Chapter 20
On the Hadronic Mass Spectrum

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Abstract We argue that the sole requirement of a self-consistent bootstrap including all hadrons up to infinite mass leads to asymptotically exponential laws for the hadron mass spectrum, for momentum distributions, and for form factors (and to a highest temperature).

Over the last few years an increasing number of hadron mass formulas and, recently, of speculations about the whole hadronic mass spectrum have been published, all of them based on group theoretical considerations, quark models, or the like. We present here a different approach, a kind of asymptotic bootstrap, resulting from the 'thermodynamical model' and dealing only with the spectral density $\rho(m)$. The model has been described in three papers [1] entitled *Statistical Thermodynamics of Strong Interactions at High Energies I, II, and III*. The present consideration is a small but basic part of it.

In the thermodynamical model we describe highly excited hadronic matter by relativistic quantum statistical thermodynamics, allowing arbitrary absorption and creation of hadrons (and antihadrons) of all kinds, including all resonances. As the spectrum of resonances cannot be limited, we take into account all of them, even the not yet discovered ones. It goes as follows: we introduce one common name 'fireballs' for all hadrons and postulate (the feedback arrow is most important!).

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Postulate. A fireball is

$$
\begin{array}{lr}
\rightarrow \text{ a statistical equilibrium of an undetermined number of all} & \\
\uparrow \text{ kinds of fireballs, each of which is in turn considered to be} & \rightarrow \\
\uparrow & \downarrow \\
\leftarrow \quad \leftarrow \quad \leftarrow \quad \leftarrow \quad \leftarrow \quad \leftarrow \quad \leftarrow \quad \leftarrow \quad \leftarrow \quad \leftarrow \quad \leftarrow \quad \leftarrow & \leftarrow
\end{array}
\tag{20.1}
$$

We forget about complications like collective motions (in non-central collisions) and imagine ideal equilibrium (realistic fireballs are discussed in Part II of [1]). One writes down the partition function $Z(V,T)$ for a gas consisting of an undetermined number of all kinds of particles (fireballs) which must be labeled, for instance, by their mass m. In calculating Z, one has to sum over all single-particle momentum states, over all possible numbers of particles (bosons $0,\ldots,\infty$, fermions 0,1), and over all possible kinds of particles (hadrons and anti-hadrons). The latter is done by introducing the number of hadron states between m and $m + \mathrm{d}m$, namely, $\rho(m)\mathrm{d}m$. With this (unknown) function $\rho(m)$, the partition function becomes (see Part I of [1])

$$Z = \exp\left[\int_0^\infty \rho(m)F(m,T)\mathrm{d}m\right] , \tag{20.2}$$

with a known function $F(m,T)$. On the other hand, Z can be written (see any book on statistical mechanics)

$$Z = \int_0^\infty \sigma(E)\mathrm{e}^{-E/T}\mathrm{d}E , \tag{20.3}$$

where $\sigma(E)$ is the number of states between E and $E+\mathrm{d}E$ of the fireball considered. As for this fireball $E = m$ (we stay in its rest frame), we can say as well that we have for our 'main' fireball $\sigma(m)\mathrm{d}m$ states in the mass interval $\{m, \mathrm{d}m\}$. Now $\rho(m)$ is the number of hadron states in the interval $\{m, \mathrm{d}m\}$ and if our postulate (20.1) above is applied, it follows that asymptotically $\rho(m)$ and $\sigma(m)$ must somehow become the same. A detailed discussion (see Part I of [1]) reveals that one cannot require more than that

$$\frac{\log \rho(m)}{\log \sigma(m)} \underset{m\to\infty}{\longrightarrow} 1 , \tag{20.4}$$

which says that, for $m \to \infty$, the entropy of a fireball is the same function of its mass as the entropy of the fireballs of which it is composed. This implies that *all fireballs are on an equal footing*.

We now equate the two expressions (20.2) and (20.3) and require simultaneously that Eq. (20.4) should be valid. It is shown in Part I of [1] that $F(m,T)$ falls off asymptotically as $m^{3/2}\exp(-m/T)$ and that therefore

$$Z \longrightarrow \exp\left[\int_0^\infty m^{3/2}\rho(m)\mathrm{e}^{-m/T}\mathrm{d}m\right] \longleftrightarrow \int_0^\infty \sigma(m)\mathrm{e}^{-m/T}\mathrm{d}m\,. \tag{20.5}$$

This is consistent with the bootstrap requirement (20.4) if and only if[1]

$$\rho(m) \underset{m\to\infty}{\longrightarrow} \frac{\text{const.}}{m^{5/2}}\mathrm{e}^{m/T_0}\,. \tag{20.6}$$

It follows that T_0 is *the highest possible temperature*—a kind of 'boiling point of hadronic matter' in whose vicinity particle creation becomes so vehement that the temperature cannot increase further, no matter how much energy is fed in.

An immediate consequence is a Boltzmann-type momentum distribution [asymptotically $\sim \exp(-p_\perp/T)$] with $T \lesssim T_0$, but never larger than T_0! This explains why the transverse momentum distribution in high energy jets is practically energy independent (for all details and possible deviations, see Part II of [1]).

Back to the mass spectrum: $\rho(m)$ counts each state (spin, etc.) separately and includes antiparticles. If one smooths out the experimental mass spectrum [2], one obtains Fig. 20.1, in which an exponential increase is seen in the region $\lesssim$1,000 MeV, i.e., in that region where we know almost all resonances. Extrapolating the experimental curve with an expression having the required asymptotic behavior, Eq. (20.6) yields

$$T_0 = 160 \pm 10\,\text{MeV}\,, \tag{20.7}$$

and with this value, excellent fits (ranging over ten orders of magnitude) to the momentum spectra and multiplicities in high energy production processes are obtained (see Part II of [1]). It is then only natural to expect [3] the form factors to decrease as $\sim \exp\big(-|t^2|^{1/2}/4T_0\big)$.

We treat hadrons as self-consistently infinitely composed of all other hadrons—this is what the postulate (20.1) says. If all hadrons are virtually contained in each of them, it is natural to assume that all phase relations between the infinitely many contributing amplitudes wash out and that therefore statistical thermodynamics is adequate to treat this asymptotic bootstrap. Although the technique is unconventional, it is not so far from the usual ones as one might think. An intimate relation between the mass spectrum and the momentum distribution in multiparticle production seems unavoidable in any theory, and the Gibbs ensemble description

[1] It is not possible to have this $\rho(m)$ cut off somewhere because this would imply two types of essentially different fireballs: one with almost exponential density of states, the other with asymptotically vanishing density of states, and both would contribute and exist on an equal footing. This is inconsistent.

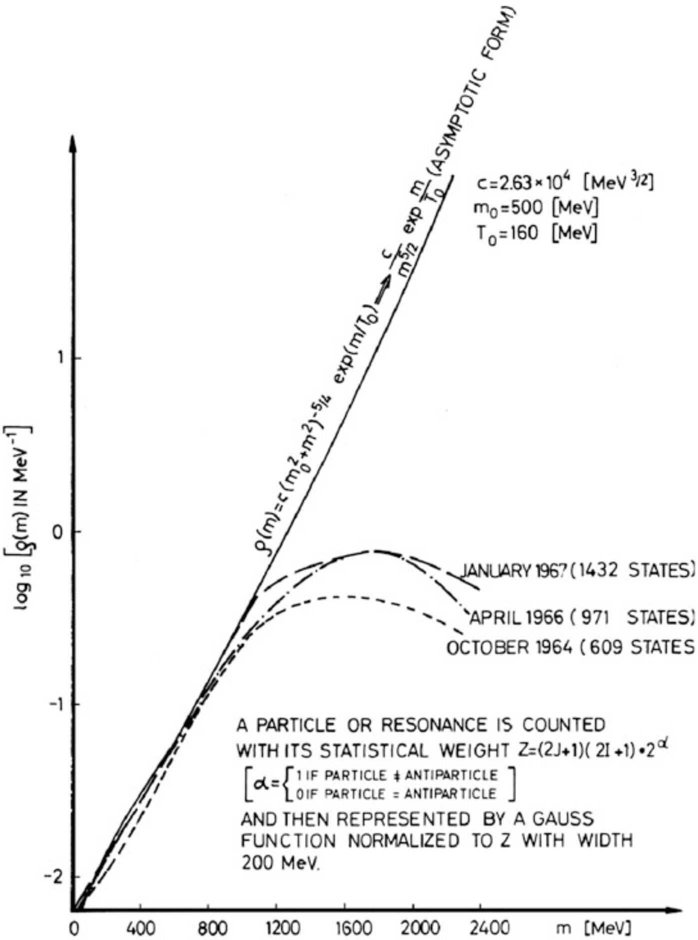

Fig. 20.1 The experimental mass spectrum smoothed by Gauss functions as indicated in figure, experimental spectra for three (1964, 1966, 1967) sets of particle data, and a fit by a simple function with the asymptotic behavior required by Eq. (20.6). The normalization constant c is a fitted parameter, m_0 is an estimated value

with fixed T somehow resembles off-shell effects because the masses of fireballs present at temperature T extend to infinity (with exponentially falling weight).

It will be impossible to prove or disprove our mass density in Eq. (20.6) by direct experiments, because the density increases exponentially and the production cross-section for each individual resonance decreases exponentially with m—the two mechanisms act in common against the experimenter.

Any 'proof' of Eq. (20.6) will be indirect, but the internal consistency of the thermodynamic model and the good agreement of its predictions with an enormous amount of experimental data (see Parts II and III of [1]) provides strong indirect support. In this respect, it is relevant that, in the applications of the model (see Parts II and III of [1]), the asymptotic mass spectrum is used explicitly in integrals over m extending to infinity.

Recently, two papers [4] have been presented which use conventional quantum mechanical techniques to construct infinitely composed, self-consistent hadrons. A variety of different model assumptions were shown to lead to one common behavior: the form factors fall off asymptotically as $\exp(-\text{const.} \times |t|^{1/2})$ in complete analogy with our result on momentum spectra. It seems then that the sole requirement of self-consistent infinite compositeness is sufficient to produce these asymptotically exponential laws for mass spectra, momentum distributions, and form factors—at least this is strongly suggested by the fact that the thermodynamical model does not make any other assumption and that, in the papers by Stack and Harte, this assumption was the only one common to their various models.

In future, one should distinguish the 'vicinity of the boiling point of hadronic matter,'[2] where $T \to T_0$ and $E \to \infty$, and where literally all hadrons merge into each other. It follows from the small value $T \approx 160\,\text{MeV}$ ($1.86 \times 10^{12}\,\text{K}$) that $E \to \infty$ means in this respect E above 10 GeV (for quantitative relations, see Part II of [1]).

We conclude this letter with a curiosity—or perhaps not a curiosity. Consider a class of fireballs $f_n^{(i)}$ with roughly equal mass $m_n^{(i)}$, composed of quarks and antiquarks (n of them altogether, with n large). As the quark has 12 states [SU(3) × SU(2) × antiparticle conjugation], this class of fireballs will have 12^n states ($i = 1, \ldots, 12^n$) if one assumes that each quark is in the ground state relative to all others (contrary to current models where, e.g., orbital momenta are discussed: here too they might be built in if one tries harder). Assume further that (as in nuclear physics) each of them contributes roughly the same and N independent amounts Δm to the average mass $\langle m \rangle_n$ of these fireballs. Then,

$$\langle m \rangle_n = \Delta m \times n \,. \tag{20.8}$$

For large n, the number of fireballs of mass $\approx \langle m \rangle_n$ becomes

$$z(m) = 12^n = \exp(n \log 12) = \exp\left(\langle m \rangle \frac{\log 12}{\Delta m}\right) \,. \tag{20.9}$$

[2]For symmetries, etc., one had better look at the 'vicinity of the freezing point', so to speak, namely, where most channels are frozen in.

The quantity Δm can be estimated by using the meson 35-plet and/or the baryon 56-plet, taking the average mass of each of these multiplets

$$\Delta m = \frac{\langle m_{35} \rangle}{2} = \frac{\langle m_{56} \rangle}{3} \,. \tag{20.10}$$

We find with $\langle m_{35} \rangle \approx 700\,\mathrm{MeV}$ and $\langle m_{56} \rangle \approx 1{,}050\,\mathrm{MeV}$,

$$z(m) = \exp \frac{\langle m \rangle}{140} \qquad (\langle m \rangle \text{ in MeV}) \,. \tag{20.11}$$

It might be an accident that this is the leading term of Eq. (20.6) with a reasonable value of T_0. (It might be no accident.)

There is no contradiction in considering a fireball as built of fireballs and at the same time as built of quarks—superfluid helium is understood only if considered as a boson liquid, but after all it 'really' consists of fermions. Such pictures are complementary.

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