Chapter 6
The Tale of the Hagedorn Temperature

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Please note the Erratum to this chapter at the end of the book

Abstract We recall the context and impact of Rolf Hagedorn's discovery of limiting temperature, in effect a melting point of hadrons, and its influence on the physics of strong interactions.

6.1 Particle Production

Collisions of particles at very high energies generally result in the production of many secondary particles. When first observed in cosmic-ray interactions, this effect was unexpected for almost everyone,[1] but it led to the idea of applying the wide body of knowledge of statistical thermodynamics to multiparticle production processes. Prominent physicists such as Enrico Fermi, Lev Landau, and Isaak Pomeranchuk made pioneering contributions to this approach, but because difficulties soon arose this work did not initially become the mainstream for the study of particle production. However, it was natural for Rolf Hagedorn to turn to the problem.

Hagedorn had an unusually diverse educational and research background, which included thermal, solid-state, particle, and nuclear physics. His initial work on statistical particle production led to his prediction, in the 1960s, of particle yields at the highest accelerator energies at the time at CERN's proton synchrotron. Though there were few clues on how to proceed, he began by making the most of the 'fireball' concept, which was then supported by cosmic-ray studies. In this approach, all the energy of the collision was regarded as being contained within a small space-time volume from which particles radiated, as in a burning fireball.

Adapted from an article which appeared in the September 2003 issue of CERN Courier.

[1]Expected for example by W. Heisenberg, see Z. Phys. **126**, 569 (1949) and references therein. We thank H. Satz for bringing this work to our attention.

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Several key ingredients from early experiments helped him to refine this approach. Among these observations, the most noticeable was the limited transverse momentum of the overwhelming majority of the secondary particles. Also, the elastic scattering cross-section at large angles was found to drop exponentially as a function of incident energy. Such behavior strongly suggested an inherently thermal momentum distribution.

However, many objections were raised in these pioneering days of the early 1960s. What might actually be 'thermalized' in a high-energy collision? Applying straightforward statistical mechanics gave too small a yield of pions. Moreover, even if there was a thermalized system in the first place, why was the apparent temperature constant? Should it not rise with incident beam energy?

It is to Hagedorn's great credit that he stayed with his thermal interpretation, solving the problems one after another. His particle-production models turned out to be remarkably accurate at predicting yields for the many different types of secondaries that originate in high-energy collisions. He understood that the temperature governing particle spectra does not increase, because as more and more energy is poured into the system, new particles are produced.

It is the entropy that increases with the collision energy. If the number of particles of a given mass (or mass spectrum) increases exponentially, the temperature becomes stuck at a limiting value. This is the Hagedorn temperature T_{H}. The value of T_{H} is hard to pin precisely, as it depends also on other parameters of the strongly interacting particles still evolving in our understanding today as exact mathematical tools, such as lattice gauge theory, mature. Hagedorn gave its value as $T_{\mathrm{H}} \simeq 150\,\mathrm{MeV}$, but it may be as low as the mass of the lightest hadron, the pion, $T_{\mathrm{H}} \approx m_{\pi} \approx 140\,\mathrm{MeV}$ and as high as $160\,\mathrm{MeV}$.

The impressive number of distinguishable hadronic states which now have to be considered at the same time leads to a rewriting of equations based on statistical physics, and introducing numerous massive hadron resonances which eventually fragment into less massive ones to yield the observed secondary particles. At the 'bottom line', this solved the problem of the pion yield. The factor $1/n!$, which originated in the quantum indistinguishability of identical particles, had plagued the statistical calculations that focused only on pions. Now it had become unimportant as each one of the many states was unlikely to have a population, n, exceeding 1. At long last agreement between experiment and statistical calculations prevailed.

6.2 The Statistical Bootstrap Model

Once these physical facts had been assembled, explanation of the observed hadron production was at hand. However, the implementation of the model required considerable fine-tuning of parameters and mathematical equations, a situation which Hagedorn in the end did not like. Hagedorn turned his attention to improving the theoretical and conceptual interpretation, in particular to present a natural

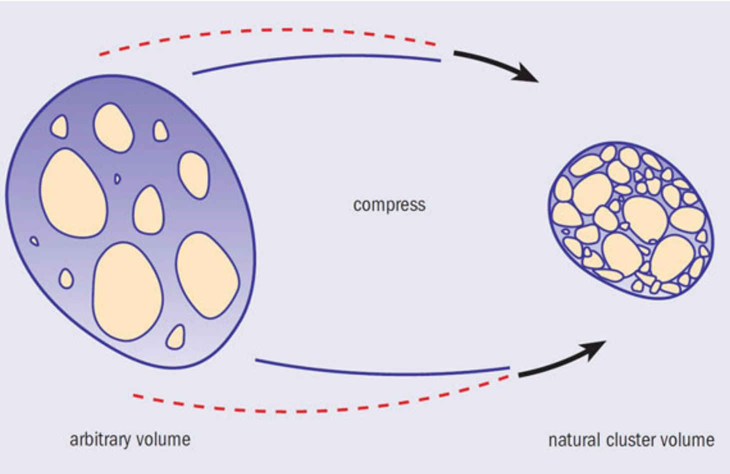

Fig. 6.1 An illustration of the statistical bootstrap idea. When a drop comprising hadronic particles and resonances is compressed to the 'natural volume', it becomes another more massive resonance. This process then repeats, creating heavier resonances, which consist of hadron resonances, which in turn consist of resonances, and so on. *Picture: CERN Courier September 2003 p. 30*

mechanism generating the numerous hadronic resonances which dominate the scattering cross-section. He proposed the statistical bootstrap model (SBM).

In a nutshell, in the SBM, each of the many resonant states into which hadrons can be excited through a collision is itself a constituent of a still heavier resonance, whilst also being composed of lighter ones. In this way, when compressed to its natural volume, a matter cluster consisting of hadron resonances becomes a more massive resonance with lighter resonances as constituents, as shown in Fig. 6.1. One day in late 1964, one of us (TE) ran into Hagedorn: this must have been soon after he had invented the statistical bootstrap. He gave the impression of a man who had just found the famous philosopher's stone, describing how fireballs turn into fireballs forever and all in a logically consistent way. Visibly, Hagedorn was aware of the importance his ideas. It was interesting to observe how deeply he felt about it from the very beginning.

Using the SBM approach for a strongly interacting system, Hagedorn obtained an exponentially rising mass spectrum of resonant states. As time progresses and new data emerges, experimental results on hadronic level counting reveal ever greater number of cataloged resonances. They agree beautifully with theoretical expectations from the SBM. As our knowledge has increased, the observed mass spectrum has become a better exponential, as illustrated in Fig. 6.2. The solid blue line in Fig. 6.2 is the exponential fit to the smoothed hadron mass spectrum of the present day, which is represented by the short-dashed red line. Note that Hagedorn's long-dashed green line of 1967 was already a remarkably good exponential. One can imagine that the remaining deviation at high mass in the top right corner of the

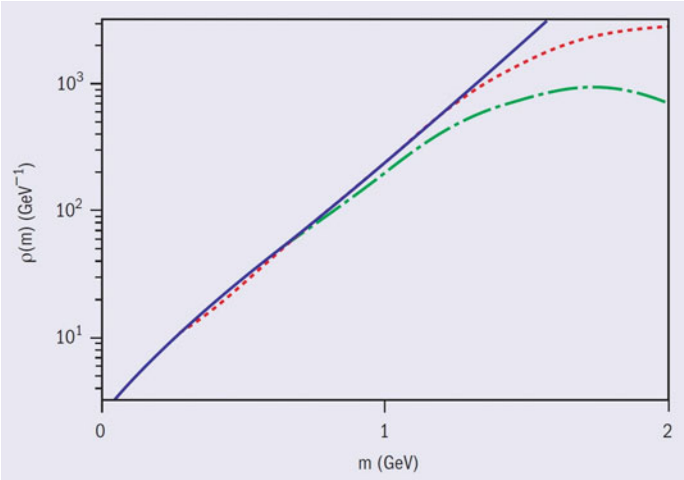

Fig. 6.2 The smoothed mass spectrum of hadronic states as a function of mass. Experimental data: *long-dashed green line* with the 1,411 states known in 1967; *short-dashed red line* with the 4,627 states of mid 1990s. The *solid blue line* represents the exponential fit yielding $T_H = 158$ MeV. Depending on the preexponential factor, a range $T_H = 150 \pm 15$ MeV is possible. *Picture: CERN Courier September 2003 p. 30*

figure originates in the experimental difficulties of discovering all these high mass states, which have a much less obvious experimental signature.

The important physics message of Fig. 6.2 is that the rising slope in the mass spectrum is the same as the falling slope of the particle momentum spectra. The momentum spectra originate in the thermalization process and thus in reaction dynamics; the mass spectrum is an elementary property of strong interactions. The SBM provides an explanation of the relationship between these slopes, and explains why the hadron gas temperature is bounded from above. Moreover, since the smallest building block of all hadronic resonances is the pion, within the SBM one can also understand why the limiting temperature is of the same magnitude as the smallest hadron mass $T_H \approx m_\pi$.

As time has passed since the discovery of the limiting temperature, this Hagedorn temperature T_H turned into a brand name. The concept of an exponentially rising mass spectrum is part of our understanding of hadron phenomena. However, considering the historical perspective, when first proposed the SBM was viewed with considerable skepticism, even within the CERN Theory Division where Hagedorn worked. Over the years, the understanding of the particle-production process that Hagedorn brought about has grown in significance and his work has become the standard model. Such is the sign of truly original work, of something that really influenced our thinking. Hagedorn's refereed article[2] presented 50 years ago for the

[2] R. Hagedorn, Nuovo Cim. Suppl. **3** 147 (1965).

first time and which introduced the statistical bootstrap model of particle production and placed the maximum temperature in the vocabulary of particle physics, has found a place among the most cited physics papers.

The accurate description of particle production, through the conversion of energy into matter, has numerous practical implications. Even in the very early days, Hagedorn's insight into the yields and spectra of the produced secondaries showed that neutrino beams would have sufficient flux to allow a fruitful experimental program, and this gave a theoretical basis for the planning of the first neutrino beams constructed at CERN.

6.3 Quark-Gluon Plasma

At the same time that the SBM was being developed, the newly discovered quarks were gaining acceptance as the building blocks of hadrons. While Hagedorn saw a compressed gas of hadrons as another hadron, in the quark picture it became a drop of quark matter. In quark matter at high temperatures gluons should also be present and as the temperature is increased asymptotic freedom ensures that all constituents interact relatively weakly. There seems to be nothing to stop a dense assembly of hadrons from deconfining into a plasma of quarks and gluons. It also seems that this new state of matter could be heated to a very high temperature, with no limit in sight. So what is the meaning of the Hagedorn temperature in this context?

In the SBM as conceived before quarks, hadrons were point particles. A subtle modification is required when considering quarks as building blocks. Hadrons made of quarks need a finite volume that grows with hadron mass. One of us (JR), upon his arrival at CERN at the end of the 1970s and into the early 1980s, worked on this extension of the SBM with Hagedorn. Much of this work is reported in this volume. We discovered that at the Hagedorn temperature, finite-size hadrons dissolve into a quark-gluon liquid. Both a phase transition and a smoother transformation are possible, depending on the precise nature of the mass spectrum.

The most physically attractive alternative was a first-order phase transition. In this case the latent heat is delivered to the hadron phase at a constant Hagedorn temperature T_{H}. A new phase is then reached wherein the hadron constituents—the quarks and the gluons—are no longer confined. The system temperature can now rise again. The presence of a true phase transition including its mathematical properties turned out to be of no deeper relevance to this concept, as long as the actual physical properties of the system change according to the model described. Therefore we chose to speak of 'transformation' of strongly interacting phases of matter.

Within the study of hot hadronic matter today, the Hagedorn temperature is understood as the phase boundary temperature between the hadron gas phase and the deconfined state of mobile quarks and gluons (see Fig. 6.3). Several experiments involving high-energy nuclear collisions at CERN's Super Proton Synchrotron (SPS), at the Relativistic Heavy Ion Collider (RHIC) at the Brookhaven National

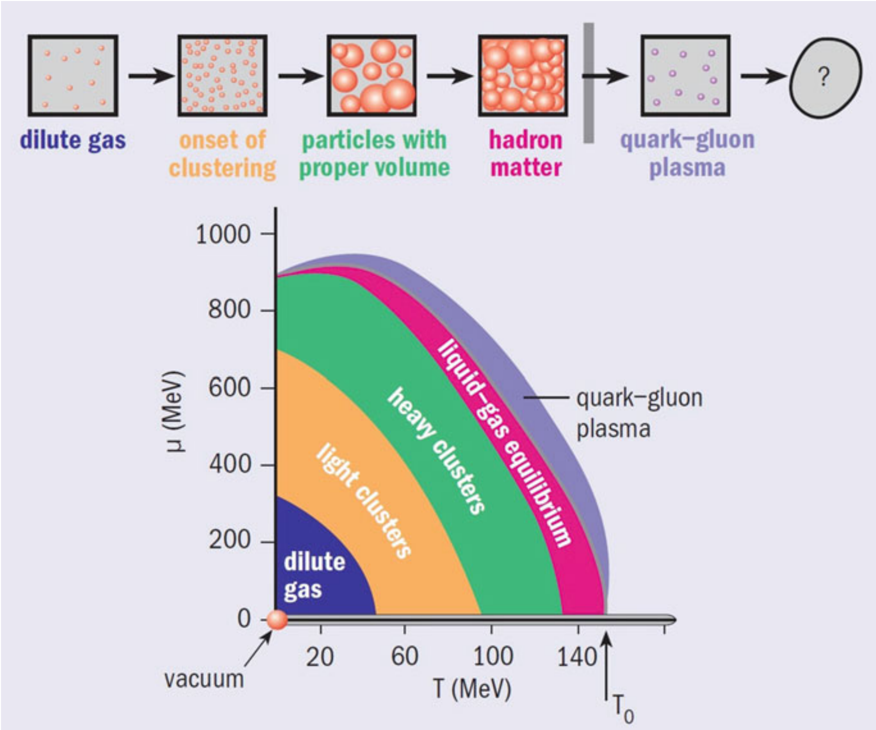

Fig. 6.3 The different regions of the statistical parameter plane (temperature, T, and the baryochemical potential μ), according to the statistical bootstrap model of hadronic matter. *Picture: CERN Courier September 2003 p. 30*

Laboratory, and at CERN's Large Hadron Collider (LHC) are testing these new concepts. Nuclei, rather than protons, are used in these experiments in order to maximize the volume of quark deconfinement. This allows a clearer study of the signature of the formation of a new phase of matter, the quark-gluon plasma (QGP).

In past years both CERN and RHIC communities have presented clear evidence for the formation of the deconfined QGP state in which the hadron constituents are dissolved. The current experimental objective is the understanding of the physical properties of this new phase of matter. This requires the use of novel probes, which respond to a change in the nature of the state within the short time available. More precisely, the heating of hadronic matter beyond the Hagedorn temperature is accompanied by a large collision compression pressure, which is the same in magnitude as the pressure in the very early Universe. In the subsequent expansion, a collective flow velocity as large as 60 % of the velocity of light is exceeded. The expansion of dense QGP phase occurs on a timescale similar to that needed for light to traverse the interacting nuclei.

In the expansion—cooling process of QGP formed in nuclear collisions, the Hagedorn temperature at which hadrons emerge is again reached after a time that corresponds to the lifespan of a short-lived hadron. A break up—that is, hadronization—then occurs and final-state hadrons emerge. Hagedorn was particularly interested in understanding the hadronic probes of QGP produced in hadronization. He followed closely the initial exploration of the strangeness flavor as a signal of QGP formation.

Looking back, already in February 2000 the totality of intriguing experimental results obtained at the SPS over several years was folded into a public announcement stating that the formation of a new phase of matter was their best explanation.[3] More on this subject is said in this volume. The key experimental results, including, in particular, strangeness and strange antibaryon enhancement, agreed with the theoretical expectations that were arrived at when one assumes that the QGP state was formed. To this day these signatures are the cornerstone of the QGP discovery.

Other signatures of QGP have been since detected. For example, over the past decade it became evident that this deconfined phase of matter is highly non-transparent to fast quarks. The majority of researchers today are convinced that the deconfined phase has been formed at the SPS, at RHIC and at the LHC. The thrust of current research depends on the range of accessible relativistic heavy ion beams. At lower energy, at SPS, and in special effort at RHIC, research addresses threshold conditions that are necessary for the onset of QGP, and the study of the phase boundary as function of baryon density. At the LHC we seek to understand the initial reaction conditions in dense matter and QGP in conditions similar to those present in the early universe is studied.

In the next few years, the study of hadronic matter near the Hagedorn temperature will also dominate experimental efforts in the field of nuclear collisions, in particular at the new international experimental facility FAIR under construction today at the GSI laboratory in Darmstadt, Germany and at the planned experimental facility NICA in Dubna, Russia. The richness of the physics at hand over the coming years is illustrated in the phase diagram in Fig. 6.3, which was obtained from the study of the SBM. Here, the domain is spanned by the temperature, T, and the baryochemical potential, μ, which regulates the baryon density.

In past 50 years the understanding of the physics related to the Hagedorn temperature has changed. In the beginning it was the maximum temperature seen in proton–proton collisions. It then became the SBM inverse slope of the mass spectrum. Today, it denotes the phase boundary between hadron and quark matter. Moreover, as recent work in string theory has shown, Rolf Hagedorn (Fig. 6.4) will not only be remembered for the physics of hot hadronic matter: his name is already attached to a more general family of elementary phenomena that originate in the methods he developed in the study of strong interaction physics.

[3]CERN Courier April 2000: *Harping on About Hadrons*; CERN Courier May 2000: *Opening the Door to the Quark-Gluon Plasma.*

Fig. 6.4 Rolf Hagedorn in his garden Fall 1978; *Photo: Johann Rafelski*