Chapter 11
Hagedorn's Hadron Mass Spectrum and the Onset of Deconfinement

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Abstract A brief history of the observation of the onset of deconfinement—the beginning of the creation of quark-gluon plasma in nucleus–nucleus collisions with increasing collision energy—is presented. It starts with the measurement of the hadron mass spectrum and Hagedorn's hypothesis of the limiting temperature of hadronic matter (the Hagedorn temperature). Then the conjecture that the Hagedorn temperature is the phase transition temperature was formulated with the crucial Hagedorn participation. It was confirmed by the observation of the onset of deconfinement in lead–lead collisions at the CERN SPS energies.

11.1 Hadron Mass Spectrum and the Hagedorn Temperature

A history of multi-particle production started with the discoveries of hadrons, first in cosmic-ray experiments and soon after in experiments using beams of particles produced in accelerators. Naturally, the first hadrons, discovered in collisions of cosmic-ray particles, were the lightest ones, pion, kaon and Λ. With the rapid advent of particle accelerators new particles were uncovered almost daily. About 1,000 hadronic states are known so far. Their density in mass $\rho(m)$ increases approximately exponentially as predicted by Hagedorn's Statistical Bootstrap Model [1] formulated in 1965:

$$\rho(m) \; = \; \text{const} \, m^{-a} \, \exp(b\,m) \; . \tag{11.1}$$

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In the case of point-like hadron states this leads to a single-particle partition function:

$$Z(T, V) \; = \; \frac{V}{2\pi^2} \int_{m_\pi}^{\infty} dm \int_0^{\infty} k^2 dk \, \exp\left(-\frac{\sqrt{k^2 + m^2}}{T}\right) \rho(m) \, , \qquad (11.2)$$

where V and T are the system volume and temperature, respectively. The m-integral exists only for $T < 1/b$. Thus, the hadron gas temperature is limited from above. Its maximum temperature $T_H = 1/b$ (the so-called Hagedorn temperature) was estimated by Rolf Hagedorn based on 1965 data to be $T_H \cong 160$ MeV. More recent estimates of the Hagedorn temperature(s) can be found in [2], for further discussion see Chaps. 20 and 21.

The first statistical model of multi—hadron production was proposed by Fermi [3] in 1950. It assumes that hadrons produced in high energy collisions are in equilibrium and that the energy density of the created hadronic system increases with increasing collision energy. Soon after, Pomeranchuk [4] pointed out that hadrons cannot decouple (freeze-out) at high energy densities. They will rather continue to interact while expanding until the matter density is low enough for interactions to be neglected. Pomeranchuk estimated the freeze-out temperature to be close to pion mass, ≈ 150 MeV. Inspired by this idea Landau [5] and his collaborators formulated a quantitative hydrodynamical model describing the expansion of strongly interacting hadronic matter between the Fermi's equilibrium high density stage (the early stage) and the Pomeranchuk's low density decoupling stage (the freeze-out). The Fermi-Pomeranchuk-Landau picture serves as a base for modeling high energy nuclear collisions up to now [6].

Hagedorn's conjecture concerning the limiting temperature was in contradiction to the Fermi-Pomeranchuk-Landau model in which the temperature of hadronic matter created at the early stage of collisions increases monotonically with collision energy and is unlimited.

11.2 Discovery of the Onset of Deconfinement

The quark model of hadron classification proposed by Gell-Mann and Zweig in 1964 starts a 15 years-long period in which sub-hadronic particles, quarks and gluons, were discovered and a theory of their interactions, quantum chromodynamics (QCD) was established. In parallel, conjectures were formulated concerning the existence and properties of matter consisting of sub-hadronic particles, soon called the QGP and studied in detail within the QCD [7].

Ivanenko and Kurdgelaidze [8], Itoh [9] and Collins and Perry [10] suggested that quasi-free quarks may exist in the centre of neutron stars. Many physicists started to speculate that the QGP can be formed in nucleus–nucleus collisions at high energies and thus it may be discovered in laboratory experiments. Questions

concerning QGP properties and properties of its transition to matter consisting of hadrons were considered since the late 1970s.

Cabibbo and Parisi [11] pointed out that the exponentially increasing mass spectrum proposed by Hagedorn may be connected to the existence of the phase in which quarks are not confined. Then Hagedorn and Rafelski [12], see Chap. 23, Gorenstein et al. [13] suggested that the Hagedorn massive states are not point-like objects but quark-gluon bags. These picture leads to the interpretation of the upper limit of the hadron gas temperature, the Hagedorn temperature, as the transition temperature from the hadron gas to a quark-gluon plasma. Namely, at $T > T_H$ the temperature refers to an interior of the quark-gluon bag, i.e., to the QGP.

In the mid-1990s the Statistical Model of the Early Stage (SMES) was formulated [14] as an extension of Fermi's statistical model of hadron production. It assumes a statistical production of confined matter at low collision energies (energy densities) and a statistical QGP creation at high collision energies (energy densities). The model predicts a rapid change of the collision energy dependence of hadron production properties, that are sensitive to QGP, as a signal of a transition to QGP (the onset of deconfinement) in nucleus–nucleus collisions. The onset energy was estimated to be located in the CERN SPS energy range.

Clearly, the QGP hypothesis and the SMES model removed the contradiction between Fermi's and Hagedorn's statistical approaches. Namely, the early stage temperature of strongly interacting matter is unlimited and increases monotonically with collisions energy, whereas there is a maximum temperature of the hadron gas, $T_H \approx 160\,MeV$, above which strongly interacting matter is in the QGP phase.

Rich data from experiments at the CERN SPS and LHC as well as at the BNL AGS and RHIC clearly indicate that a system of strongly interacting particles created in heavy collisions at high energies is close to, at least local, equilibrium. At freeze-out the system occupies a volume which is much larger than a volume of an individual hadron. Thus, one concludes that strongly interacting matter is created in heavy ion collisions [6].

The phase transition of strongly interacting matter to the QGP was discovered within the energy scan program of the NA49 Collaboration at the CERN SPS [15, 16]. The program was motivated by the predictions of the SMES model. The discovery was based on the observation that several basic hadron production properties measured in heavy ion collisions rapidly change their dependence on collisions energy in a common energy domain [17], see Fig. 11.1.
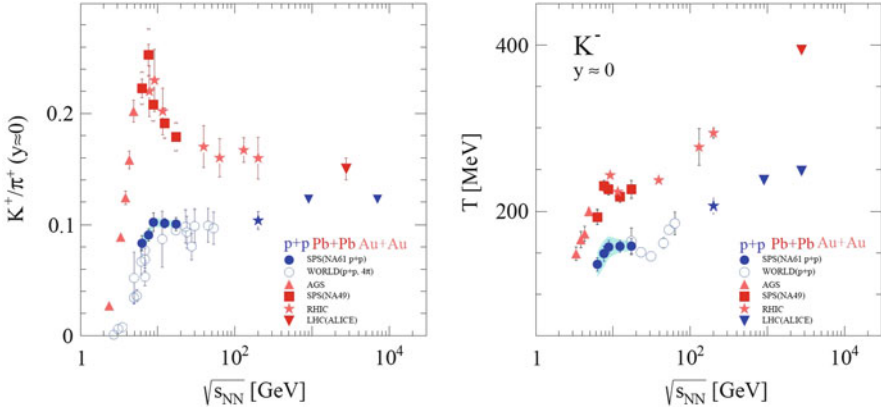
Fig. 11.1 Recent results on the observation of the phase transition in central Pb+Pb (Au+Au) collisions [18]. The horn (*left*) and step (*right*) structures in energy dependence of the K^+/π^+ ratio and the inverse slope parameter of K^- $m_\perp$ spectra signal the onset of deconfinement located at the low CERN SPS energies

Fig. 11.2 Marek Gaździcki (facing to right off *center*) at Hagedorn Divonne Fest, June 30, 1994. *Photo: Lucy Carruthers*

The first ideas which resulted in formulation of the SMES model were presented by one of us [19] (Fig. 11.2) at the Workshop on *Hot hadronic matter: Theory and experiment*, which took place in Divonne, France in June 1994. The workshop was dedicated to 75th birthday of Rolf Hagedorn.

Dear Dr. Gaździcki,

it was a pleasure to listen to your presentation of entropy production in nuclear collisions. It would be interesting to see your interpretation of the overproduction in S+S collisions at 200 GeV/c/A confirmed by future data and by independent theoretical estimates of the change of entropy at the phase transition. A lot of work remains to be done before we know there is a (one, maybe two?) phase transition and what are the details. Good luck for the future!

Yours Rolf Hagedorn

Fig. 11.3 The letter of Rolf Hagedorn to Marek Gaździcki commenting the first talk on the onset of deconfinement in nucleus–nucleus collisions at the low CERN SPS energies [19] presented in June 1994 at the Divonne workshop dedicated to Rolf Hagedorn on occasion of his 75th birthday

Hagedorn's letter on the presentation is reprinted in Fig. 11.3 in lieu of a summary.

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