas for the elementary and secondary level.

In addition, we will discuss various teacher professional development programs which are implemented by the 16 metropolitan/local education offices. Typical professional development programs include the pre-employment training program, the 'first-level teacher' training program for teachers with more than 3 years of teaching experiences, and various in-service training programs. We will also explain the master teacher system, teaching consulting programs, and the classroom assessment system, which are designed to develop teachers' professionalism.

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Mathematics Education in Singapore

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Abstract Mathematics education in Singapore is a shared responsibility of the Ministry of Education (MOE) and the National Institute of Education (NIE). The MOE overseas the intended, implemented and attained curriculum in all schools while the NIE is involved in teacher preparation and development and also research in mathematics education. Therefore this report has two sections respectively, the first describes the education system and school mathematics curricula while the second briefly provides relevant information on teacher preparation and development and mathematics education research in Singapore.

Keywords Singapore · Mathematics education · Curriculum · Teacher education · Research

Introduction

Mathematics education in Singapore is a shared responsibility of the Ministry of Education (MOE) and the National Institute of Education (NIE). MOE develops the national mathematics curriculum and oversees its implementation in all schools, while the NIE is involved in teacher preparation and development and also research in mathematics education. This report comprises two sections: the first describes the education system and school mathematics curricula while the second provides relevant information on teacher preparation and development and mathematics education research in Singapore.

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The Education System and School Mathematics Curricula

Education in Singapore has evolved through a continual process of change, improvement and refinement since the country gained independence in 1965. Today, all children receive at least 10 years of general education in over 350 primary, secondary and post-secondary schools. There are diverse pathways and opportunities for students to discover their talents, realize their potential, and develop a passion for life-long learning. Singapore's education system largely follows a 6-4-2 structure, with 6 years of primary (Grade 1–6), 4 years of secondary (Grade 7–10) and 2 years of pre-university (Grade 11–12) education (MOE 2012).

Mathematics is a compulsory subject from Primary 1 up to the end of secondary education. In the early grades, about 20 % of the school curriculum time is devoted to mathematics so that students build a strong foundation to support further learning in later years. The mathematics curriculum is centrally planned by MOE. However, flexibility is given to schools to implement the curriculum to best meet the abilities and interests of students. The mathematics curriculum is reviewed every 6 years with consultation of key stakeholders and partners to ensure that it meets the needs of the nation.

The mathematics curriculum aims to enable students to acquire and apply mathematical concepts and skills; develop cognitive and metacognitive skills through a mathematical approach to problem solving; and develop positive attitudes towards mathematics. A single mathematics curriculum framework (MOE 2007) unifies the focus of the mathematics curriculum for all levels from primary to pre-university. The focus is on developing students' mathematical problem-solving abilities through five integral components namely, concepts, skills, processes, attitudes, and metacognition.

A spiral approach is used in the design of the mathematics syllabuses from primary to pre-university. At every level, the syllabuses comprise a few content strands (e.g. number and algebra, geometry and measurement, statistics and probability), facilitating connections and inter-relationships across strands. The content in each strand is revisited and taught with increasing depth across levels. There is differentiation in the content, pace and focus among syllabuses within the same levels to cater to different student profiles.

Primary 1–4 students follow a common mathematics syllabus, covering the use of numbers in measurements, understanding of shapes and simple data analysis. At Primary 5–6, there are two syllabuses: the Standard Mathematics syllabus builds on the concepts and skills studied in Primary 1–4, whereas the Foundation Mathematics syllabus revisits some of the important concepts and skills taught earlier. At the secondary level, there are 5 different syllabuses for students in the Express, Normal (Academic) and Normal (Technical) courses. These syllabuses include concepts and skills in number and algebra, measurement and geometry, and statistics and probability. Calculus and trigonometry are covered in the additional mathematics syllabuses for Secondary 3–4 students who are more mathematically-inclined. At the pre-university level, mathematics is an optional subject. Three syllabuses

(H1, H2 and H3) are available to prepare students for different university courses and the use of graphing calculators is expected.

There are also programmes to support the slow progress students and stretch those talented in mathematics. Primary 1 students (about 5 %) who lack age-appropriate numeracy skills are given support through the Learning Support Programme for Mathematics where they are taught in small groups by specially-trained teachers. For gifted learners, there is an enriched mathematics curriculum that emphasizes problem solving, investigations, making conjectures, proofs and connections among concepts. The NUS High School of Mathematics and Science also offers mathematically talented students a broad-based 6-year programme that includes undergraduate level topics and a mathematics research component.

For the teaching of mathematics at the primary levels the Concrete-Pictorial-Abstract (C-P-A) approach, introduced in 1980, is prevalent. Since 1990s, it has been used together with activity-based learning to encourage active participation by students in the learning process. In the early 1980s, MOE also developed the model method for solving word problems at the primary level (MOE 2009). This method provides a visual tool for students to process and analyse information and develop a sequence of logical steps to solve word problems. The model method is also used with algebra to help students formulate algebraic equations to solve problems in lower secondary mathematics. This facilitates the transition from a dominantly arithmetic approach at the primary level to an algebraic one at the secondary level. At the secondary and pre-university levels, teacher-directed inquiry and direct instruction are common. These approaches are used with other activities and group work to engage students in learning mathematics.

Resources are critical to curriculum implementation and effective delivery of mathematics lessons. Textbooks are essential materials to help teachers understand the emphases and scope of the syllabuses, and for students to learn independently. In the late 1990s, MOE devolved textbook writing to commercial publishers to allow for a greater variety of textbooks. Quality is assured through a rigorous textbook authorization and approval process by MOE. Besides textbooks, MOE also produces additional materials to support teachers especially at the primary levels.

Teacher Preparation and Development, and Research in Mathematics Education

The NIE and Teacher Education

The National Institute of Education (NIE) is an autonomous institute within the Nanyang Technological University and sole teacher education institution in Singapore. It offers both pre-service and in-service education programmes ranging from diploma to doctorate levels. Its present model of Teacher Education for the 21st century (TE^{21}) is unique and has six foci intended to enhance the key elements of teacher education. The foci are the Values^3, Skills and Knowledge (V^3SK) model, the Graduand Teacher Competencies (GTC) framework, strengthening the theory-practice nexus, an extended pedagogical repertoire, an assessment framework for 21st century teaching and learning, and enhanced pathways for teacher professional development (NIE 2009). In particular the V^3SK model explicates three dimensions of values for the teacher, viz. learner-centredness, teacher identity and service to the profession and community, without which the beginning teacher may easily lose her focus in an increasingly technological and knowledge-driven world. The GTC framework makes clear the competencies to which the student teacher should aspire to attain or be aware of in his studies at NIE. This is a distinct attempt to state what must be achieved in one's pre-service teacher education and also what should be reasonably accomplished only after some years of experience as a teacher.

Pre-service Education of Mathematics Teachers

Pre-service education provides the crucial initial training that can have long-term impacts on the quality of future teachers in an education system. Besides education courses and the practicum, trainee teachers take mathematics-related courses called Curriculum Studies (methodology), Subject Knowledge (deeper understanding of school mathematics), and Academic Studies (tertiary mathematics). These courses are taught by mathematicians, mathematics educators, and "mathematician educators" (those with expertise in both areas) from the same Mathematics and Mathematics Education Academic Group. These courses stress the rigour of mathematics contents and relevance to local school contexts and school mathematics, in particular, the model method used in problem solving. Locally developed resources (Lee and Lee 2009a, b) used in these courses combine local experience and research with international "best practices". Blended learning is used in teacher education courses in response to the significant roles of ICT in instruction as well as the changing characteristics of the trainee teachers. Findings from IEA's Teacher Education and Development Study in Mathematics (TEDS-M) (Tatto et al. 2012) show that NIE trainee teachers scored above international average in mathematics content and pedagogical content knowledge, and most of them expressed strong commitment to the teaching profession as their life-long career.

Professional Development of Mathematics Teachers

Since 1998 all teachers in Singapore are entitled to 100 h of training and core-upgrading courses each year to keep abreast with current knowledge and skills. The Professional Development (PD) is funded by the MOE. Teachers have different pathways to upgrade their knowledge and skills through the Professional Development Continuum Models (PCDM) of the MOE. The MOE works closely with NIE to design courses for practicing teachers. Numerous academic courses offered by NIE lead to postgraduate degrees. For example, in order to upgrade mathematics teachers' content knowledge, a unique master degree programme MSc (Mathematics for Educators) is offered by NIE. The mathematics chapter of the Academy of Singapore Teachers (AST), the Association of Mathematics Educators (AME) and the Singapore Mathematical Society (SMS) are also actively engaged in the PD of teachers. They hold relevant annual meetings, seminars and conferences for teachers. Teachers may also attend international conferences or study trips to widen their perspectives on mathematics education. Lastly, teachers are also engaging in professional learning and development by participating in research projects at the school level. Examples of two such projects are the Enhancing the pedagogy of mathematics teacher (EPMT) project (Kaur 2011) and the Think-Things-Through (T3) project (Yeap and Ho 2009).

Mathematics Education Research in Singapore

Research is undertaken by graduate students and university scholars. Since 2002, the MOE through the Office of Education Research (OER) at NIE has funded research to inform policy and practice so as to improve education in Singapore. Some of the projects in mathematics education that have been funded and completed are as follows: An exploratory study of low attainers in primary mathematics (Kaur and Ghani 2012); The Singapore mathematics assessment and pedagogy project (Wong et al. 2012); Individual differences in mathematical performance: social-cognitive and neuropsychological correlates (Lee and Ng 2011); Mathematical problem solving for everyone (Toh et al. 2011), Student perspective on effective mathematics pedagogy (Kaur 2009), and Teaching and learning mathematical word problems: A comparison of the model and symbolic methods (Lee et al. 2011). These projects were carried out by university scholars in collaboration with students, teachers and research staff at NIE. Research studies undertaken by graduate students almost always culminate in dissertations, thesis or academic reports, all of which are available at the NIE library repository.

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National Presentation of the United States of America

Rick Scott

The United States of America was honored to be invited to make one of the National Presentations at the 12th International Congress on Mathematical Education in Seoul, Korea. The United States National Commission on Mathematics Instruction (USMC/MI) oversaw the U.S. participation. (The USNC/MI advises the National Academy of Sciences-National Research Council (NAS-NRC) in all matters pertaining to the International Commission on Mathematical Instruction.) Significant financial support was supplied by the National Science Foundation (NSF) and important logistical support was provided by the National Council of Teachers of Mathematics (NCTM). The main activities of the U.S.A. National Presentation were the

- National Presentation sessions,
- U.S.A. Exhibit,
- *Capsule Summary Fact Book*, and
- U.S.A. Reception.

The National Presentation Sessions

The National Presentation highlighted the uniqueness and important features of mathematics education in the U.S. in two 90-min sessions with a total of five presentations. The first three presentations provided *An Overview of Math Ed in the*

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U.S.: Curriculum Reform. The last two focused on *Teaching Mathematics in the United States.*

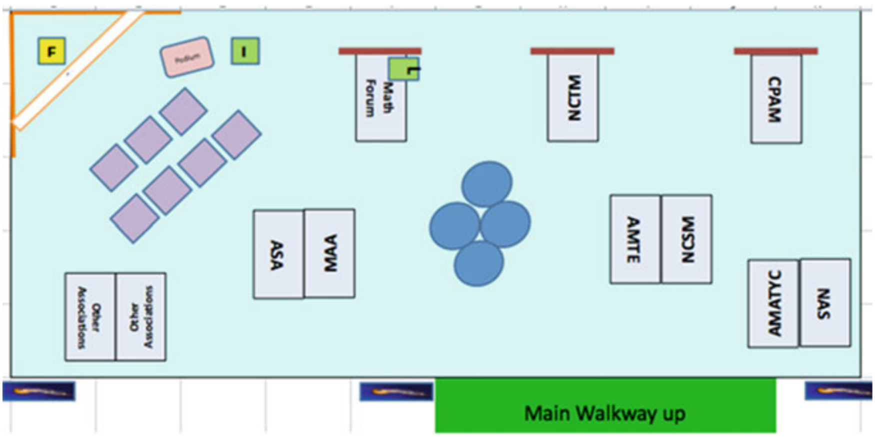
1. *Mathematics Education in the United States 2012*—Katherine Halvorsen, Smith College
   This presentation provided an overview on the system of education in the U.S., including the size of the educational enterprise, governance, intended curriculum, implemented curriculum, attained curriculum, the Common Core Standards in Mathematics (CCSSM), programs for special populations, and teacher education and professional development. More details can be found below in the section entitled ***A Capsule Summary Fact Book***.
2. *Evolution and Revolution: From the NCTM Standards to the Common Core State Standards in the U.S.*—Michael Shaughnessy, Immediate Past President of the National Council of Teachers of Mathematics (NCTM)
   This short history on mathematics standards in the U.S. pointed to the main NCTM documents and the major themes in those documents (1989 and 2000 NCTM Standards, Curriculum Focal Points, and the high school Reasoning and Sense Making Initiative). The case was made that the CCSSM represents an evolutionary step, anchored in NCTM's Process Standards and the *Adding It Up* proficiencies, drew on Curriculum Focal Points series, and is closely tied to NCTM's Reasoning and Sense Making effort. The revolutionary step is that the CCSSM is *common.* The history of prior adoption and implementation mathematics standards in the US has been on a state by state basis, with local control and local decision making. For states to adopt a set of Common Standards is quite different for our nation.
3. *Research Perspectives on Mathematics Standards Reform in the U.S.*—Mary Kay Stein, University of Pittsburgh
   The presentation began with a short introduction that identified the roles that research can and has played in past standards-based eras. Then attention turned to some key areas in which research might shed light in this era of the CCSSM. After referring to the NSF report, *A Priority Research Agenda for Understanding the Influence of the CCSSM*, four features of the CCSSM were outlined that set them apart from past standards: Fewer, clearer, higher standards; learning progressions; the positioning of mathematical practices, and their commonness. Theories-of-action associated with how each of these features is expected to contribute to improved mathematics instruction and student learning were described. Suggestions were made regarding how research could help us monitor the extent to which the theories-of-action play out as expected and whether there are any unintended consequences.
4. *The "Mathematics Studio": Sustainable School-Based Professional Learning*—Linda Foreman, President of Teachers Development Group
   The design of the "Mathematics Studio" professional development model is guided by a robust body of research on effective mathematics learning, teaching, professional development, and school leadership. Implementation of the model over time produces a powerful school-based culture of professional learning in

which sense making about meaningful mathematics instruction is continuous for all teachers and school leaders. At the heart of this work is engaging all mathematics teachers and leaders from a school together—during live classroom teaching episodes—in publicly coached rehearsals of well-defined, research-based “mathematically productive teaching routines.” Grounded by the premise that all students are capable mathematical thinkers, this work fosters leaders’ and teachers’ habitual use of practices that yield all students’ internalization of mathematical habits of mind typified by the *Common Core State Standards for Mathematical Practice*. This presentation provided a glimpse of the Mathematics Studio model’s design, implementation, and impact.

5. *Challenges of Knowing Mathematics for Teaching in the United States*—Deborah Ball, University of Michigan
   Teachers’ mathematical knowledge is a concern in many countries around the world. However, several features of the U.S. educational context present special challenges for ensuring that teachers know mathematics well enough to teach it. This presentation examined the mathematical knowledge needed for teaching and analyzed the special challenges presented by the U.S. context. Questions for participants were: How unique are these U.S. challenges for mathematical knowledge for teaching? Do other countries share these challenges?

The U.S.A. Exhibit

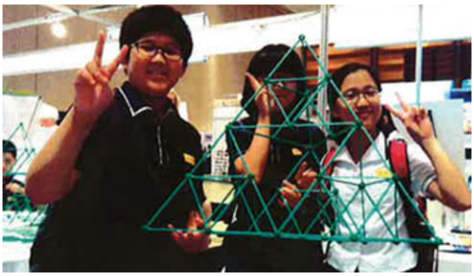
The U.S. Exhibit showcased not only the uniqueness of math education in the U.S., but also the diversity and variety of products, key players, and stakeholders involved in the practice. The U.S. exhibit included speakers, videos, materials, and interactive experiences.



Since the number and diversity of the professional organizations devoted to various aspects of mathematics education was judged to be something unique about the United States all such organizations were asked to exhibit information about their activities. The figure on the right, which was one of the banners at the entrance to the Exhibit, indicates the organizations that participated in the U.S.A. National Exhibit.

A unique aspect of the Exhibit was that a corner was set aside with a projector and a screen so that presentations could be made. Those presentations led to many productive international conversations.

Another corner of the Exhibit area had a monitor that showed videos of mathematics classes in the United States. Coordinated by the Council of Presidential Awardees in Mathematics (http://cpam.teachersdg.org/), participants were assisted in using an iPad app called “Common Core Look-fors (CCL4s)” that can be used as a non-evaluative assessment tool to determine the extent to which teachers and students are engaged in aspects of the Common Core Mathematical Practices (http://www.corestandards.org/). Both an English transcription and a translation of the transcription into Korean were provided for the video clips. Those transcriptions helped many visitors understands the clips and use the iPad app.



Particularly popular among both teachers and the crowds of Korean students who attended ICME-12 were Zome Tools (http://zometool.com/), an innovative manipulative that challenges students to creatively explore geometry and informally introduces them to concepts of topology. In addition to working at the tables set up in the middle of the exhibit, teachers and students were given individual packets of Zome Tools so they could continue with their explorations.

Besides USNC/MI members and representatives from the professional organizations in the Exhibit, U.S. mathematics educators who had received travel grants from NCTM with funding from the National Science Foundation (NSF) also took turns at the Exhibit discussing mathematics educations with visitors to the Exhibit.

A Capsule Summary Fact Book

It has become a tradition for NCTM to commission for each ICME a document entitled *Mathematics Education in the United States: A Capsule Summary Fact Book*. For ICME-12 it was prepared by John Dossey, Katherine Halvorsen, and Sharon McCrone. Attendees at the National Presentation and the National Exhibit were given a copy of the *Fact Book* on a USB drive. The following two paragraphs from the "Preface" give a very good indication of what can be found in the complete report:

> This document begins with some general information about education in the United States. The three kinds of curricula identified in the Second International Mathematics Study—intended, implemented, and attained—are then described (McKnight et al. 1987). A special focus is given to the emergence of a common K–grade 12 curriculum that has been adopted by forty-five states and the District of Columbia. This curriculum, the Common Core State Standards for Mathematics (CCSSM), was developed by a consortium consisting of state governors and chief state education officers (National Governors Association Center for Best Practices and Council of Chief State School Officers [NGA Center and CCSSO] 2010). The adoption of such a set of common outcomes, matching assessments, and similar instructional materials is expected to bring to U.S. mathematics education a level of uniformity that it has never before seen.

> As in earlier editions, this publication has sections dealing with programs for high-achieving students, programs for mathematics teacher education, and resources for additional information about U.S. mathematics education. One message that comes through repeatedly in these descriptions is the variety of available programs and thus the inability to characterize them adequately in a brief document like this one. Another message is that all levels of the educational system exhibit great flux, and even though we have attempted to provide the latest available information, we realize that the information presented here will quickly become dated. By listing our sources, we hope to enable the interested reader to obtain updated information.

The report is available for download at www.nctm.org/about/affiliates/content.aspx?id=16955.

The U.S. Reception

A U.S. reception was held at the ICME-12 on July 10th for 150 international attendees. The reception was intended to foster international collaborations between U.S. math educators and their international peers. A Speed Networking session facilitated the networking experience between the attendees. The reception was

NCTM NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

AMATYC

ASA AMERICAN STATISTICAL ASSOCIATION

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sponsored by the National Council of Teachers of Mathematics (NCTM), the American Mathematical Association of Two-Year Colleges (AMATYC), The American Statistical Association (ASA), and the Conference Board of the Mathematical Sciences (CBMS).

National Presentation of India

K. Subramaniam

The National Presentation Process

A proposal to make a national presentation on the status of mathematics education in India at ICME-12 was sent by the Indian National Science Academy to the Chairman of the International Programme Committee in February, 2011. After the proposal was accepted, a Steering Committee was formed to oversee the preparation of the National presentation. A national initiative was launched to identify initiatives and innovations in mathematics education in India and to bring diverse groups working to improve mathematics education together on a common platform. The National Initiative on Mathematics Education (NIME) aimed to develop a vision about the changes necessary in mathematics education policy, the need for research studies on mathematics education, and ways of implementing system wide improvement and transformation of the practice of mathematics education.

Under the NIME initiative five regional conferences on mathematics education were held in 2011 across India to provide a wide participatory platform. This was followed by a National Conference on mathematics education in early 2012. The aim of the conferences was to bring together the important and significant innovations and efforts to improve mathematics education in school and in higher education in the diverse regions of the country and to build awareness of such efforts in the community of mathematics educators. The proceedings of the conferences formed an input for the Indian National Presentation.

The Indian National Presentation at ICME-12 includes the following components:

1. A book on *Mathematics Education in India: Status and Outlook*, containing key articles on the themes identified for the National Presentation.

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S.J. Cho (ed.), *The Proceedings of the 12th International Congress on Mathematical Education*, DOI 10.1007/978-3-319-12688-3_23

2. The slides of the presentations made at ICME-12 in two slots: one sub-plenary slot of 90 min, and a parallel session of 90 min.
3. Four video films (1 long and 3 short) on the challenges and hope giving initiatives in mathematics education in India.
4. An exhibition covering historical aspects of mathematics and mathematics education, the challenge of diversity, basic data about mathematics education in India and information about some initiatives displayed in a stall in the exhibition area of ICME-12.

All these components, except the long video film, are available on the NIME website for free download under an open access license: http://nime.hbcse.tifr.res.in. The website also contains links to the NIME regional and national conference websites, and the research papers presented by Indian participants at ICME-12.

Mathematics Education in India

The Indian National presentation was organized under the following four major themes:

1. Historical and Cultural aspects of mathematics and mathematics education
2. Systemic and Policy aspects of education
3. Mathematics Curriculum and Pedagogy at the elementary, secondary and tertiary levels (including nurture and enrichment programmes)
4. Teacher education and development

We provide below a summary of the presentations made for the Indian National Presentation (presentation slides available on the NIME website). The book on *Mathematics Education in India: Status and Outlook*, may be consulted for more details. The first presentation session covered the first two themes, while the remaining themes were covered in the second presentation session.

Historical and Cultural Aspects of Mathematics and Mathematics Education

There were two presentations under this theme, the first on the history of Indian mathematics and the second on the history of Indian mathematics education. Ramasubramaniam presented an overview of the historical tradition of mathematics in India. He described the main contributions made by Indian mathematicians in the Ancient Period (prior to 500 CE), the Classical Age (500–1200 CE) and the Medieval Period (1250–1750 CE). The examples that he described included the knowledge of geometry used to identify the cardinal directions and methods of finding irrational quantities such as square roots in the *Śulbasūtras* ($\sim$500 BCE), Aryabhaṭa's recurrence relation for sine values as well as the table of sine differences (499 CE), the summation of series and finding the sums of sums in the Classical Age, and the infinite series for π and the fast converging approximations developed in the Kerala school of mathematics in the Medieval Period. He pointed out that the Indian approach to mathematics laid emphasis on the development of algorithms and on practical applications. Indian mathematical texts typically illustrated a principle or rule with a large number of examples drawn from the practical world. He also pointed out the role of memory in communicating mathematics and the organization of the texts in compressed verse form (*sūtras*). He argued that inclusion of the history of mathematics in mathematics education would help eliminate euro-centrism and biases, and also introduce a cross-cultural perspective on mathematics.

Senthil Babu spoke about the indigenous traditions of mathematics education in pre-British South India. Indian merchants, traders and craftsmen were renowned for their facility in arithmetic and computational ability. They learned to carry out a variety of complex computations grounded in practical contexts in indigenous schools called "*pāṭhaśālas*". The curriculum in these schools was grounded in the needs of the economy and society. The objective of the *pāṭhaśāla* education was to produce competence and skill in dealing with numbers and letters. A primary mode of learning was recollective memory, which combined knowledge of tables and series of numbers and quantities with problem solving. Public display of competence and skill was a celebrated part of *pāṭhaśāla* learning. The encounter with the Colonial British government and the efforts to introduce modern education gradually led to the *pāṭhaśālas* being appropriated and replaced with a curriculum and education system that was disconnected from the life of the community. Babu pointed to lessons that this may hold for the problems surrounding mathematics education in contemporary India.

Systemic and Policy Aspects of Education

There were three presentations under this theme covering respectively school education, the assessment culture and nurture programs for high achievers in mathematics. Anita Rampal presented an overview of the challenges and policy initiatives in mathematics education at the school level. Although India is a country with strong mathematical traditions, it is grappling with multiple challenges emerging from endemic poverty and large numbers of children not completing school. The systemic challenges include restructuring the education system to ensure an equitable education of high quality to a huge young population with high aspirations. Rampal presented an overview of the institutional structures and organization of mathematics education at various levels in India. She described two major policy initiatives in school education—the National Curriculum Framework of 2005 and the Right to Education Act of 2009. The new curriculum framework, which emphasized learning through activity and exploration and making the child free from fear and anxiety, had major implications for mathematics education. The new Act has set in place assessment reforms that can have a major impact. The new textbooks at the primary level aim to build on how children think and integrate themes from work, crafts and cultural hertiage. Rampal argued for redesigning secondary education curricula to meet the needs of a diversity of learners and called for a culturally responsive critical pedagogy of mathematics education.

Shailesh Shirali spoke on the role of assessment in mathematics education in India. He described the high stakes, highly competitive examination environment that students in India face at the end of schooling in order to gain entry to higher education. Such intense competition has led to a pervasive culture of private coaching and has shaped assessment practices right down to the primary school level. Shirali discussed sample questions from some of the most competitive entrance examinations, which tend to emphasize procedure and manipulative skills, and heavy dependence on memorization. He described the recent initiative on "continuous and comprehensive evaluation" as promising, but as critically dependent on adequate teacher preparation. He also called for research on assessment tools and models.

Kumaresan described the training programs at different levels aimed at high achievers in mathematics in India. He grouped the training programs under three categories: (i) those at the undergraduate level (ii) those at the graduate and Phd level and (iii) those aimed at National Olympiad toppers. The most significant program at the undergraduate level is the Mathematics Training and Talent Search (MTTS) program. This program aims to move students out of a pervasive culture of rote learning towards discovering mathematics by inquiry, to awaken their thinking abilities, to expose them to the excitement of doing mathematics, and to change the teaching of mathematics in the country in the long run. The MTTS sessions are highly interactive, where students are trained to observe patterns, formulate conjectures and develop proofs. The Advanced Training in Mathematics (ATM) schools address the needs of graduate and Phd level students. Olympiad toppers are trained through a special nurture program.

Mathematics Curriculum and Pedagogy at the Elementary, Secondary and Tertiary Levels

There were four presentations on the mathematics curriculum and pedagogy at different levels of education in India. The first two presentations described the curriculum and the pedagogical challenges at the primary level. The remaining two presentations analysed the curriculum at the secondary and tertiary levels of education. Amitabha Mukherjee described the changes introduced in the primary mathematics curriculum and textbooks following the revision of the school curriculum framework by contrasting the new curriculum and the traditional curriculum along several dimensions. The new curriculum emphasises concrete experience as the basis for learning mathematics, and encourages multiple approaches to solving problems. Topics not emphasised in traditional curriculum such as shapes and space, measurement, data handling and patterns have been given space in the new curriculum. Jayasree Subramanian analysed the limitations of well-intentioned reforms in primary mathematics education. She pointed out that activity based approaches need resources in the form of teaching-learning materials, which are not available in most schools catering to children from low socio-economic backgrounds. "Drill and practice" still dominates classroom teaching of mathematics. Jayasree Subramanian cautioned that curriculum and pedagogy alone cannot ensure mathematics for all in a society fractured by several inequities.

Jonaki Ghosh presented an overview of the secondary mathematics curriculum, where the central focus is on consolidation of concepts learned earlier and exploring wider connections. There is an emphasis on the structure of mathematics as a subject and mathematical processes such as argumentation and proof, logical formulation, visualization, mathematical communication and making connections. While assessment is largely summative, a new initiative by the Central Government has shifted the emphasis towards continuous and formative assessment by removing the mandatory requirement of a final public examination at the end of Grade 10. The senior secondary stage (Grades 11–12) is dominated by the culture of high-stakes examinations, and Ghosh identified areas where a change of approach is needed, especially of making students familiar with the power of applications of mathematics in solving real world problems. She also emphasised the role of technology and the need to apply it thoughtfully to overcome the challenge of resource-poor classrooms.

Geetha Venkataraman spoke on the challenges facing mathematics education at the tertiary level where there are about 400 Universities and 18,000 colleges offering undergraduate courses in mathematics. Although syllabus reforms have taken place in the undergraduate curriculum since the late 1960s and 1970s, further reform is needed at the present time. The recommendations and model syllabus of the University Grants Commission failed to provide leadership in terms of applicability of mathematics and the use of information technology in mathematics education. The pedagogy followed is largely one of demonstrating content by stating and proving theorems with minimal student interaction. Assessment typically requires students to reproduce from memory rather than to think, analyse and solve problems. There are almost no pre-service training or inservice training programs available for faculty to learn about teaching methods and tools. Geetha Venkataraman also called for better links between the community of research mathematicians and mathematics educators.

Teacher Education and Development

There were two presentations on teacher education and development, one focused on the organization of pre and in-service teacher eduation and one on innovations and initiatives in teacher education. Ruchi Kumar presented an overview of the structures and institutions implementing teacher education in India and the place of mathematics education in the teacher education curriculum. The new school curriculum framework demands a revisiting of teachers' beliefs, a strengthening of content knowledge of mathematics and a better understanding of the psychology of young children. Ruchi Kumar gave some examples of what aspects of teacher education need change. She concluded that research on teachers' beliefs and knowledge and the relation of these to student learning was greatly needed but largely absent in the Indian context. Subramaniam spoke about the trends of change in mathematics teacher education in India and the sources of change. He cited the example of the innovative program for preparing primary mathematics teachers launched in the 1990s by the Indira Gandhi National Open University. The curriculum of this program aimed at addressing teachers' beliefs about mathematics and its learning and at giving them experiences of exploring interesting mathematics. He also mentioned some in-service teacher development initiatives as harbingers of change in mathematics teacher education. He emphasized the role of teacher associations in bringing about change in mathematics teaching at the secondary level and called for greater participation by the associations in framing curricula for pre-service teacher education.

The final presentation by Rakhi Banerjee presented a brief review of research in mathematics education in India, which is still a highly under-developed research domain in the country. Traditional studies typically follow psychometric models aiming to identify learning difficulties in mathematics or factors responsible for poor achievement in mathematics. She described some promising new trends in mathematics education research in the country which include intervention studies aimed at alternative learning trajectories for key concepts such as whole numbers, fractions or algebra. Research studies on teacher education and development are very much needed, but are nearly absent. She criticised traditional studies for failing to provide insights into the nature of the problem or possible solutions. The new research studies often lack methodological rigour or a strong theoretical framework. She also pointed to the lack of adequate support in the universities for mathematics education research and the isolation of education departments from subject disciplines as factors hindering the growth of mathematics education research in the country.

Three short video films, screened at the end of the first presentation session, had the following titles: (i) Legacy of maths at work and play (ii) Diverse learners multiple terrains (iii) Initiatives to transform maths learning. These video films and the presentation slides can be found at http://nime.hbcse.tifr.res.in.

Spanish Heritage in Mathematics and Mathematics Education

L. Rico

On the occasion of the 12th ICME, the Spanish Committee of Mathematics Education decided to prepare a national presentation entitled Spanish Cultural Heritage.

The presentation takes the form of a series of posters, each of which has a special focus, showing relevant historical events identified according to time and institutions. As a whole, the posters outline a comprehensive historical trajectory devoted to the Hispanic Heritage.

The relevance of mathematics in the relations between Spain and America has remained unbroken since its beginning 520 years ago. Julio Rey Pastor emphasizes the importance and scope of this heritage for its scientific and technological use and its benefits since the discovery of America. Since then, throughout 520 years of continuous cultural cooperation, the mathematical background shared by Spain and the American Republics, people and countries, that have remained solid and permanent.

To present the Spanish Heritage in the ICME 12 of Seoul (Korea), from the Education Commission of the CEMAT (Spanish Mathematics Committee) have been prepared 27 posters, which set out key moments, characters and events in the history of mathematics.

The list of themes chosen is as follows:

1. Spanish Heritage in Mathematics and Mathematics Education.
2. Mathematics and Science in the Discovery of America.
3. The Founding of the First American Universities.
4. First Mathematical-Scientific Publication in the New World.
5. The House of Trade: Navigation, Cartography, and Astronomy.
6. The 16th-Century Mathematics Academy: Philip II, Siliceo, Juan de Herrera.
7. Science and Technology in the 16th Century.

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8. Mathematics in the Baroque Period in Spain.
9. Scientific Policy of the First Bourbons. The Jesuits and Mathematics.
10. Enlightenment Mathematics. The Reforms of Charles III.
11. José Celestino Mutis. An Enlightened Scientist in the New World.
12. Jorge Juan and Antonio de Ulloa. Meridian measurement in Quito.
13. Educational Reforms in Hispano-America, based on the 1812 Constitution.
14. The *Compendium of Pure and Mixed Mathematics* by D. José Mariano Vallejo.
15. 19th-Century Mathematics.
16. The Metric System in Textbooks in the Second Half of the 19th Century.
17. The Mathematicians of Scientific '98.
18. Andrés Manjón and the Ave María Schools at the end of the 19th Century.
19. The Spanish Republican Exile: the Mathematicians in America.
20. Researching Together: Return Journeys.
21. The Iberoamerican Mathematical Olympiad.
22. Research Centers.
23. Journals, research and collaboration in Mathematics Education.
24. ICME 8 Seville (Spain), July 1996.
25. Miguel de Guzmán (1936–2004) Academic, scientific, and educational legacy.
26. Mathematical Research in Ibero-America, Spain and Portugal.
27. Spanish Mathematics: the last 20 years.

The posters has been prepared by:

- M. de León and A. Timón, from the Institute of Mathematical Research (ICMAT).
- J. Peralta, from University Autonomous of Madrid.
- A. Maz; N. Adamuz; N. Jiménez-Fanjul; M. Torralbo and A. Carrillo de Albornoz, from the University of Cordoba.
- L. Rico; E. Castro-Rodríguez; J. A. Fernández-Plaza; M. Molina; M.C. Cañadas; J.F. Ruiz-Hidalgo; J.L. Lupiáñez; M. Picado; I. Segovia; I. Real and F. Ruiz, University of Granada.
- I. Gómez-Chacón; M. Castrillón and M. Gaspar, from the University Complutense of Madrid.
- M. Sierra and M.C. López, from University of Salamanca.
- B. Gómez, L. Puig and O. Monzó, University of Valencia.

The main objective of this work was to present the joint activity on mathematics and mathematics education, thought and written in Spanish, conducted by Spanish and American in more than 500 years of history and shared culture. We will stress the links established between Americans and Spaniards, as demonstrated by the information presented. We will underscore the scientific, technological, or cultural value of these events, their subsequent implications, and the social impact they produced in their time.

As there is a common language, a shared history and culture, there are ways of thinking and doing math based on that language, that culture and that history. This work aims show and claim the shared heritage in mathematics and mathematics education in this community. We have done a selection of the information presented in the posters and we will comment it here.

We have organized the posters considering five general comprehensive periods:

1. Discovery and colonization.
2. The Creole society.
3. The Century of Independence.
4. 20th Century: mutual assistance and help.
5. Current cooperation.

Summary of key moments and ideas of the above mentioned periods.

1. Mathematics and Science in the Discovery of America.

On October 12, 1492, a Spanish expedition commanded by Admiral Christopher Columbus arrived at the island of Guanahani and took possession of the land in the name of their Majesties Isabella of Castile and Ferdinand of Aragon. This act of Discovery is essential to the birth of historical modernity and of science. It marks the origin of a sociocultural community, the Ibero-American community, based on the unique relationship between Spaniards and Americans. Columbus's goal was to reach Asia, that say, the island of Cipangu (Japan), which was thought to be at the same latitude as the Canary Islands. Columbus was not trying to discover a new continent, but rather to "reach the East by sailing West." The information that Columbus used involved several significant errors and to understand them it was needed a big change of ideas. Toscanelli's map, reflects the ideas of many navigators and geographers of the period, describes the route that Columbus believed he had travelled. He thus believed that the distance from the Canary Islands to Japan was 800 leagues by west (4,500 km), when it was actually about 3,500 leagues (19,500 km). These data were sufficient grounds for undertaking his first and the subsequent voyages. When Columbus arrives in the Antilles, he was convinced that he has reached the western coast of Asia hence his naming these lands the West Indies (Fig. 1).

Institutions: the first Universities. Starting with their first years in the colonies, the Crown, the Church, and the religious orders intervene in the area of education to teach and train clergy, government officials, and the middle classes. Their knowledge was classified into study in the trivium (grammar, rhetoric, and logic) and the quadrivium (arithmetic, geometry, music, and astronomy). Founding the Universities and Colleges in America is a historical feat and cultural phenomenon of prime importance, particularly in the first half of the 16th century. The first universities were the universities of Santo Domingo, Lima and Mexico, that were respectively founded in 1538 (Santo Domingo) and in 1551 (Lima and Mexico).

MATHEMATICS AND SCIENCE IN THE DISCOVERY OF AMERICA

- Authors:
- Dr. Luis Rico. University of Granada. lrico@ugr.es Spanish ICMI Representative (CEMat)
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On October 12, 1492, a Spanish expedition commanded by Admiral Christopher Columbus arrived at the island of Guanahani and took possession of the land in the name of their Majesties Isabella of Castile and Ferdinand of Aragon. This act of Discovery is essential to the birth of historical modernity and of science. It marks the origin of a sociocultural community, the Hispano-American community, based on the unique relationship between Spaniards and Americans.

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The information that Columbus used involved several significant errors.
It assumed that the earth had a maximum circumference of 29,000 km., estimated according to Posidonius and the length of a terrestrial degree, which was based on Arabic, not the shorter Italian, mile. Posidonius calculated the earth's circumference as less than three fourths of its real size, and this figure had been accepted as scientific truth since the time of Eratosthenes.
On the other hand, interpreting the information from the voyages of Marco Polo, Columbus estimated the width of Eurasia to be 225º, subtracting only 135º of ocean.

He thus believed that the distance from the Canary Islands to Japan was 800 leagues by west (4500 km), when it was actually about 3500 leagues (19,500 kilometers). But these data were sufficient grounds for undertaking his first and the subsequent voyages. When Columbus arrives in the Antilles, he is convinced that he has reached the western coast of Asia hence his naming these lands the West Indies.

Columbus's subsequent voyages demonstrated the crucial importance of the new lands and cultures discovered for the construction and grounding of new knowledge. The exploration of the New World established a contrast between reality and speculative ancient doctrines on the size and shape of the earth's geography. This stimulated the spirit of free investigation and empirical contrast of hypotheses. Geography, cartography, cosmography, navigation, geodesy, and the physics of the globe ceased to be practical techniques and began their careers as physical-mathematical sciences.

The problem of geographical longitudes also had to be solved to respond to the need for nautical maps. The demand for greater precision led to the emergence of new techniques and instruments for measurement, which required a considerable advance in science, particularly mathematics.

Although Columbus made four voyages, explored the islands of Guadalupe and Puerto Rico, reached the mouth of the Orinoco River, navigated the islands of Chacachacare and Margarita, Tobago and Granada, and landed in region of Panama, he died without knowing that he had discovered the "New World" for Europeans. In 1507, the German cartographer Waldseemüller published a geography book that included a map. The book represented the new lands and included the tales of a Florentine navigator, Amerigo Vespucci, who stated that the discoveries were not Asian lands, but a new continent. In Vespucci's honor, the 1507 map called the new lands America.

CEMat Comité Español de Matemáticas

Spanish Heritage ICME12 SEOUL

Fig. 1 Mathematics and science in the discovery of America

2. The Creole Society.

To talk about the Creole society we have fixed our attention in the following ideas:

- Scientific Policy of the First Bourbons. The Jesuits and Mathematics.
- Enlightenment Mathematics. The Reforms of Charles III.
- José Celestino Mutis. An Enlightened Scientist in the New World.
- Jorge Juan and Antonio de Ulloa. Meridian measurement in Quito.

At the beginning of the 18th century, the Jesuits assumed responsibility for educating the nobility, through Seminaries for Nobility, which began in Madrid. This institution's model of teaching spreads to Barcelona, Valencia, Gerona, and other cities. Based on its model, new centers are founded throughout the 18th century. In Mexico, the Royal Academy of San Carlos of the Noble Arts of New Spain is founded. These centers also trained the American elites. The Seminaries of Nobility become one of the most important centers of teaching and research in America. The Jesuits authored fertile textbooks in mathematics.

José Celestino Mutis y Bosio (Cadiz 1732; Bogotá 1808) Botanist, doctor, astronomer, and mathematician. Mutis developed important scientific work on American soil. He gave the inaugural speech for the Chair of Mathematics at the College of Our Lady of the Rosary in Santa Fe de Bogotá. There, he held the positions of rector and director. He determined the coordinates of Santa Fe de Bogotá, observed an eclipse of a satellite of Jupiter, and was one of the observers of the Transit of Venus on June, 1769.

Jorge Juan y Antonio de Ulloa. They participated in the expedition from 1735 to 1744 to measure an arc of 1^{o} of latitude near the equator and one near the pole, to determine the lemon or orange shape of the earth.

The Royal Academy of Sciences decided to undertake the task of obtaining precise data from two meridian positions at two locations on Earth: Lapland (North Pole) and the Viceroyalty of Peru (the equator). To do this, two expeditions were organized. If the measurements obtained by both expeditions were the same, the Earth was sphere-shaped. If the measurement was greater at the pole, there was flattening at the poles.

If the polar measurement was smaller, the French were right and the lemon shape shall be the model. To carry out the expedition to the cities of Quito and Cuenca, located today in the Republic of Ecuador (Fig. 2).

3. The Century of independence. We have fixed our attention on the following subjects.

- Educational Reforms in Hispano-America, based on the 1812 Constitution.
- The *Compendium of Pure and Mixed Mathematics* by D. José Mariano Vallejo.
- 19th-Century Mathematics.
- The Metric System in Textbooks in the Second Half of the 19th Century.
- The Mathematicians of Scientific '98.

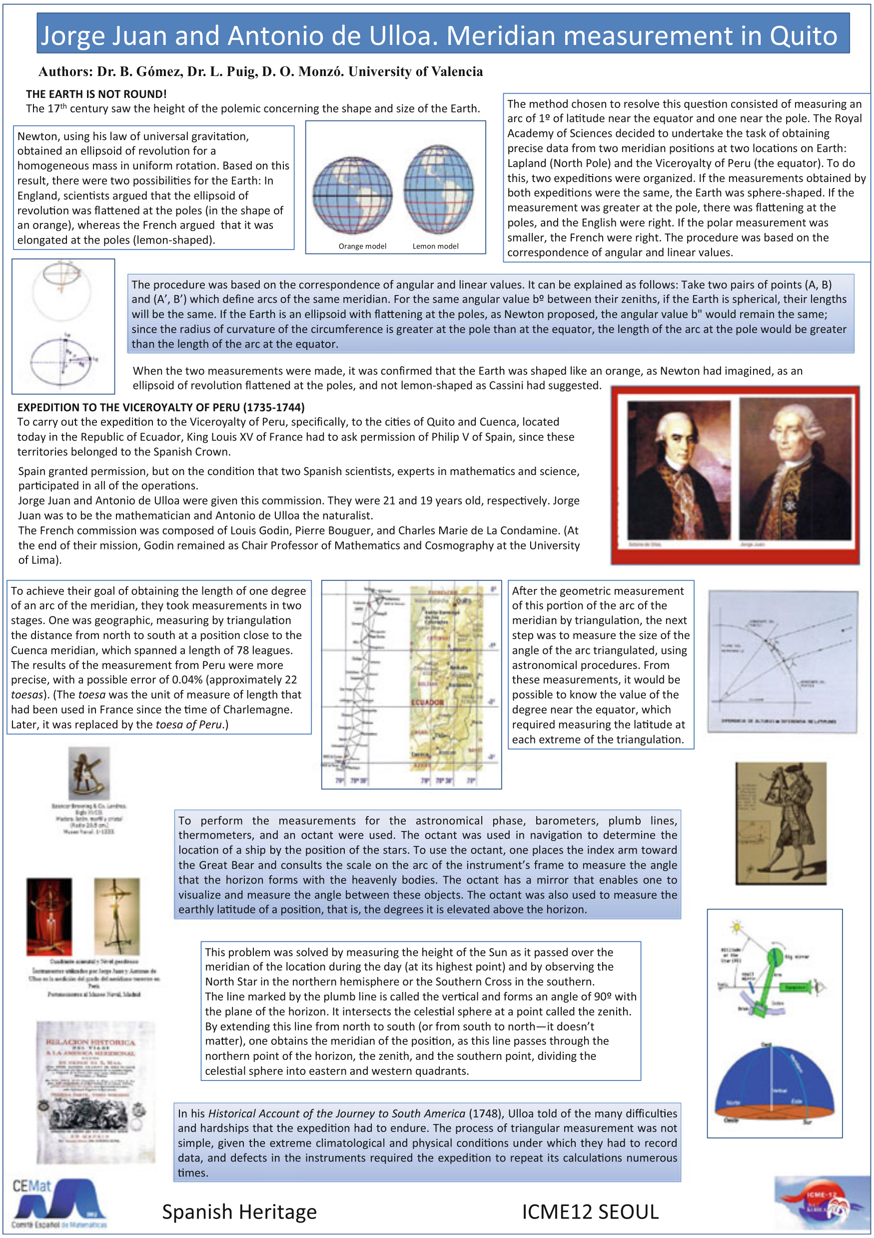

Fig. 2 Jorge Juan and Antonio de Ulloa. Meridian measurement in Quito

Conception of education. Spanish resistance to Napoleon's invasion in 1808 led to the formation of the Courts of Cadiz, which developed the Constitution of 1812, oriented ideologically to exalt and safeguard individual liberty.

The Constitution's ninth title, dedicated to education, articulated the liberal idea of education, defending the idea of general, uniform education for all citizens and the need to form a Council for Public Education. This Council prepared a report that can be considered the most representative document on liberal ideology in matters of education. The report was published in Cadiz on September 9, 1813, under the following title: *Report of the Council created by the Regency to propose the means for proceeding to regulate the various branches of Public Education*.

It was established that "education must be universal, uniform, public, and free, and it must enjoy liberty." This Report formed the basis and origin of the educational reforms put into effect throughout Hispano-America after the revolutions that led to the independence of the Spanish colonies.

The 19th century is a turbulent period in the history of Spain. It begins with the invasions of Napoleon's armies in 1808 and ends with the Spanish-American War in 1898, known as the *disaster of '98*. The beginning of the 19th century brings the independence of the former colonies in America, giving rise to the new American republics. Spain loses its status as world power. The 19th century ends with the loss of Cuba, Puerto Rico, and the Philippines and the defeat of the Spanish fleet in Santiago, Cuba. Spain concludes its political presence in America. Spain is aware of its cultural and educational decline, a feeling aggravated by the loss of its colonies. It is thought that the military defeat was caused in good part by the country's scientific and technical backwardness: *the crisis of '98* (Fig. 3).

4. 20th Century: mutual assistance and help.

Where we are? Who are the leaders? We choose five points to reflect about our common work during the 20th Century.

- Andrés Manjón and the Ave María Schools at the end of the 19th Century.
- The Spanish Republican Exile: the Mathematicians in America.
- Researching Together: Return Journeys.
- The Iberoamerican Mathematical Olympiad.
- ICME 8 Seville (Spain), July 1996. Current research and cooperation (Fig. 4).

5. Current cooperation.

To describe this point we selected the following reflections:

- Research Centers.
- Journals, research and collaboration in Mathematics Education.
- Miguel de Guzmán Ozámiz (1936–2004) Academic, scientific, and educational legacy.
- Mathematical Research in Ibero-America, Spain and Portugal.
- Spanish Mathematics: the last 20 years (Fig. 5).

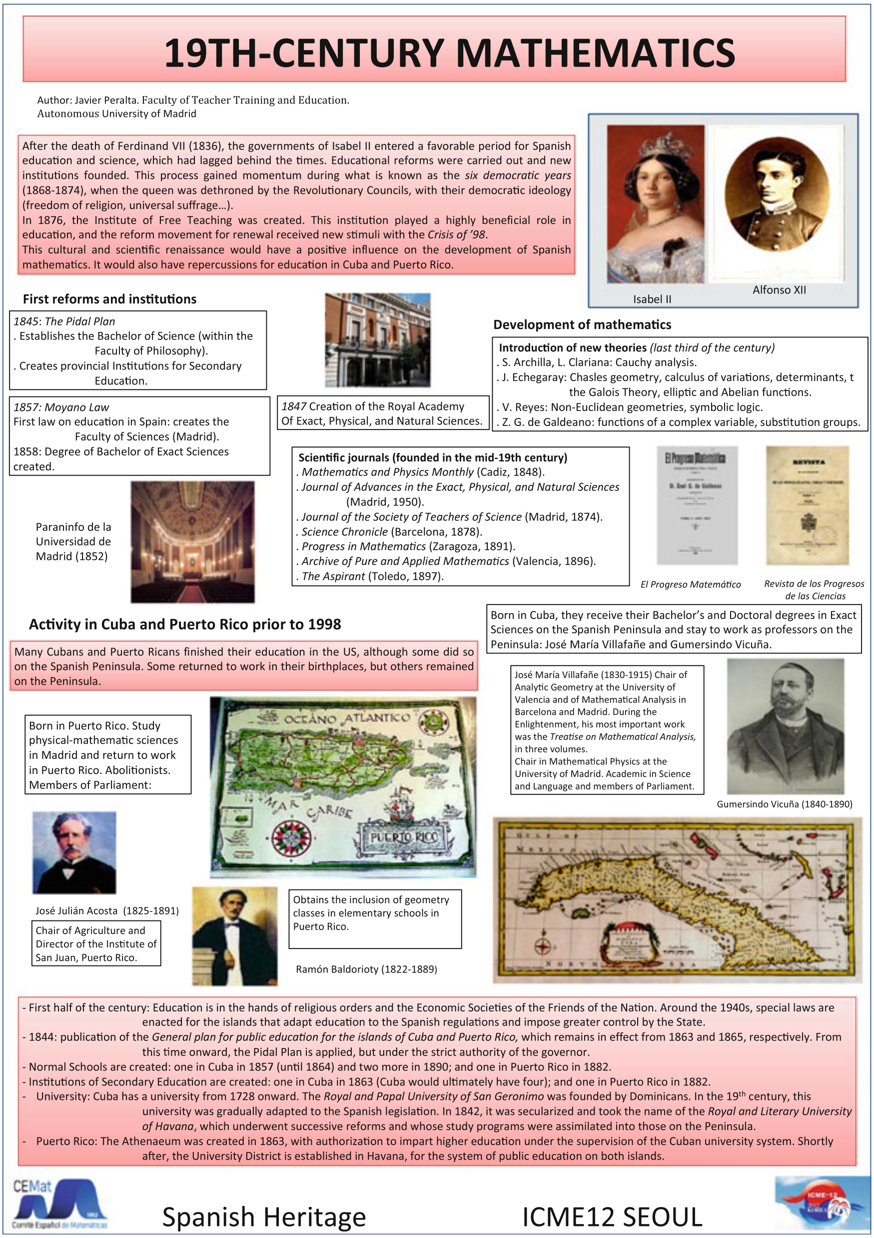

Fig. 3 19th-century mathematics

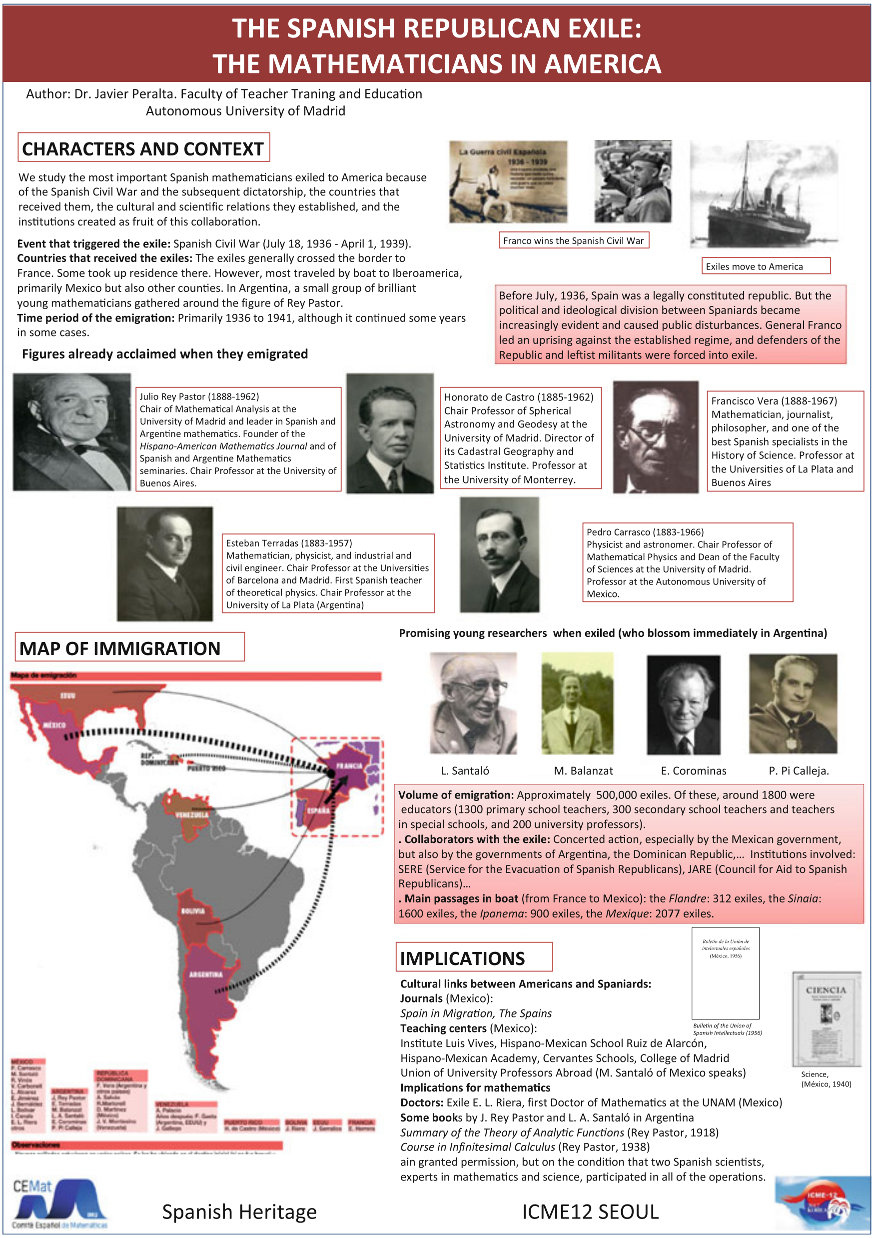

Fig. 4 The Spanish Republican exile: the mathematician in America

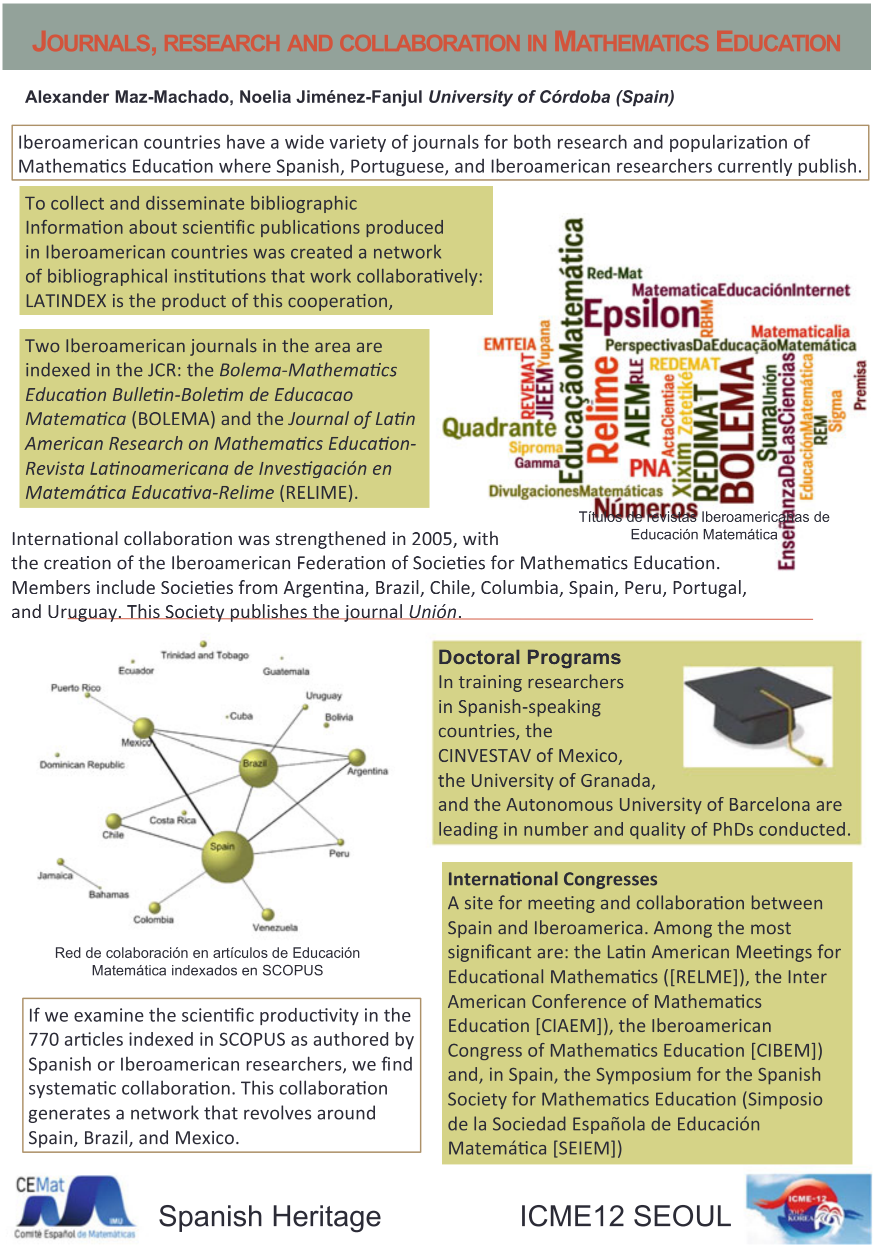

Fig. 5 Journals, research and collaboration in mathematic education

The goal of the presentation was to show and underline the relevance of the cultural and scientific cooperation in mathematics and mathematics education between Spain and the American Republics over the last 500 years. The presentation seeks to publicize this common mathematical heritage by emphasizing its importance and the far-reaching influence these relationships have had and continue to have for science, technology, and education in our countries.