# Chapter 11
# Purposefully Relating Multilingual Registers: Building Theory and Teaching Strategies for Bilingual Learners Based on an Integration of Three Traditions

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## 11.1 Introduction

The purpose of this chapter is twofold: to build theory as well as suggesting teaching strategies that could be used with multilingual mathematics learners. The teaching strategies are based on the approach of purposefully relating multilingual registers by which we integrate three research traditions and approaches.

In the mathematics education research literature there are three different strong ideas related to different language registers and discourses that have not been closely linked. Indeed they are often treated as quite distinct entities. These are code-switching between first and second languages, transitions between informal and academic (mathematical) forms of language within a given language, and transitions between different mathematical representations. Exploring the overlap between these three ideas, and in particular by articulating their interconnections, new insights and implications are gained. For this, we draw upon the sociolinguistic construct of registers as functional varieties of language use being associated with different contexts. Rather than three apparently discrete sets of ideas, an integrated set of ideas for teachers and researchers is presented that has the potential to drive theory, curriculum, and teaching developments.

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R. Barwell et al. (eds.), *Mathematics Education and Language Diversity*, New ICMI Study Series, https://doi.org/10.1007/978-3-319-14511-2_11

Mathematical ideas are not always obvious to the learner; indeed often they are quite opaque and nonintuitive. Once the move away from traditional teaching that promoted rote learning gathered pace, teachers turned to strategies which would bridge the gulf from what starts as being nonintuitive, but in the end becomes obvious to students. One of the enduring strategies over the last five decades is the use of manipulatives. These can act as excellent scaffolding devices that reflect the targeted mathematical relationships, and may well bring students to a place where they are predisposed or begin to engage in particular learning. But for most students, the learning experience needs to be focused by using more scaffolding than that provided by just manipulatives. This additional scaffolding is normally a purposeful entanglement with manipulatives of language and pictorial representations of the ideas.

The intertwining of such components for good teachers just seems to happen, but young teachers need to learn what emphasis to give each component at appropriate times. We suggest that the teacher's role is not to encourage students from moving from sloppy language to a language of precision, but rather to recognize that students will often need to use everyday language as they grasp for the underlying mathematical relationship that they are trying to understand. Hence the teacher will privilege the dynamic of students moving between everyday, school and technical registers, as they start to understand more deeply the central core of a mathematical idea, the limits of the idea, and its relationship to other mathematical and nonmathematical ideas that already exist within their ideas network. To play this role, the teacher needs a thorough knowledge of the mathematics under study and the appropriate language forms so that their support is encouraging, generative, and productive as the students engage in sustained thinking.

Guidance for promoting the use of the technical mathematical register has been developed in the literature over the last decade or so. One main thrust of the Cognitive Guided Instruction (CGI) project was to encourage teachers to use language that guided students to using an appropriate mathematical register to discuss mathematics, with which they were engaged (Carpenter, Fennema, Franke, Levi, & Empson, 1999). Watson and Mason's (1998) different categories of questions, and the use of open-ended questions (Sullivan & Lilburn, 2004), prompt the use of technical language. However there seems little in the literature that guides teachers in transitioning their language from the everyday to the school register, although this has received increasing importance in the discussion about linguistic difficulties (see below). The emphasis is to move directly to a technical language register without looking in depth at better ways of moving students through these transitions, let alone privileging the dynamic between them with an expectation that this dynamic will be an ongoing feature of the classroom, with students free to move between registers as they feel the need to do so.

We return to these deliberations later, but at this point it is useful to consider one of two specific teaching snapshots that will allow the later theoretical argument to be grounded. The two empirical snapshots from very different language contexts (India and Germany) suggest that the teaching strategy of purposefully linking appropriate registers seems to have a high potential to initiate substantial student activities that lead to deeper understanding. It is interesting to reconstruct how teachers and students gradually developed their strategies to clarify the mathematical relations by using different representational registers. In particular, the graphical register supported the processes of linguistic and conceptual clarification. Although only a very short insight into the rich data can be given here, these snapshots show that the teaching strategy adopted from integrating language and mathematics education experiences seems to be promising as it focuses directly on the conceptual core of linguistic and mathematical challenges. We choose to present one snapshot here to ground our discussion, follow this by our theoretical argument, and then return to a second snapshot to position the argument fully in the context of a classroom.

## 11.2 Relating Registers for Common Multiples: Snapshot from India

The context for the following snapshot was a camp held at the Homi Bhabha Centre for Science Education, Mumbai (HBCSE, Bombay). The students were low-achieving sixth graders in the age-group of 10–11 years from a neighborhood English-medium school. The camp classes were held over a period of 2 months every Thursday for around one and a half hours. Altogether 21 students (12 boys and 9 girls) participated in the camp. They were selected by their class teachers on the basis of their performance in a school mathematics test with less than 40 % marks. All the students knew Hindi and English while some also spoke Marathi and Tamil. Marathi is the official language of the state of Maharashtra of which Mumbai is the capital city. The teaching for the entire camp was undertaken by an HBCSE member-researcher who speaks English, Hindi, and Marathi fluently. In urban India, the phenomenon of code-mixing is very common and is a regular feature in daily-life conversations (similar to urban Pakistan as described in Halai, 2009). However, most English-medium schools (including the one these students came from) promote a monolingual English practice, and the use of other languages in the classrooms is actively discouraged. The medium of instruction in the camp was English, but teacher and students also used *Bambaiya Hindi* (the local dialect of Hindi used in and around Mumbai), which we consider here as the students' first language.

The following two Episodes, A and B, are from two different camp lessons that depict interesting student–teacher exchanges involving transitions between verbal registers with significant amounts of code-switching and mixing. The lessons were videotaped, and selected clips were transcribed for the analysis. For the two transcripts, an English translation has been provided on the right of each utterance, and the numeral before each utterance indicates the line number in the respective transcript. In the right-hand translation, the underlined script is the translation, and the non-underlined script is English words used in the original verbal exchange. "T" and "C" stand for teacher and student respectively.

### 11.2.1 Tasks and Situation in the Camp Lessons

In the classroom Episodes A and B, the teachers wrote the following word problems on the blackboard and then started explanations and discussions with the students:

- *Problem A*. A cat take [sic] a jump of 4 and a rat takes a jump of 2. Now, (the) cat is at number 8 and (the) rat is at number 20, will (the) cat (be able to) catch the rat?
- *Problem B*. In the year 1997, price of a rubber ball is [sic] Rs. 7. If the price increases every year by Rs. 0.25, then in which year the price of the ball will be Rs. 9?

The analysis focuses on how the teachers and students interactively made sense of the word problems.

### 11.2.2 Code-Switching Between Registers for Better Explanation

The teacher began by writing the "warm-up" Problem A as the problem of the day on the board in English. Then she started a verbal exchange with the students to ensure they have understood the task well:

| | | | | |
|---|---|---|---|---|
| a38 | T | samajh mein aaya kya likha hai? (*indicates the warm-up problem for the day on the black board*) | T | understood what I wrote? (*indicates the warm-up problem for the day on the black board*) |
| a39 | T | hum Hindi mein bolenge. | T | We'll speak in Hindi |
| a40 | T | cat kitna jump leti hai ek saath? | T | how much jump does cat take at one go? |
| a41 | C | Four | C | Four |
| a42 | T | four ka? aur rat kitna jump leta hai? | T | in fours? And how much jump rat takes? |
| a43 | C | Do | C | Two |
| a44 | T | do ka/ | T | in twos/ |
| a45 | T | to samjho aisa yeh ek number-line hai/ haan, aisa ek tree hai number wala/ | T | then understand this way a number-line is taken/ yes, there is a tree of numbers |
| a46 | T | one, two, aisa wala/ | T | one, two, likewise/ |
| a47 | T | to abhi cat kidhar hai? | T | so where is the cat now? |
| a48 | C | four, four pe/ | C | four, at four/ |
| a49 | T | nahin cat is at number eight/ | T | No cat is at number eight/ |
| a50 | C | eight/ | C | eight/ |

The teacher used a code-switch from English to Hindi to ensure that the students clearly understood the task, and that they were able to work on their own. But while doing so, she used technical terms (suitable for the problem-tasks) in English such as, “numbers,” “number-line,” and some number names. Interestingly she also used English for “cat” and “rat” which although not mathematical-technical terms, are crucial for understanding the context of the original English written problem. Hence her code-mixing addresses the technical register as well as relevant parts of the everyday register. By code-mixing (using the mathematical-technical terms but in English, embedded in Hindi sentences) the teacher tried to ensure that the students can understand and appropriately use these key terms in English.

### *11.2.3 Use of Gestures and Nonverbal Cues*

The presentation of the problem-task was accompanied by gestures and drawings on the board for everyone to see. In Problem A, the teacher used her hand-gestures indicating a wave-like flow of “jumps” (read: numbers) that the rats and cats take; these gestures can be assigned to the base level of concrete representations (Fig. 11.1). She also tried helping students visualize the jumps by drawing a similar “wave” number-line on the board, and marked the initial positions of the cat and the rat. In the graphical representation, the nonverbal cues supported the process of understanding by visualizing the subsequent jumps and the point where the cat meets the rat.

At one early point in the lesson, the following interaction occurred:

| | | | | |
|---|---|---|---|---|
| a55 | T | idhar samjho cat hai? hmm.. aur twenty main nikalti nahin/ wahan pe rat hai/ | T | Consider cat to be here? Hmm.. and I haven’t marked twenty/ Rat’s there/ |

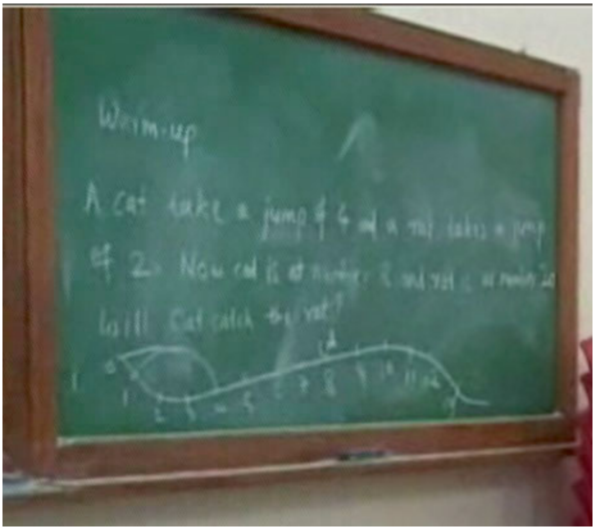



**Fig. 11.1** Text and drawing on the board (screenshot)

| | | | | |
|---|---|---|---|---|
| a56 | T | to kya cat, aise woh kud raha hai aur/ hamesha rat kitna jump le raha hai? do/ | T | will cat then, she's jumping like this/ and how much jump does rat take always? Two/ |
| a57 | T | aur cat kitna le rahi hai? Four/ | T | and cat takes how much? Four/ |
| a58 | T | to kya cat rat ko pakad sakti hai? Karke dekho/ | T | then can cat catch the rat? Show by doing/ |
| a59 | C | yes teacher/ yes teacher/ pakad sakti hai/ | C | yes teacher/ yes teacher/ can catch/ |

Importantly the style of verbal language changed immediately when a drawing was introduced into the teaching context. The introduction of a graphical representation allowed the language to change to a more deictic form ("she's there," line a56). The comment by the teacher "Show by doing" (line a58) allowed the linguistic load to be lightened for the students, and allowed them to develop first their solution ideas which they later expressed in a more elaborate verbal language. In this way adequate language in the school and/or technical registers were built successively. However, the existence of and the relation between these registers all took place without the teacher's explicit reference to them.

### *11.2.4 Use of Technical and Everyday Registers*

In Episode A, the teacher connected the technical and the everyday register by repeatedly using terminologies likes, "number-line" and "jump," "catch," etc. In contrast, in Episode B, she encouraged using everyday registers that involved the use of nonformal units like "anna," which are old currency-units no longer in use. However, such nonformal currency-units are still part of the everyday register and people refer to them quite often (1 rupee = 100 paise = 16 anna; 50 paise = 8 anna; 25 paise = 4 anna). This transcript shows how the use of these informal "everyday" units helped the students to make sense of the abstract value 0.25 (that is to be added to 7 until reaching 9):

| | | | | |
|---|---|---|---|---|
| b67 | C | teacher, maine na, inn charon ko aath anna, char anna, sabko plus kiya tho equal to one rupees aaya/ | C | teacher, I took all these four, eight anna, four anna, I did plus and it came equal to one rupees/ |
| b68 | T | kaise? | T | how? |
| b69 | C | yeh char anna - char anna se aath anna aaya/ isko bhi char anna - char anna se aath anna aaya/ one rupees/ | C | this four anna-four anna make eight anna/ this too gave four anna-four anna eight anna/ one rupees/ |
| b70 | T | means? | T | means? |
| b71 | T | nahin, yeh dono milake kitna hua? | T | No, how much these two taken together? |
| b72 | C | aath anna/ | C | eight anna/ |
| b73 | T | hmm, aur yeh aage ke do? | T | hmm, and this further two? |
| b74 | C | aath anna/ | C | eight anna/ |

| | | | | |
|---|---|---|---|---|
| b75 | T | tho kitne aath aane milane hain tumhe? Do/ | T | then how many eight anna do you have to put together? Two/ |
| b76 | C | do aath aane/ tho ek rupaya banta hai/ | C | two eight anna/ then that makes one rupee/ |
| b77 | T | hmm, ek rupaya banta hai/ | T | hmm, makes one rupee/ |

By using transitions between the mathematical-technical and everyday registers, and by transitions between their languages (code-switching and mixing), students were able to arrive at the solutions. Such transitions appeared to help in reducing the cognitive load for the students. Hence this seems to have encouraged all the students to take part in the classroom activity and contributed to the solving process, correct or not. This also helped in building confidence among the students.

With this analysis of the Indian snapshot in mind, we now return to the theoretical issues that have been used and discuss them in more depth.

## 11.3 Revisiting Three Traditions of Reflecting on Linguistic Transitions

### *11.3.1 Transitions Between First and Second Languages: Code-Switching*

The role of language in thinking and learning is well accepted. In Post-Piagetian constructivism, it is argued that a child's mind is not a *tabula rasa*. Children come to school with prior knowledge drawn from their environment and everyday experiences, including knowledge of languages, both home language and/or local language, which in many multilingual societies are different languages. In many countries like India, Pakistan, South Africa, etc. it is not unusual to find children entering elementary grades with knowledge of more than two languages (Bose & Choudhury, 2010; Halai, 2009; Setati, 2005). A natural "movement" between languages occurs in the forms of speech and thought during the conversation flow between conversant bilinguals (or multilinguals). This phenomenon of movement between languages or language-switching is known as "code-switching": the practice of switching between two or more languages in a conversation or an utterance (Farrugia, 2009). Code-switching is generally accompanied by "code-mixing" where the latter refers to the case when only "one or few words" are borrowed from one language and embedded into a sentence predominantly in the alternate language (Farrugia, 2009). Code-switching is widely recognized as a social phenomenon that can facilitate effective mathematical learning since both mathematics and language are socially constructed and hence the linguistic and social nature of mathematics allows it to develop together with language (Barton, 2009; Choudhury & Bose, 2011).

Different contexts of mathematics classrooms where learners and teachers are engaged in classroom activities, and therefore also engaged in language negotiations, have been carefully described in terms of code-switching (see Barwell, 2005;

Bose & Choudhury, 2010; Clarkson, 2007; Farrugia, 2009; Halai, 2009; Setati, 2005, 2008). Such instances occur while engaging with difficulties in comprehension of the problem-tasks or keywords and phrases therein; search for alternate strategies; presentation of explanations; achievement with correct solutions accompanied by exuberant exclamations with a few words/phrases drawn from a different language; and while using mathematical terminologies. In addition, teachers code-switch while providing scaffolding to make the subject-matter comprehensible or to develop certain mathematical abilities among the students; to reduce students' cognitive load; and while enforcing authority and discipline in the classroom. Code-switching is also used to facilitate the connection of verbal languages with visual representations (see Sect. 11.2.3). Furthermore, the use of code-switching by either teacher or students seems to allow simultaneous learning of both languages, as well as mathematics (Barwell, 2005; Bose & Choudhury, 2010; and many others).

Building upon this empirical work, it is time not only to argue that code-switching should be *allowed* and descriptively analyzed concerning its conditions and effects, but it is also the time to develop and promote teaching strategies that make use of code-switching and other links between first and second languages more purposefully. In particular, teachers should be enabled to guide students to make conscious choices to use transitions between their languages as a possible solution strategy for co-learning mathematics. An aspect of such a model will be to privilege students' competences in all their languages.

### *11.3.2 Transitions Between Everyday and Technical Language*

When turning to the second key idea of this chapter, that of students moving between informal and formal registers within their dominant language, we extend this idea to multilingual learners. Within the mathematics education research literature, Pimm (1987), Freudenthal (1991) and many others have advocated a careful transition from everyday language to the technical language of mathematics as an important teaching strategy that enhances conceptual understanding of mathematical concepts and ideas. Empirical studies (e.g., Van den Heuvel-Panhuizen, 2003) show the power of this teaching strategy.

However, some students seem to experience serious difficulties in making such a transition and it appears that the source of the problem is connected to an intermediate register between the informal everyday register and the formal technical language. For a long time, this intermediate register has been underestimated. For a theoretical explanation of this issue it is useful to use Cummins' distinction between Basic Interpersonal Communicative Skills (BICS) and Cognitive Academic Language Proficiency (CALP). Cummins (2000) has suggested that there is a distinction between surface fluency in an everyday language register and the language skills needed in a context of high cognitive academic demands. He developed the construct BICS to describe the situation when there are contextual supports for language. Face-to-face conversations, for example, provide actions with hands and eyes, instant feedback, and other cues to support meaning. Such situations are said to be "context embedded" (Koch & Oesterreicher, 1985) and conceptually oral, and surface language skills are sufficient.

On the other hand, where higher-order thinking skills such as analysis and evaluation are required (for example in problem solving contexts), language becomes "disembedded" from a meaningful supportive context and becomes more abstract. Often, it appears conceptually written even if used orally. Such a situation can be thought of as "context reduced" (Koch & Oesterreicher, 1985; Schleppegrell, 2004) and needs more explicit linguistic means to be mastered. The skills required to become fluent in this style of language falls in the domain of CALP. The corresponding language register is here termed the "school register." By "school register," we refer to the term language of schooling as explored by Schleppegrell (2004) and discussed in many political contexts (as in the European Council; see Thürmann, Vollmer, & Pieper, 2010).

Although many overlaps exist, we can, for analytical reasons, distinguish the school register from the everyday register (in which Pimm's informal language and Cummins' BICS is located), as well as the technical register (which comprises mathematical technical language of school mathematics), hence giving a three-tiered model as shown in Fig. 11.2.

Most teachers are aware that the technical register needs to be acquired in school, whereas the school register (to which students of privileged socioeconomic background are often already acquainted) is sometimes treated as *a learning condition*, instead of *a learning goal*. This distinction suggests quite a different set of teaching strategies. One crucial implication for learning is that students with weaker language background, either because they come from a lower socioeconomic background, or they are from a migrant community not speaking the language of schooling, or both, experience difficulties.

Hence for language acquisition (of the everyday register, of the school register as well as of the technical register), it turns out to be important not only to transit once from the everyday register using the school register to the technical register, but to move flexibly forward and backward between all the three, as emphasized by Freudenthal (1991) and elaborated by Clarkson (2009). While extending the model for multilingual learners,

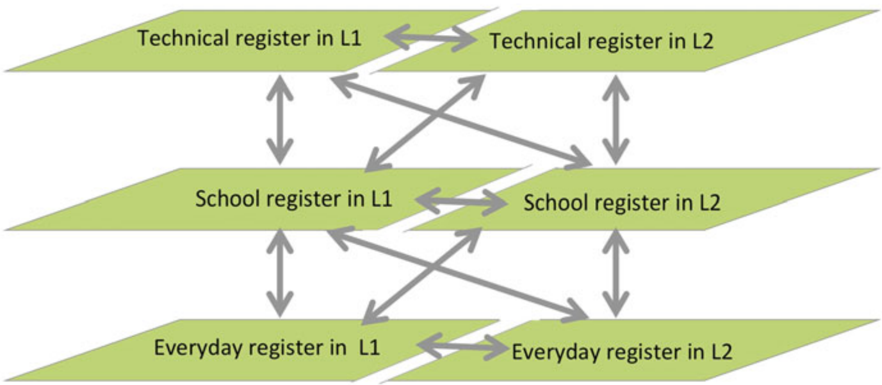



**Fig. 11.2** Three-tiered model of registers (adapted from Clarkson, 2009)

Clarkson (2009) adds the important perspective that the three registers might exist in more than one language for multilingual learners (see Fig. 11.2).

He suggested that as well as multilingual students switching codes, they should also be encouraged by teachers to recognize and move between everyday register(s), school register(s), and technical register(s). It is not enough, he has argued, to concentrate only on the mathematical-technical register, as has so often happened in the past. Students bring to their mathematical learning their own lived experiences outside the classroom, which will be encoded mainly in their everyday language, which in turn may well be in their first language (L1). This is why a dynamic transitioning between registers and in switching codes needs to be encouraged by teaching strategies.

Figure 11.2 clearly outlines many possible connecting paths. Whether all pathways are useful for some students is an open question. For example, if a student's L1 does not support some aspects of school mathematics, then the top left hand cell may be empty. Some pathways that students use have been identified in the literature (Halai, 2004). But which of these pathways should be encouraged by the teacher is still a question for research and is probably quite bound to the specific language context.

### *11.3.3 Transitions Between Different Mathematical Representations: Relating Mathematical Registers*

Bruner (1967), Dienes (1969), Lesh (1979), Duval (2006) and many others have pointed out that relating different mathematical representations is an important activity for developing students' conceptual understanding (Fig. 11.3). Many examples from the research literature indicate how this is employed as a fruitful teaching strategy, for example in task design (Swan, 2005).

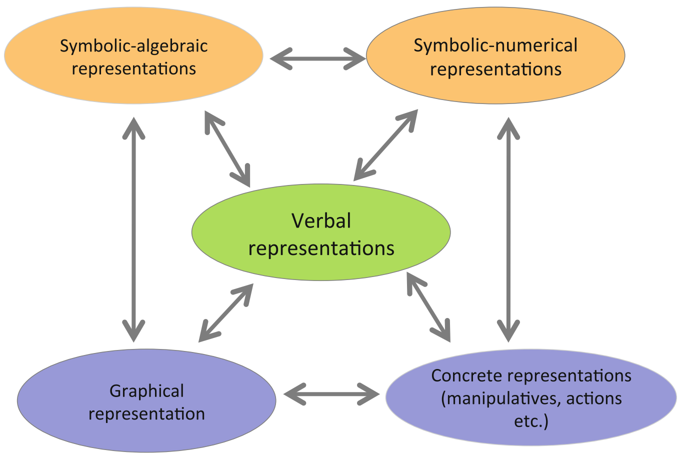



**Fig. 11.3** Transitions between different representations (Bruner, 1967; Duval, 2006; Lesh, 1979)

Historically, the crucial notion of using manipulatives, an influence of Dienes (1969) in particular, in teaching mathematics, was a huge step forward in the 1960s that changed the very nature of teaching mathematics. Coinciding with this was a growing emphasis on visualizing, and making or using pictorial representations of mathematical ideas in teaching, an influence of Bruner's (1967) enactive, iconic and symbolic modes of representation.

These new ideas began to offer an alternative to the traditionally accepted teaching style when the teacher delivered small portions of mathematics that needed to be learnt for a particular lesson followed by students completing individually symbolic-based exercises. With the introduction of manipulatives and the growing emphasis on visual representations, learning by discovery and with understanding emerged as an alternate teaching practice, and has now become the dominant paradigm in the discourse on teaching mathematics.

More recently, Duval (2006) in particular has given a semiotically grounded theoretical foundation for why transitions between different modes of mathematical representations are crucial for the acquisition of conceptual understanding. He has emphasized that the abstract nature of mathematical concepts is one reason for these necessities.

Although from the early days in the 1960s, language was seen to be an integral part of mathematical semiotic registers, often the manner in which teacher and students interacted with language, both verbally and in written modes, was left unexamined. In particular the "linguistic" registers were not differentiated for L1 versus L2 and hence notions such as code-switching that might occur could not be recognized. Nor were the differences between everyday registers, school registers, and technical registers included in their models to any extent. These differences need to be acknowledged as yet another variation within the communications between classroom participants.

### *11.3.4 Integrating Three Transitions Between Languages, Registers, and Representations*

Following Leisen (2005), Prediger and Wessel (2011) formulated an integration of the above three different perspectives on language transitions as the "relating register approach" (see Fig. 11.4). In Fig. 11.4, the different registers and representations are ordered hierarchically according to their abstractness (as proposed by both Leisen, 2005 and von Kügelgen 1994). The type face used in both the upper and the base levels are printed in grey in order to sketch that they are not always used: primary and lower secondary classrooms do not refer to the symbolic-algebraic register, upper secondary classrooms usually do not refer to concrete artefacts and manipulatives.

Recently this integrated model has proved to be very useful as a heuristic tool that was used to guide the practical design and support of learning processes for multilingual learners (Leisen, 2005; Prediger & Wessel, 2011) as well as its empirical investigation (Prediger & Wessel, 2011). However, the model requires further theoretical and empirical exploration.

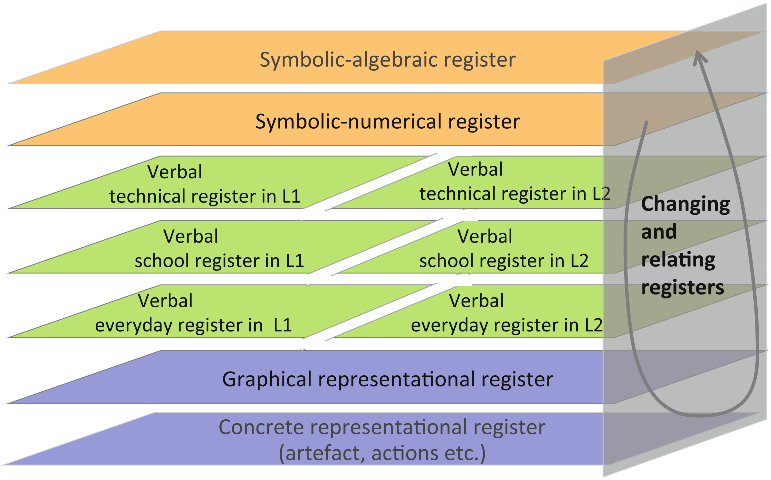



**Fig. 11.4** An integrated model which relates the three transitions (Prediger & Wessel, 2011)

## 11.4 Theoretical Exploration: Representations, Registers, or Languages?

### *11.4.1 Different Conceptualizations*

In the first integrated model (Fig. 11.4), the different registers and representations are ordered hierarchically according to their increasing degree of abstractness (as proposed by Leisen, 2005; von Kügelgen 1994). However, it has become apparent in Prediger and Wessel (2011) that the degree of abstractness also depends on the mathematical topic under study and the concrete representations used. As well it would appear that the different levels are not of the same quality. For building an adequate theoretical conceptualization, different authors have suggested different theoretical constructs including the following:

- The linguist von Kügelgen (1994, p. 34) offered the construct "concept levels" with a strong emphasis on the hierarchy which has not been proved to be useful for all cases and which has too narrow a focus on words without grasping the complexity of language.
- The psychologist Bruner (1967) suggested the classical conceptualization of different "modes of representation" (enactive, iconic, symbolic), which has proved to be useful for designing learning sequences. However the construct "representation" lacks important dimensions such as references to contexts, functions, and social embeddedness. Hence the construct "mode of representation" might be understood in the way that there are one-to-one translations between all modes of representations without shifts in meaning and function. This is definitely not the case.

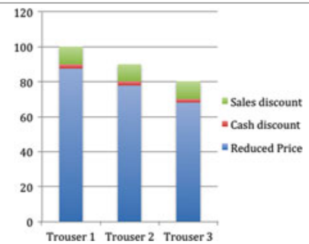
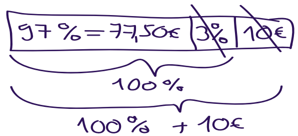
- The language education researcher Hallet (2012) focuses on "symbolic languages" and emphasizes their different semiotic functions to describe and explain the world in specific ways. This has proved useful for formulating this latest model.
- Some mathematics education research focuses on different semiotic functions. In his construct of "semiotic registers," Duval (2006) emphasizes that the meaning ("content") of a mathematical object can change with a shift of representation: "The content of a representation depends more on the register of the representation than on the object represented. That is the reason why passing from one register to another changes not only the means of treatment, but also the properties that can be made explicit" (p. 111). With this observation, Duval points to an aspect that is equally important for the sociolinguistic construct of "register." This distinction has proved to be useful to us.
- The sociolinguist, Halliday (1978), defines register as "set of meanings, the configuration of semantic patterns, that are typically drawn upon under the specific conditions, along with the words and structures that are used in the realization of these meanings" (p. 23). Halliday distinguishes registers from dialects by defining dialects as ways of saying the same things differently (which in a certain way also applies to representations), whereas he describes registers as "ways of saying different things" (p. 35). As a consequence, a change of register also implies a shift in meanings. Additionally, Halliday emphasizes the social embeddedness of the communication situation for characterizing registers: "A register can be defined as the configuration of semantic resources that a member of a culture typically associates with the situation type. It is the meaning potential that is accessible in a given social context" (p. 111). Hence, registers are characterized by the types of communication situations, their field of language use, and the discourse styles and modes of discourse. This has also been useful in the present model building exercise.

### *11.4.2 Registers with Different Representations*

The everyday register, the school register, and the technical register can thus be characterized as registers in Halliday's sense that are used in different communication situations (with of course some overlapping, especially between the technical register and the school register). Graphical and symbolic representations can also be conceptualized as registers, but only in Duval's sense. The decision not to mix the different conceptualizations of registers led us to give up the symmetry of all levels in Fig. 11.4, and to develop a more complex $3 \times 4$ matrix structure (Table 11.1) that shows how the different representations are used in the different registers.

The everyday register has been characterized by its contextual embeddedness and lacking mathematical explicitness in face-to-face-communication. It encompasses verbal and concrete representations, but rarely graphical and symbolic representations. An example is given in Table 11.1 (adapted from Meyer & Prediger, 2012).

**Table 11.1** Linguistic registers and their representations—illustrated for percentages in a shopping situation (from Meyer & Prediger, 2012)

<table>
<tr><th></th><th>Everyday register</th><th>School register</th><th>Technical register</th></tr>
<tr><td>Representation in words</td><td>Yesterday I was at a sale, in my favourite shop. The sale meant I received a 10 € discount for the trousers. Since I paid in cash, the sales clerk gave me another discount of 3 %. In all I only paid 77.50 €. How much was the original price?</td><td>In a sale, a pair of trousers was reduced by 10 €, and further 3 % discount was offered if you paid in cash. Hence, the final price was 77.50 €. What was the original price of the trousers?</td><td>If the original value is reduced by 10 € and then by 3 %, the new value is 77.50 €. What was the original value?</td></tr>
<tr><td>Graphical representation</td><td>
</td><td>
</td><td>
</td></tr>
<tr><td>Symbolic-numerical representation</td><td>–</td><td><table>
<tr><th></th><th>Original price (€)</th><th>Price with sales discount (€)</th><th>Price with 2nd discount (€)</th></tr>
<tr><td>Trouser 1</td><td>100</td><td>90</td><td>87.30</td></tr>
<tr><td>Trouser 2</td><td>90</td><td>81</td><td>78.57</td></tr>
<tr><td>Trouser 3</td><td>80</td><td>72</td><td>69.84</td></tr>
</table></td><td><table>
<tr><th></th><th>Base value</th><th>First reduction</th><th>Second reduction</th></tr>
<tr><td>Trial 1</td><td>100</td><td>90</td><td>87.3</td></tr>
<tr><td>Trial 2</td><td>90</td><td>82</td><td>78.57</td></tr>
<tr><td>Trial 3</td><td>80</td><td>72</td><td>69.84</td></tr>
</table></td></tr>
<tr><td>Symbolic- algebraic representation</td><td>–</td><td>–</td><td>$(G - 10\,€) \times 0.97 = 77.50\,€$</td></tr>
</table>

The school register has been characterized by its context-disembeddedness, higher explicitness, less personal references, and higher complexity of its grammatical structures (Schleppegrell, 2004). It appears in newspapers, books, and in mathematics classrooms especially in textbooks. Although predominantly written, this register clearly is found in the verbal speech of the teachers. It uses not only verbal and graphical, but also numerical representations. In contrast, symbolic-algebraic representations are rarely used in the school register.

The technical register can be characterized by similar communication situations and similar modes of discourse as the school register, but it is optimized for even higher economy and unambiguousness for very restricted purposes; in the case of mathematics, for structural and quantifiable relations. In mathematics classrooms, the technical register is mostly used for intra-mathematical contexts, or for grasping the mathematical structures in real-world situations. The mathematical-technical register encompasses all registers, including the symbolic-algebraic register, which is exclusively used in the mathematical-technical register. Only the symbolic-numerical and symbolic-algebraic registers allow the symbolic treatments (e.g., in formal algorithms) from which mathematics gains its calculation and logical power.

The example, Table 11.1 shows that by using Halliday's construct "register," we can emphasize different situations of language use: whereas the situation in the everyday register is very concrete and vivid, it is more depersonalized in the school register, and further decontextualized in the technical register. For example, talking about lucky sales would not be appropriate in the school or technical register, but in contrast using the symbolic-algebraic representation of this problem found in the technical register in the shop window is highly unlikely to increase sales of the trousers. Thus the example sketches how acquiring language proficiency is indeed connected to situation-adequate use of registers and representations.

### *11.4.3 Some Teaching Implications*

We suggest that developing quite specific examples to show moves from one register to another can be a useful exercise both for researchers and teacher educators. But giving such fine-grained specific advice to teachers, particularly young teachers, is probably not very helpful in changing their practice for the better. Expecting teachers to employ such micromanaging will make the task of teaching unmanageable. It may be better to suggest particular holistic ways of using language in the mathematics classroom and then, over an extended period of time, work with teachers on examples from their own classrooms to examine such dynamic language use in a context of which they have ownership. But this point needs further exploration by intense development and detailed research.

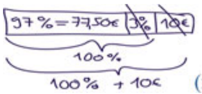
Asking students to swap between their languages is not simple. The model (Fig. 11.4) suggests not only a vertical and horizontal movement in its middle language component, but implies diagonal movement is also possible, with switching between language registers happening rapidly. No teaching strategy we know of seems to have attempted to guide teachers through this complex maze. For making practical use of these general theoretical considerations in mathematics classrooms, extensive further development and research is needed to learn how purposefully relating registers can be elicited in classrooms. We make some suggestions in the following.

A first step involves the specification of concrete activities by which students are encouraged to relate registers. By using activities that students find easy to relate to, it will be easy to ask students to translate from one register into another, like the example given in Table 11.1. Hence:

- Verbal everyday register L1 → graphical register: Here is a shopping situation (Table 11.1). Draw a picture that shows you the relevant quantities.
- Verbal technical register L2 → verbal everyday register L1/L2: The situation is explained very succinctly (Table 11.1). Retell the story, using your preferred language, giving more details.
- Symbolic → picture: Find or draw a picture for the equation $(x - 10\,€) \times 0.97 = 77.50\,€$.

As Swan (2005) has shown, more guided activities can also be fruitful; for example finding the same mathematical content in different registers or examining if different representations fit. For example:

- Does this drawing fit with this symbolic expression for the calculation? Explain why (or why not).




$(x + 10\,€) \times 0.97 = 77{,}50\,€.$

- On these 15 file cards, you find symbolic expressions, texts with situations, and drawings for percentage tasks. Group those that belong together. Devise and then add any missing cards.

Further examples of concrete activities, which have been investigated by Prediger and Wessel (2013), cover:

- Translation from one register into another (freely chosen or determined)
- Finding and fitting registers
- Examining and then deciding whether correct registers fit given situations
- Explaining how to find a mathematical relation or structure in a certain register
- Collecting and reflecting on different means of expression within one register

These activities can be used by teachers in the preparation of lessons, and also spontaneously for initiating students' moves between registers and representations during teaching. If the teacher has a model such as in Fig. 11.4 available as a core referent in their suite of stable teaching strategies, then they will be able to make further spontaneous interventions that allow students to bring all their language abilities to bear in the moment of engagement.

We now turn to the second snapshot from the classroom that will give more insight into the theoretical position we have outlined.

## 11.5 Transitions Between Registers for Developing Conceptual Understanding of Fractions: Snapshot from Germany

### *11.5.1 Research Context*

The following empirical snapshot from the MuM-Project[1] has been documented by Prediger and Wessel (2011). In the German MuM-Project, the teaching strategy "relating registers" is developed and researched within the paradigm of design research by means of design experiments (Gravemeijer & Cobb, 2006). The empirical research focus is on the situational potential to initiate substantial mathematical and linguistic activities of students.

Twenty-five design experiments in interview settings were conducted with pairs of students in grade 6. Although these students grew up in Germany, their parents were immigrants. The students had Turkish as their first language and good German BICS. The teacher only spoke German. The interviews were video-recorded, transcribed and analyzed qualitatively. The processes of relating registers were coded and then analyzed with respect to the opportunities and challenges for conceptual understanding.

The learning situation started with a text in the second language school register that encodes complex information of a UN-report about rates of analphabets, i.e., people who cannot read (see Fig. 11.5). The main conceptual challenge of this text was to refer 2/3 to the adequate referent whole, which is neither all adults nor all women, but the group of all analphabets (being a quarter of all adults).

To give students the opportunity to construct the necessary conceptual relations, five activities of relating registers were initiated:

Step 1: Translate the given difficult text (in school register L2) into own words (everyday register L1/L2).
Step 2: Check if the text matches to another utterance of a fictitious student, Tobias, "Wow, two-thirds of all women cannot read? Is that possible?"
Step 3: Translate the texts in school register and in everyday register into own drawing.
Step 4: Check if the given drawing (Fig. 11.5, right side) matches their own text and picture.
Step 5: Assign simpler texts, pictures, and fractions (this step is not discussed here).

[1] Within the long-term project MuM ("mathematics learning under conditions of multilingualism"), the study "Understanding fractions for multilingual learners. Development and evaluation of a language- and mathematics-integrated teaching strategy by relating registers" (Prediger & Wessel, 2013) was funded by the ministry BMBF (Grant 01JG1067).

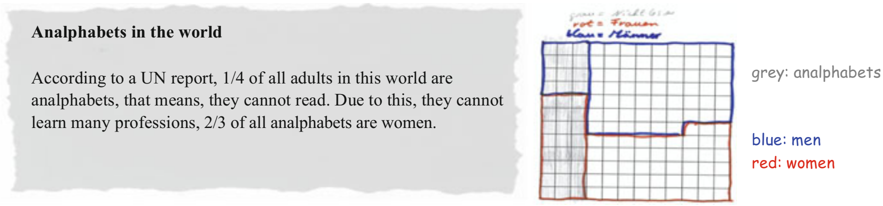



**Fig. 11.5** Text and drawing by a fictitious student

## *11.5.2 The Case of Amir and Ekim: Challenges and Insights While Relating Registers*

The case of Amir and Ekim, two 12-year-old boys in grade 6, illustrates some exemplary phenomena that were reconstructed in many cases. Having read the text (in Fig. 11.2) and contextualized it by talking about analphabets, the boys were asked to find a simpler formulation for it. Amir wrote down: "1/4 of all adults cannot read. That is why they cannot get many jobs." The first excerpt of the transcript (translated from German) starts when Ekim suggested the next reformulations for the phrase "2/3 of all analphabets are women":

59 Ekim: Oh wait, shortly. One quarter [*whispers something not understandable*]
60 Amir: Loud!
61 Ekim: Of one quarter are two-thirds
62 Amir: Women
63 Ekim: Who cannot read. Eh, we will write: thereof are two-thirds
64 Amir: Shall I comma? Mhm. [*negating his own question*]
65 Ekim: No, don't think so. Thereof are tw-two-third [*whispering*], [*louder:*] twothirds women, who cannot read.
66 Amir: [*writes down a slightly changed phrase: "Thereof are two-thirds* ***of*** *women who cannot read"*]

Ekim understood which whole the two-thirds must refer to in line 59, and formulated it explicitly in line 61: The two-thirds refer to the quarter. In his second suggestion in line 65, he correctly substituted "the quarter" by the undetermined adverb "thereof" and specified them linguistically by juxtaposing "women who cannot read."

When his partner Amir wrote the phrase, he added another preposition "of." By this little word, he referred two-thirds to two different wholes, so his sentence was linguistically and mathematically incoherent. The subtlety of these details becomes visible in (the nonprinted) line 70: When Ekim read the modified text aloud, he did not seem to realize the divergence embedded in his own suggestion.

In the second step, the boys were asked to evaluate Tobias's fictitious wrong interpretation ("Wow, 2/3 of all women cannot read? Is that possible?"):

89 I: And if—ehm—you think about, that he has the same data as you have, Tobias, he has the same data as you. That means, the same text or an easier text. Does the data match the text then? The numbers?
90 Amir: [*break 11 sec*] Yes, doesn't it? [to *Ekim*]
91 Ekim: Well, of one quarter are two-thirds of—ehm—of all women cannot read and then [*break 8 sec*]
92 I: Let us just draw it. [….]

Ekim correctly explained the relation between part and whole (line 91), but related the two-thirds to the group of all women as the whole by adopting Tobias's wording ("of all women" in line 91). He did not yet succeed in identifying Tobias's mistake. The episode shows the challenge of mentally constructing adequate relations between the part and its whole, and of finding an adequate verbalization for it.

In the third step, the boys were asked to draw the situation in a square (reconstructed in Fig. 11.6).

164 I: Yes, that is very good. And when we refer it back to the text now, can you explain to me what it means for the situation in the text? Again with the information—ehm—that the whole square are all humans in the world?
165 Ekim: Well this [*hints to the whole square*] are all adults and that [*hints to the red quarter*] are all adults who—eh
166 Amir: who cannot read
167 Ekim: exactly. who cannot read. And thereof, now two-thirds are women who cannot read.
168 I: Mhm. [*agreeing*] Do you draw that, too? Can you draw that into it?
169 Ekim: Two-thirds
170 Amir: Thirds. [*break 4 sec*] Yes.
171 Ekim: Shall we do that here? [*hints to the red quarter*, *but the interviewer does not react. Ekim answers himself without any break*] Yes, don't we? We must do that.

Ekim's verbalizations in lines 165 and 167 show how the transfer to the graphical register strengthened his understanding and he verbalized the structural relationship successfully. In line 165, he hinted at the graphical elements and used deictic means ("this" and "that") for expressing his ideas about the part and the whole precisely, although not yet explicitly. Using the drawing to clarify his thinking, he found a successful explicit verbalization of the structural relation between part and whole in line 167. The scene indicates how the struggle for explicit verbalization is associated closely with a stabilization of their mathematical insights. After that, they draw the picture as scanned in Fig. 11.7. They subdivided the quarter into three parts (intended to be of equal size) and colored two of them in blue. Their own drawing helped to transfer the situation to the context of an alphabet and to translate the mentally constructed relations into verbal formulations.

**Fig. 11.6** First drawing


**Fig. 11.7** Amir's and Ekim's drawing

In the end, the boys succeed in assigning all three representational registers and thus solved a cognitively demanding task cooperatively. When coming back to Tobias's fictitious utterance, Amir could now explain what Tobias did wrong: "Mmh, he has not the one quarter… drawn." (line 354, not printed here). Still having difficulties in expressing his thinking in the technical register, he emphasized that the quarter must not be neglected; hence he constructed the essential meaning through the long process of relating registers.

## 11.6 Final Remarks

In this chapter, we have outlined three language registers: the *everyday register*, *school register*, and *technical register*. In the mathematics classroom these registers are used in different communication microsituations, but are often used in combination across the context of a whole teaching session. It is important to stress that although the core of each register is clearly distinguishable, the boundaries between them are not hard and rigid, but permeable and at times quite fluid. This is due to the clear overlap between the contexts within which each register should be used, especially between the technical register and the school register. We also note that with the recent emphasis on authentic and/or real problems, which we applaud, teachers often deliberately seek to have students' transition between their everyday language register in describing and understanding such "real" problems before reconceptualizing it in the technical register. We encourage this, but clearly one of the key messages of this chapter is that such transitions by students do not always happen naturally. We need to look for teaching strategies that identify the language registers that the student may be using at any one time, and show teachers that there can be deliberate choices by students concerning what language register they wish to work in and what they could work in. Once a problem is solved mathematically, then a transition transporting the solution from the technical language back into everyday language is required so that students can appreciate the full power of their solution by linking it and embedding it into the network of their everyday experiences.

We have also commented on the movement between languages that multilingual students will naturally undertake (code-switching), whether school policy approves of such transitions or not. We have noted that such code-switching may occur when using everyday, school or the technical registers of language. Transitions between registers can be within the same language or between languages.

It is our contention that many mathematics teachers do not appreciate the complexity of the language, landscape their students' travel through mathematics lessons. Complex though this is, far from being a necessary impediment to learning, we contend that this rich landscape offers affordances that can be grasped to the advantage of deep student learning. We have noted that these affordances are often clearer when teachers plan to use manipulatives and visualization to contextualize the learning environment they attempt to create.

This first practical and empirical exploration shows the potential of bringing the three transitions together, theoretically as well as in terms of curriculum development and teacher strategies. Nevertheless this beginning brings with it a range of questions, both, theoretical and practical. We therefore conclude by asking a number of these questions which may prove to be crucial for an ongoing research agenda:

- Do students who are taught using this model develop better understanding of the subject-matter and/or perform better than other students? (Some empirical evidence has already been collected that pertains to this question: see Prediger & Wessel, 2013).
- What are the similarities and differences in language processes/strategies that students use while moving between L1 and L2 as compared to moving up the tiered model?
- Which teaching strategies can possibly guide/encourage students to use multiple languages and language forms?
- Are there central teaching principles at the core of different teaching strategies, with the strategies changing in response to the differing language contexts found in different countries and classrooms?
- Are there effective teaching strategies already in the literature that can be adapted to meet the conditions of this model? For example, can open questions, authentic questions, and an adapted use of think boards/place mats (Cunningham, 2002), be used as teaching strategies that are useful/effective in implementing this model?
- Are strategies aimed at making "language" more explicit in mathematics classrooms, such as, for example, using displays such as "word walls," helpful in focusing students' attention on language issues?
- What changes to mathematics curriculum documents, beyond including vocabulary lists and glossaries, will make issues such as language and the way it is taught and used for both monolingual and multilingual students one of the core components around which content and procedures coalesce, rather than the present structure that has content as the central component of such documents?

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