Chapter 18
About 'Distinguishable Particles'

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Abstract I present the context in which Hagedorn withdrew his first work on limiting temperature, show his withdrawal note, and discuss what was lost from view for 50 years while the manuscript shown in Chap. 19 lingered in the archives. I close describing the contemporary meaning of Hagedorn temperature.

18.1 Withdrawn Manuscript

In early 1978 Rolf Hagedorn shared with me a copy of his unpublished manuscript 'Thermodynamics of Distinguishable Particles: A Key to High-Energy Strong Interactions?', a preprint CERN-TH-483 dated 12 October 1964. He said there were two copies; I was looking at one; another was in the CERN archives. A quick glance sufficed to reveal that this was, actually, the work proposing a limiting temperature and the exponential mass spectrum. Hagedorn explained that upon discussions of the contents of his paper with Léon Van Hove, he evaluated in greater detail the requirements for the hadron mass spectrum and recognized a needed fine-tuning. Hagedorn concluded that his result was therefore too model-dependent to publish, and in the CERN archives one finds Hagedorn commenting on this shortcoming of the paper. These comments are printed below, and can be appreciated in full after reading Chap. 19.

However, Hagedorn's 'Distinguishable Particles' is a clear stepping stone on the road to a better understanding of strong interactions and particle production. The insights gained in this work allowed Hagedorn to rapidly invent the Statistical Bootstrap Model (SBM). The SBM paper 'Statistical Thermodynamics of Strong Interactions at High Energies', preprint CERN-TH-520 dated 25 January 1965, was published in Nuovo Cim. Suppl. **3**, pp. 147–186 (1965).

In the SBM model, a hadron exponential mass spectrum with the required properties is a natural outcome. However, it took time for the SBM model to be recognized. Arguably, this was so because the stepping-stone 'Distinguishable Particles' manuscript had not seen the light of day. The need for the 'right' mass spectrum

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was not evident to the reader of the SBM paper. SBM is presented in this book both conceptually and with mathematical detail as relating to hot nuclear matter in Chap. 23, and the relation to phase transition to quark matter is further developed in Chap. 27. A historical SBM perspective that discusses the role of distinguishable particles is offered in Chap. 17. The unpublished 'Distinguishable Particles' paper, motivating SBM, is published here for the first time in the following Chap. 19.

18.2 Note by Rolf Hagedorn of 27 October 1964

The logical difficulty mentioned on p. 213 has been removed as follows, and the result is disappointing. Let α and β label possible momenta (in a two-dimensional box of volume $V_0^{2/3}$) and kinds of particles, respectively. Then

$$\log Z = \sum_{\alpha,\beta} \log\left(1 - \mathrm{e}^{-\sqrt{p_\alpha^2+m_\beta^2}/T}\right)^{-1} .$$

We replace

$$\sum_\alpha \longrightarrow \frac{V_0^{2/3}}{2\pi}\int_0^\infty p\mathrm{d}p \ldots , \qquad \sum_\beta \longrightarrow \int_0^\infty \rho(m)\mathrm{d}m \ldots ,$$

and expand the logarithm:

$$\log Z = \frac{V_0^{2/3}}{2\pi} T^2 \sum_{n=1}^{\infty} \frac{1}{n^3} \int_0^\infty \rho(m)\mathrm{d}m \left(1 + \frac{nm}{T}\right) \mathrm{e}^{-nm/T} .$$

Now everything depends on the asymptotic behaviour of the mass spectrum $\rho(m)$:

1. If $\rho(m)$ grows faster than exponentially, $\log Z$ diverges for all $T > 0$. No thermodynamics is possible.
2. If $\rho(m)$ grows $\sim \mathrm{e}^{m/T_0}$, then $\log Z$ diverges for $T > T_0$. Whether it diverges for $T = T_0$ and if so, how it diverges for $T = T_0$ depends on the detailed behaviour of $\rho(m)$. In any case, an upper bound T_0 exists.
3. If $\rho(m)$ grows less than exponentially, no upper bound T_0 exists; if $\rho(m)$ grows exponentially up to some large M and afterwards grows less than exponentially, then the system would—over some energy range which depends on M—behave as if a highest temperatures existed.
4. In the 'distinguishable particles' model, Z diverges as $T_0/(T_0 - T)$ for $T \to T_0$ (two-dimensional case). Such behaviour can be obtained from a particular choice of $\rho(m)$, e.g.,

$$\rho(m) \longrightarrow \frac{1}{1 + m/T_0}\frac{2}{m}\sinh\frac{m}{T_0}$$

and with $V_0 = (2\pi)^{3/2}/T_0^3$. It is seen, therefore that every type of exponential growth leads to an upper bound T_0, but only a very particular type leads to a behaviour which, for $T \to T_0$, coincides with that of our model. The model describes therefore a highly singular case[1] and should be abandoned. *Its main point, the highest temperature T_0 has still a chance to survive*, maybe only as an apparent ‘highest temperature’ in a large energy range (this, at least, is suggested by the experiments).

5. As the upper bound T_0 does not depend on the volume any more the simple description by means of a longitudinal and a transverse temperature must be abandoned.

18.3 From Distinguishable Hadrons to SBM

The beginning of a new idea in physics often seems to hang on a very fine thread: was anything lost when ‘Thermodynamics of Distinguishable Particles’ remained unpublished? And what would Hagedorn do after withdrawing his first limiting-temperature paper? My discussion of the matter with Hagedorn suggests that his vision at the time of how limiting temperature could be justified evolved very rapidly. Presenting his final insight was what interested Hagedorn and motivated his work. Therefore, he opted to work on the more complete theoretical model.

While the withdrawal of the old, and the preparation of an entirely new paper seemed to be the right path to properly represent the evolving scientific understanding, today’s perspective is different. In particular the insight that the appearance of a large number of different hadronic states allows to effectively side-step the quantum physics nature of particles within statistical physics became essentially invisible in the ensuing work. Few scientists realize that this is a key property in the SBM, and the fundamental cause allowing the energy content to increase without an increase in temperature, as Hagedorn explains in the withdrawal note, see also the end of Sects. 17.2 and 19.1.

The loss of relevance of quantum physics in hot hadronic matter is the scientific fact that we lost sight of after ‘Distinguishable Particles’ was withdrawn. To the best of my knowledge the dense, strongly interacting hadronic gas is the only physical system where this happens. Normally, the greater the density of particles, the greater the role of quantum physics. After surfacing briefly in Hagedorn’s withdrawn ‘Thermodynamics of Distinguishable Particles’ paper, this finding faded from view. This indeed was a new idea in physics hanging on a very fine thread which ripped.

On the other hand, the Hagedorn limiting temperature lived on. Within a span of only 90 days between the withdrawal of his manuscript, and the date of his new CERN-TH preprint, Hagedorn formulated the Statistical Bootstrap Model. Its salient feature is that the exponential mass spectrum arises from the principle that

[1]Editor’s note: today called ‘fine-tuned’.

hadrons are clusters comprising lighter (already clustered) hadrons. The key point of this second paper is a theoretical model based on the very novel idea of hadrons made of other hadrons. Such a model bypasses the need to identify constituent content of all these particles.

Models of this type are employed in other areas of physics. The simplest one is the statement that while atomic nuclei are made of individual nucleons, a great improvement in the understanding of nuclear structure is achieved if we cluster four nucleons (two protons and two neutrons) into an α-particle introducing the α-substructure, and so on. Hagedorn simply made all hadrons be clusters of lightest mesons, pions. The difference to the nuclear α-model is that the number of pions and more generally of all strongly interacting particles is not conserved. That turns out to have a big consequence and is the origin of the limiting temperature behavior.

Clustering pions into new hadrons and then combining these new hadrons with pions, and with already preformed clusters, and so on, turned out to be a challenging but soluble mathematical exercise. The outcome in this new Statistical Bootstrap Model (SBM) was that the number of states of a given mass was growing exponentially. Thus, in SBM, the exponential mass spectrum required for the limiting temperature arose naturally ab-initio. Furthermore the model established a relation between the limiting temperature, the exponential mass spectrum slope, and the pion mass, which provides the scale of energy in the model.

18.4 Hagedorn Temperature as a General Physics Concept

The presentation of the original limiting temperature article Chapter 19 is in part motivated by the reccent developments which adopt the concept in domains of physics that are entirely unrelated; it is the physics principle that leads the way. Examples taken from search of the name 'Hagedorn' in title of a publication unrelated to the domain of physics in which Hagedorn worked are for example:

- T. Biswas, T. Koivisto and A. Mazumdar, "Atick-Witten **Hagedorn** Conjecture, near scale-invariant matter and blue-tilted gravity power spectrum," JHEP **1408**, 116 (2014) [arXiv:1403.7163].
- A. Arslanargin and A. Kaya, "Open Strings on D-Branes and **Hagedorn** Regime in String Gas Cosmology," Phys. Rev. D **79**, 066013 (2009) [arXiv:0901.4608].
- R. H. Brandenberger, A. Nayeri, S. P. Patil and C. Vafa, "Tensor Modes from a Primordial **Hagedorn** Phase of String Cosmology," Phys. Rev. Lett. **98**, 231302 (2007).