

# Chapter 8

## The Hagedorn Spectrum and the Dual Resonance Model: An Old Love Affair

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**Abstract** In this contribution I recall how people working in the late 1960s on the dual resonance model came to the surprising discovery of a Hagedorn-like spectrum, and why they should *not* have been surprised. I will then turn to discussing the Hagedorn spectrum from a string theory viewpoint (which adds a huge degeneracy to the exponential spectrum). Finally, I will discuss how all this can be reinterpreted in the new incarnation of string theory through the properties of quantum black holes.

### *Preamble*

My first (virtual) “encounter” with Rolf Hagedorn dates back, I think, to my University studies when I read his CERN yellow report on relativistic kinematics and phase space. I remember finding his notes particularly clear and instructive. Much later, I had the privilege of being his colleague in the TH-Division at CERN, and of benefiting from his insight into physics for many years, even after his retirement, when he would still attend regularly the theory seminars. But I was particularly pleased by one of our last (real this time) encounters:

Shortly after the 1994 Divonne Conference we met each other at CERN. He thanked me for my contribution and added something like this:

The dual resonance model (or string theory) gives a microscopic explanation for the spectrum I arrived at using my bootstrap arguments. There is an amusing analogy here to what statistical mechanics does to thermodynamics by providing a microscopic interpretation of entropy.

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## 8.1 A Surprise That Should Not Have Been One

There is a simple (a posteriori!) physical argument for the necessity of a Hagedorn-like spectrum of excited states in any model that satisfies duality (possibly including QCD itself). Let me remind you that, by definition of (Dolen-Horn-Schmit) duality, the asymptotic behavior of (the imaginary part of) any scattering amplitude should be correctly described by **either** Regge pole exchange in the t-channel **or** by resonance formation and decay in the s-channel.

The Regge-pole description gives a power-like behavior, hence, if we work to exponential accuracy, a constant amplitude at high energy. The resonance description yields instead (say, for an elastic two-body collision):

$$\text{Im}A_{\text{el}} \simeq \frac{1}{E} \sum_{\text{R}} \Gamma_{2\text{b}}^{\text{R}} = \sum_{\text{R}} \frac{\Gamma_{\text{T}}^{\text{R}}}{E} B_{2\text{b}}^{\text{R}} \leq N(E) \cdot \bar{B}_{2\text{b}}, \quad (8.1)$$

where  $\Gamma_{2\text{b}}^{\text{R}}$  is the partial width of the resonance ‘R’ in the chosen two-body channel ‘2b’, and we have assumed the total width  $\Gamma_{\text{T}}^{\text{R}}$  to be smaller than the mass  $M_{\text{R}} \sim E$ . This gives a bound on  $\text{Im}A_{\text{el}}$  in terms of the number of states  $N(E)$  at energy  $E$  and of their average branching ratio  $\bar{B}_{2\text{b}}$  into the particular two-body channel under consideration. By asking for consistency of the two descriptions we thus find, to exponential accuracy:

$$N(E) \geq \bar{B}_{2\text{b}}^{-1} \quad (8.2)$$

Making now the (reasonable) assumption that  $\bar{B}_{2\text{b}}$  behaves like the ratio of two-body phase space to total phase space, and using the fact that the latter becomes exponentially small at high energy, we arrive at the “prediction” of an exponentially growing spectrum for duality-fulfilling resonances. The (crucial) power of  $E$  appearing in the exponent is not fixed, however, by this argument.

We may ask why this conclusion was not immediately reached in the very early days of duality. The reason was, I believe, that one was accustomed to associate each resonance with a separate pole in the scattering amplitude; now, the number of poles occurring in the dual-model amplitudes was **not** growing exponentially with increasing energy. All one could see was that the  $N$ th pole contained resonances of spin up to  $N$ , a fact that, in itself, could only account for a power-like growth of the number of states, not for an exponential one.

Of course we know very well now (and for at least 45 years!) the answer to this apparent paradox. The exponential growth of the number of states in the dual resonance model is hidden behind an exponential degeneracy! This degeneracy, which is neither predicted by Hagedorn’s arguments, nor implied by the duality-based reasoning given above, is the truly new ingredient brought in by the dual resonance model. As I shall now argue this degeneracy is directly related to an underlying string picture for the resonances appearing in dual models.

## 8.2 From $T_H$ to the String

When, in the summer of 1968, I first told Sergio Fubini in Torino about my new ansatz for the scattering amplitude his reaction was: “very nice, but... what about negative norm states?” Two months later, I had barely landed in Boston/MIT that we started counting and labeling states, clearly a necessary preliminary step before we could compute their norm.

By early 1969 we had learned how to count (it took longer to answer Sergio’s original question about the norm, but eventually people proved, under certain restrictions, the celebrated no-ghost theorem). The rather unexpected result that Sergio and I (and independently Bardakci and Mandelstam) found was that the individual states were labeled by a set of integers  $\{N_1, N_2, \dots\}$  with the mass of the state given by the simple formula:

$$\frac{\alpha' M^2}{\hbar} = N = \sum_{n=1}^{\infty} n \cdot N_n, \quad (8.3)$$

where  $\alpha'$ , the universal Regge-slope parameter, sets the energy scale of the theory at  $\Lambda \equiv \sqrt{\hbar/\alpha'}$ .

The Hagedorn spectrum then simply comes from the observation that a given (allowed) mass  $M = \sqrt{N/\alpha'}$  can be obtained via Eq. (8.3) in as many ways as the number of ways in which the integer  $N$  can be written as a sum of integers. This “partitio numerorum” number is known to grow like  $\exp(c\sqrt{N})$  (with  $c$  a known constant) hence like  $\exp(c\sqrt{\alpha'/\hbar}E)$ . This gives immediately a Hagedorn temperature  $T_H = c^{-1}\Lambda$ .

In the operator reformulation of the dual resonance model the mass-square operator can be written as

$$\frac{\alpha' M^2}{\hbar} = \sum_{n=1}^{\infty} n a_n^\dagger a_n, \quad (8.4)$$

where  $a_n^\dagger, a_n$  are an infinite set of ordinary harmonic oscillator creation and destruction operators satisfying the usual commutation relations:

$$[a_n, a_m^\dagger] = \delta_{n,m}. \quad (8.5)$$

The high degree of degeneracy of the spectrum obviously comes from the presence of the higher harmonics ( $n = 2, 3 \dots$  in Eq. (8.4)), but this is just what characterizes a vibrating string!

We conclude that the combination of the Hagedorn spectrum and of degeneracy leads straight into strings. Conversely, if a string picture is assumed for hadrons, one immediately predicts:

- (a) duality as implied by drawing (duality) diagrams in which string splitting and joining are the basic processes underlying hadronic reactions.

- (b) a linear relation between mass and entropy (another way of defining Hagedorn's spectrum) coming from the fact that the energy stored in the string is proportional to its length ( $l = \alpha' \cdot E$ ), while there is a unit of entropy per bit of length.

The unit (bit) of length,  $\lambda_s$ , is a quantum object and is related to  $\alpha'$  by:

$$\lambda_s = \sqrt{\alpha' \hbar} . \quad (8.6)$$

For the hadronic string  $\lambda_s$  is of the order of  $10^{-13}$  cm: there is about one bit of information for every fermi of string length.

### 8.3 Crisis, Reinterpretations

One of the main motivations (successes) behind Hagedorn's model was the exponential fall-off of the transverse spectrum of produced particles in high energy collisions:

$$d\sigma/dp_{\perp} \simeq \exp(-p_{\perp}/T_H) . \quad (8.7)$$

This holds well in a sizable region of  $p_{\perp}$ . Not surprisingly, a similar behavior was found to occur in the dual model (in string theory).

The discovery of hard constituents inside the hadron, revealing themselves, e.g., through the power-like drop of jet (or exclusive) cross sections, came as a serious blow to both Hagedorn's model and to the hadronic string.

Amusingly, they both survived, with some reinterpretation, in the emerging new theory of strong interactions, QCD. The Hagedorn temperature was reinterpreted as a deconfining phase transition (rather than as an ultimate) temperature, while strings become an effective description of hadrons as composite systems of quarks which are kept together by a thin tube of chromoelectric field.

Around the time that QCD took over, Joel Scherk and John Schwarz came up with the daring proposal that fundamental strings should be relevant for describing all fundamental interactions (including gravity) at much shorter scales than  $10^{-13}$  cm. The natural scale for the new string is simply the Planck length:

$$\lambda_P = \sqrt{G_N \hbar} , \quad (8.8)$$

where  $G_N$  is Newton's constant. Numerically,  $\lambda_P \sim 10^{-33}$  cm.

Obviously, also the new string has a Hagedorn temperature: it is simply shifted upward by some 18 orders of magnitude to about  $10^{17}$ – $10^{18}$  GeV. In the last part of this talk I shall give one example of some new uses of Hagedorn's temperature in this new context: I shall argue that, in analogy with the reinterpretation of the old  $T_H$  as deconfining temperature, the new  $T_H$  will play the role of a limiting temperature for Black holes, a kind of gravitational-deconfinement temperature, if you like.

## 8.4 Many Years Later ...

There is a (so far undisproved) conjecture by J. Bekenstein that the entropy  $S$  of any physical system of energy  $E$  and size  $R$  cannot be arbitrarily large, i.e.

$$S \leq S_{\text{BB}} \equiv \frac{E \cdot R}{\hbar} . \quad (8.9)$$

This is called the Bekenstein bound. Let us consider now a black hole, i.e. a system of energy  $E$  contained in a spherical region of space of radius  $R < G_{\text{N}} \cdot E$  (this is just the definition of a collapsed state in gravity).

Let us now compare the entropy of the black hole,

$$S_{\text{BH}} = \frac{G_{\text{N}} E^2}{\hbar} , \quad (8.10)$$

with  $S_{\text{BB}}$  in Eq. (8.9), allowing the maximal size for  $R$ ,  $R = G_{\text{N}} \cdot E$ . We see that the bound is precisely saturated, something suggesting that a black hole maximizes the entropy of a system of given energy and spatial extension.

We may now ask if a string of energy  $E$  satisfies the Beckenstein bound. As said before

$$S_{\text{string}} = \frac{\alpha' \cdot E}{\lambda_{\text{s}}} , \quad (8.11)$$

which satisfies the Beckenstein bound Eq. (8.9) only if the size of the string is larger than  $\lambda_{\text{s}}$ , a conclusion that can be reached also by independent considerations.

I have told you that  $\lambda_{\text{s}}$  is of the same order as  $\lambda_{\text{P}}$ , but the more precise relation is actually

$$\lambda_{\text{P}} = \alpha_{\text{gut}}^{1/2} \lambda_{\text{s}} , \quad (8.12)$$

expressing the physical fact that, in string theory, gravitational and gauge interactions become identical at the (distance) scale  $\lambda_{\text{s}}$ .

We can finally compute the ratio between the string and black hole entropies and find

$$\frac{S_{\text{BH}}}{S_{\text{string}}} = \frac{G_{\text{N}} E \lambda_{\text{s}}}{\alpha' \hbar} = \frac{E}{M_{\text{P}}} \cdot \frac{\lambda_{\text{P}}}{\lambda_{\text{s}}} = \frac{\alpha_{\text{gut}}^{1/2} \cdot E}{M_{\text{P}}} . \quad (8.13)$$

This ratio becomes 1 at  $E = \alpha_{\text{gut}}^{-1/2} M_{\text{P}}$ . At this energy both entropies are of order  $\alpha_{\text{gut}}^{-1}$ , thus probably between 10 and 100. Above this energy, entropy considerations favor the black hole while below a string state is favored. It is easy to see that such a state is not collapsed at all since its physical size, as we argued above, has to be larger than  $\lambda_{\text{s}}$  which, in turn, is larger than the gravitational radius  $G_{\text{N}} \cdot E$  of the system.

As Hawking has shown, black holes evaporate, losing mass while increasing their temperature. In ordinary gravity this process would continue, until a curvature singularity (and an infinite temperature) is reached. The arguments given above (and due to several people) suggest that, in string theory, black hole evaporation should stop at a certain point, leaving behind a non-collapsed string system of entropy  $\mathcal{O}(\alpha_{\text{gut}}^{-1})$ .

The Hawking temperature of the black hole at this point is just the Hagedorn temperature of the string theory under consideration. We can thus say that an interpretation of  $T_H$  in the new incarnation of string theory is that of a “decollapse” temperature if you allow me to use such a word for the gravitational analogue of deconfinement. On the other hand, the analogue of the quark-gluon-plasma phase of QCD in quantum string gravity is still clouded with mystery (is space-time itself “melting”?).

There could be also a cosmological analogue of the limiting temperature for black holes, as it is known that there is a (Hawking) temperature associated with the event horizon of inflationary cosmologies and easily expressible in terms of the Hubble parameter during inflation. This could lead to new insights in the way string-Hagedorn models deal with the very beginning of the Universe.

## Conclusion

The love affair between dual and Hagedorn models is still well and alive after many years: and it seems it will last forever (Fig. 8.1).

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# Hagedorn's Reincarnation in String Theory

John Ellis

## Abstract

After a review of Hagedorn's first incarnation in the quark-hadron phase transition, universality properties of the state level density in string theory are discussed, as well as the possible transition to a new phase beyond the string Hagedorn temperature. Studies of string black holes suggest that the concepts of space and time are likely to disappear in this reincarnation of Hagedorn.

**Fig. 8.1** A love affair to one is reincarnation to another

