Chapter 13
Language Diversity and New Media: Issues of Multimodality and Performance

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13.1 Language Diversity and New Media

The discussion document of ICMI Study 21 on Mathematics Education and Language Diversity proposed the following central questions:

> Across the world, teaching and learning of mathematics occurs in contexts of linguistic and cultural diversity. How do we work with, and work within, this diversity to enhance the learning and teaching of mathematics? In particular, how can the range and complexity of learners' language backgrounds be most effectively used to promote their mathematical learning? (ICMI Study 21, p. 3—see Appendix)

We want to expand the discussion of linguistic diversity to include the role of new media in shaping how mathematics is, and how it might be, communicated. Students' out-of-school world is increasingly populated by digital and multimedia texts. Buckingham (2010) notes that,

> The term 'media' includes the whole range of modern communications media … Media *texts* are the programmes, films, images, web sites (and so on) that are carried by these different forms of communication … Media texts often combine several 'languages' or forms of communication—visual images (still or moving), audio (sound, music or speech) and written language. (pp. 3–4)

We want to consider these media and media texts as part of students' natural language environment and to explore the implications for mathematics teaching and learning. As we emphasize in this chapter, our focus is on the pedagogic production

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of digital and multimodal texts for the communication and representation of mathematical ideas through the use of the performance arts.

We want to bring to light the contrast between students' out-of-school experiences with new media (Cummins, Brown, & Sayers, 2007), with multimodal forms of communication aimed at a wide audience, and the continued predominance in most mathematics education of textual and symbolic communication aimed at a narrow classroom audience. We focus on multimodality and performance for a wider audience as two important affordances of new media and their role in shaping language diversity in mathematics education. We show these affordances in action, through two case studies from our work in mathematics classrooms, in Canada and in Brazil.

13.1.1 Humans-with-Media

Borba and Villarreal (2005) suggest that humans-with-media form a collective where new media serves to disrupt and reorganize human thinking. Likewise, Levy (1993) sees technology not simply as a tool used by humans, but rather as an integral component of a *cognitive ecology* of the humans-with-technology *thinking collectives*. Levy (1998) claims "as humans we never think alone or without tools. Institutions, languages, sign systems, technologies of communication, representation, and recording all form our cognitive activities in a profound manner" (p. 121). As Levy (1993) suggests, technologies *condition* thinking.

According to Tikhomirov (1981), computers do not replace, substitute, or merely complement humans in their intellectual activities. Processes mediated by computers *reorganize* thinking. Tikhomirov argues that computers play a mediating role in thinking as language does in Vygotsky's theory. Regarding the nature of human–computer interaction in terms of feedback, the dimensions involving computational mediation provide new insights in terms of learning, development, and knowledge production. Tikhomirov claims that:

> With regard to the problem of regulation we can say that not only is the computer a new means of mediation of human activity but the very reorganization of this activity is different from that found under conditions in which the means described by Vygotsky are used. (p. 273)

Borba and Villarreal (2005) use Tikhomirov's theory to argue how the notion of mediation by computers is qualitatively different to the mediation involving paper and pencil, for example. Through digital mediation, information technologies reorganize mathematical thinking. Media shape meaning and knowledge production and transform mathematics. The idea that media tools we use condition our thinking is not new and it should not be surprising. McLuhan (1964) made this link several decades ago with his often quoted assertion that "the medium is the message."

Scucuglia (2012) argues that students-with-media engage in thinking and feeling collectives when they produce multimodal texts to communicate their ideas using drama and/or songs, as arts-based communication forms engage both cognitively and emotionally. The pedagogic synergy involving the use of new media and the arts offer an interesting cognitive and affective scenario for students' mathematical communication and digital representation at the elementary school level.

13.1.2 Multimodality

Pahl and Rowsell (2005) posit that the word *multimodal* "describes the way we communicate using a number of different modes to make meaning" (p. 27) in effect providing a context of language diversity. Rowsell and Walsh (2011) state that "multimodality is the field that takes account of how individuals make meaning with different kinds of modes" (pp. 55–56). According to Walsh (2011), *multimodality* is "a study of the communicative process, particularly how meaning is communicated through different semiotic or meaning-making resources and in different social contexts" (p. 105):

> Multimodality as in comprehension and competence with language through a variety of modes such as image, sound, touch, multi-dimensions, is the principle upon which digital environments work. This principle of multimodality needs to be understood for educators to apply and assess new modes of learning as a part of everyday classroom practice. (Rowsell & Walsh, 2011, p. 54)

In contrast to the increasingly multimodal nature of the web, many school experiences, especially in mathematics, continue to rely on discourses that are monomodal or bimodal (in cases where diagrams or graphs are employed). Kress and Van Leeuwen (2001) suggest that in a digital environment, "meaning is made in many different ways, always, in the many different modes and media which are copresent in a communicational ensemble" (p. 111). The shift from text-based communication to multimodal communication is not simply a quantitative change. It is not just a case of having more communication modes. It is a qualitative shift, analogous to the change that occurred when we moved from an oral to a print culture (Gadanidis, Hoogland, & Hughes, 2008). Print culture, for instance, supported the creation of fixed media and records.

The New London Group (1996) proposes a model formed by six designs. These are: Linguistic Design, Visual Design, Audio Design, Gestural Design, Spatial Design, and Multimodal Design. The "Multimodal Design is of a different order to the other five modes of meaning; it represents the patterns of interconnection among the other modes" (p. 78). Some elements of Linguistic Design are:

Delivery:	Features of intonation, stress, rhythm, accent, etc.
Vocabulary and Metaphor:	Includes colocation, lexicalization, and word meaning.
Modality:	The nature of the producer's commitment to the message in a clause.
Transitivity:	The types of process and participants in the clause. Vocabulary and metaphor, word choice, positioning, and meaning.

Nominalization of processes:	Turning actions, qualities, assessments, or logical connection into nouns or states of being […]
Information structures:	How information is presented in clauses and sentences.
Local coherence relations:	Cohesion between clauses, and logical relations between clauses (e.g., embedding, subordination).
Global coherence relations:	The overall organizational properties of texts (e.g., genres) (p. 80).

The Visual Meanings refer to images, page layouts, and screen formats. The Audio Meanings refer to music and sound effects. The Gestural Meanings refer to body language, embodiment, and facial expressions. The Spatial Meanings refer to the meanings of environmental spaces/architectural spaces. However, the Multimodal Design "is the most significant, as it relates all the other modes in quite remarkably dynamic relationships" (The New London Group, 1996, p. 80). It is this component of the model that draws the other five together to form a whole.

More recently, Walsh (2011) theorizes how classrooms can become multimodal learning environments when students interact, collaborate, and produce multimodal texts in schools. Walsh actually emphasizes the role of intertextuality (combination of print-based and digital/multimodal texts) and dialogue in meaning production within educational and social purposes.

13.1.3 Performance for a Wider Audience

The multimodal texts that increasingly populate our digital environment are also conditioned by the pronounced sense of audience provided by new media. Using the Internet, and especially such tools as Youtube and Facebook, we can easily share our multimodal texts with the wider world and potentially reach a large audience. In a print culture, before the pervasiveness of the Internet, individuals had very limited access to a wide audience. Texts were produced by publishers to be consumed by us, the audience, without a reciprocal power to publish. With digital media, the audience now has access to production itself, through the availability of multimodal authoring tools (like blogs and Youtube) and access to a wide audience for self-published or collaborative work. The sense of audience that is associated with new media creates what Hughes (2008) has labeled as a "performative pull." We do not want to simply share our multimodal texts. We want to share them in performative ways. Hughes suggests that the new media that is infusing the web draws us into performative relationships with and representations of our "content." To use new media is to, in part, adopt a performative paradigm. The popularity of Youtube, whose slogan is "broadcast yourself," is a good example of the performative pull of new media.

Kress and Van Leeuwen (2001) also note that the multimodal nature of new media offers performative affordances. This is evident in the multimedia authoring tools used to create online content, which often use performance metaphors in their programming environment. For example, you program on what is referred to as the "stage," you use "scenes" to organize "actors" or "objects" and their relationships, and you control the performance using "scripts." Even more generally, Denzin (2003) notes that "We inhabit a performance-based, dramaturgical culture." Thus new media is helping amplify our cultural inclination for performance.

In the next section, we present two international cases of mathematics teaching and learning that illustrate the use and nature of multimodality and performance. Hill (1998) sees a performance-based pedagogy as a method of collaboratively making sense of experience. A performance-based pedagogy, coupled with the audience affordances of new media, position students as community storytellers of personal learning and growth and offers opportunities for them to experience "narrative reconstruction" as they reflect on their lives, their learning, their choices, their past experiences, and their goals for the future (Hull, 2003, p. 232). As Hull points out, "The ability to render one's world as changeable and oneself as an agent able to direct that change is integrally linked to acts of self-representation through writing" (p. 232). When students are given opportunities to share their "identity texts" with peers, family, teachers, and the general public through media, they are likely to make gains in self-confidence, self-esteem, and a sense of community belonging through positive feedback (Cummins et al., 2007). Hull (2003) urges collaboration among educators, researchers, and community organizations to "find space and time to think expansively about the interface of literacy, youth culture, multi-media, and identity" (p. 233).

There is ample research on the role of narrative in the construction of personal agency and identity (see Ochs & Capps, 2001). Bruner's (1994) studies of narrative indicate that changes in conceptions and representations of self are typically associated with "turning points" in personal narratives. Bruner identifies turning points as "thickly agentive … whose construction results in increasing the realism and drama of the Self" (p. 50). There is a dialogical relationship between narrative and self: to shape our narrative is to shape ourselves, and vice versa. There is also a dialogical relationship between narrative/identity and community. Narratives are social artifacts and "the narrated self is constructed with and responsive to other people" (Miller & Goodnow, 1995, p. 172). Stories change depending on the audience, and a personal knowledge story aimed for a school-based audience can change when the audience is the wider community. When the audience is the community, the narrative becomes more of a public performance. Hull and Katz (2006) note "the power of public performance in generating especially intense moments of self-enactment" (p. 47). Digital (unlike oral or solely print-based) stories, and hence more often than not using a language diversity context in the sense outlined in the first section of this chapter, potentially enhance the power of narrative to transform as they can be easily broadcast, creating a stronger sense of audience and performance (Hughes & Gadanidis, 2010).

13.2 Case Studies

In the case of mathematics education, Gerofsky (2006) notes:

> It is unusual (and energizing) to link mathematics and math education with performance, in no small part because many of the things that make a performance distinctive and interesting go squarely against many of the long-held traditions of mathematics. [It is important to] explore the human necessity of performance in mathematics. (p. 2)

In the two case studies that follow, one from Canada and one from Brazil, we explore the role that the new media affordances of *multimodality* and *performance for a wider audience* may play in shaping language diversity in mathematics classrooms. New media entered the classrooms in these case studies through the teachers' commitment to share a performance of student learning at the online Math and Science Performance Festival (available at www.mathfest.ca), which increased the sense of audience for both students and teachers. In addition, there was a focus in these classrooms on relating a good mathematics story. From our experience, when we ask parents how children respond when asked "What did you do in math today?" the common replies shared are "Nothing" and "I don't know" (Gadanidis, 2012). As we will discuss later in the chapter, the sense of audience for mathematics learning and the desire to relate a "good" mathematics story go hand-in-hand.

13.2.1 Case A: Grades 2 and 4 Students in Canada Explore Optimization

In Grades 2 and 4, students learn about perimeter and area at different levels of abstraction. Typically, these concepts are taught as definitions and formulas, with some concrete and pictorial representations. In the case below, Grades 2 and 4 students explore area and perimeter in the context of optimization. For example, "What is the largest possible rectangular area for a rectangular pen made with 12 m of fence?" and "What is the least amount of fence needed to enclose a rectangular pen whose area is 14 m^2?" Although such pedagogical directions are supported by reform curriculum documents, they are often not implemented. In a professional development session for Grades 1–6 Ontario teachers offered by the first author, all of the approximately 50 teachers present agreed with the statement that "rectangles with the same area must have the same perimeter," indicating that they most likely do not explore such relationships in their classrooms.

Grades 2 and 4 students explored optimization in the context of area and perimeter relationships: (1) "If you put 16 tables in a rectangular array, which arrangement fits the fewest chairs all around?" and (2) "What are the dimensions of the biggest rectangular pen that can be created with 20 m of fencing?"

In Grade 2 students first investigated rectangular arrays for numbers, using the story *The Doorbell Rang* (Hutchins, 1986). In this story, 2, then 3, 4, 6, and 12, children investigate how to share 12 cookies. Students used 12 linking cubes to

Fig. 13.1 Grade 2 song

We made 12 in a story

We made 12 in a Story
2 by 6 and 6 by 2
We made 12 in a Story
Have a look at the pictures we drew.

12 Kids on a bus
Have a look from the birds eye view
See my caterpillar
It has 12 polka dots too.

We made 12 in a story
3 by 4 and 4 by 3
We made 12 in a story
Look close at the pictures you see.

A cell phone with 12 buttons
12 dogs on a water slide
12 wheels on a bus
Boy can those dogs glide.

model their own solutions. They also used their imagination to illustrate the various arrays as everyday objects. For example, the 1×12 array might be a snake or a caterpillar (see Fig. 13.1), and the 3×4 array might be a calculator or a cell phone. They then used this knowledge to construct rectangular arrays for 16 patio stones (area), modeled using 16 linking cubes, and determine which arrangement would need the most or least fencing (perimeter), modeled using a piece of string. They explored different ways that 16 tables could be arranged to form a rectangle, and the number of chairs that would fit all around, and recorded their work on chart paper (see Fig. 13.3). Students also read the story *Wolf gets Hurt* (Gadanidis & Gadanidis, 2010), in which the Three Little Pigs capture the Big Bad Wolf and build a sturdy

Fig. 13.2 Grade 4 song

Need to fence a pen?

need a 20 metre fence
to make a sturdy pen
keep the dog in the yard
or catch bunnies off guard?

call us now, call us first
we're the professionals
we use math to optimize
no one beats our price

1 by 9, 2 by 8, 3 by 7, 4 by 6
rectangular designs
with four straight lines

but if times are hard
and funds are short
then order the 5 by 5 square
save money don't despair

rectangles are pretty
rectangles are nice
but to save you money
squares is what we advise
squares is what we advise

rectangular pen to hold him captive until the authorities arrive. The Three Little Pigs want to use the shortest fence possible for the pen with the 16 patio stones that they have available to build its base (so Wolf can't dig his way out). One of the songs written based on student thinking is shown in Fig. 13.2.

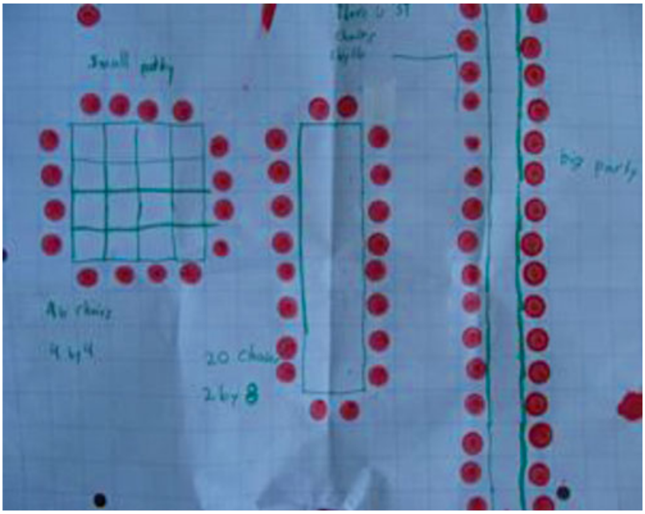


Fig. 13.3 Tables-and-chairs problem

In Grade 4, students also read the story *Scruffy's New Home* (Gadanidis, 2008), in which a young girl and her grandfather want to create the largest rectangular pen (in terms of area) using the 18 m of fence that they have available. Students graphed area vs. length and noticed that their solution (largest area) was the highest point on the graph. This was in contrast to their graph for the Wolf pen, where they graphed perimeter vs. length and noticed that their solution (shortest perimeter) was the lowest point on the graph. Students used the knowledge they learned to create an advertisement for a fencing company, and their ideas were compiled to create the class song shown in Fig. 13.2. Students also worked in small groups to author and practice performing dialogues (as skits) that they might have at home when someone asks: "What did you do in math today?"

A documentary of these Grades 2 and 4 activities is available at www.researchideas.ca/pen.html, with videos of teacher interviews and classroom action, music videos of the class songs, lesson plans, and an artistic representation of the activity (see Fig. 13.4). This documentary may be seen as a mathematics education research performance and as a mirroring of the classroom focus on mathematical performance.

The activities described above incorporated a variety of multimodal communication forms:

- Drawings
  - Illustrated children's literature used as a context for the activities.
  - Drawings of real-file objects based on arrays.
  - Illustrated advertisements for fencing companies.

Fig. 13.4 Artistic rendering of optimization

- Diagrams
  - Diagrams of number arrays.
  - Diagrams of tables-and-chairs arrangements.
- Tables
  - Tables that organized data from area–perimeter problems.
- Graphs
  - Bar graphs of area vs. length and perimeter vs. area.
- Songs
  - Lyrics based on student writing. (In this case, the lyrics were compiled by the teacher, and the students offered edits. In some other cases, students worked in small groups to summarize their learning in the form of stanzas, which were then compiled to form a song.)
  - A song set to music by a music teacher and performed by the students. (In some cases, students use common melodies like *Row*, *Row your Boat* or older students from the school set the songs to music for younger classes.)
- Videos
  - A music video of the song was shared publicly at the Math Performance Festival.

The activities also incorporated a variety of performative communication forms:

- Story
  - Illustrated children's literature as a context for the activities. (Story is a common way to organize or structure a performance.)

- Skits
  - Students created skits in preparation for sharing their knowledge with family and friends.
  - Math-based student advertisements based on their knowledge.
- Musical performance
  - Student performance of a song that summarized their learning.
- Video performance
  - Student musical performance shared with the wider world through the Math and Science Performance Festival.

What is the role of new media in connection to these multimodal and performative communication forms? We do not suggest that new media's affordances of multimodality and performance *caused* the students and teachers to use multimodal and performative forms of mathematical communication. Rather, we offer two important observations.

First, some of the communication forms listed above are already common in some classrooms and supported by curriculum documents, while others are atypical. For example, in a Canadian context, although it is not uncommon for a Grade 2 teacher to use a story as a starting point for learning, it is less common that it is used for learning of mathematics and it is generally less common as we move up the grades. Also, the use of performative forms of communication, such as skits and songs, is much less common. That is, there is a linguistic diversity in the case study classrooms, in terms of modes of communication, which is not common in mathematics classrooms. Most of these examples of multimodality or diversity can be done with other "old" media rather than with "new" digital media. However, old media does not lend itself as easily to such forms. To draw an analogy, it was possible for people relying on horse and buggy for transportation to live say 50 km away from their work, but this was a rare occurrence until the widespread adoption of the automobile.

Second, although Grades 2 and 4 teachers share student work with parents without the use of new media (by sending tests home or assigning take-home projects that involve parents), they typically do not do so with a wider audience, as did the teachers in these classrooms by engaging their students to create mathematical performances to be shared on a publicly available web site (www.mathfest.ca). In this context, it is also safe to assume that teachers want to share their students' best work. But what is "best work" changes with the audience. Narrow audiences, such as classrooms, tend to create work that is based on well-defined problems, fixed meanings, and symbolic representations. If the audience is widened beyond the mathematics classroom, to include "just plain folks," then the focus shifts to "emergent problems," "negotiated meaning," "causal stories," and generally narrative forms of communication (Brown, Collins, & Duguid, 1989). So, when the audience becomes the "just plain folks," what is "best work" is judged less in terms of right or wrong and more in terms of being a good story or performance.

Boorstin (1990), writing about movies, notes that good stories have the following characteristics (paraphrased to suit our mathematics context): they provide new mathematical perspectives; they offer mathematical surprise; they engage emotionally; and they help us experience visceral mathematical sensations, such as mathematical fit, pattern, and beauty. Without claiming causality, it is interesting to note the parallels between the goal of a good math performance and the multimodal and performative forms used in the activities. For example, by introducing constraints and focusing on optimization, the teacher created opportunities for students to experience the new and surprising idea (for young students and even many parents) that the area can remain constant while the perimeter varies (and vice versa) or that optimal solutions involve squares (assuming we are dealing with rectangular arrangements). Watson and Mason (2007, p. 4) "see mathematics as an endless source of surprise, which excites us and motivates us [...] The challenge is to create conditions for learners so that they too will experience a surprise."

Gadanidis and Borba (2008) and Scucuglia (2012) suggest that surprises are fundamental components for the production of conceptual mathematical performances. Mathematical surprises (at least surprises that are deeply and not superficially mathematical) require complex mathematical ideas and deep conceptual relationships, which is what the case study classroom focus on optimization provides. In contrast, a typical worksheet that asks students to calculate area or perimeter for rectangular shapes offers no mathematical surprise and little mathematical pleasure. By using stories and skits, students can experience emotional mathematical moments through the mathematical "adventures" of the characters involved. And, visceral sensations are introduced by engaging students with physical, diagrammatic, and tabular pattern and fit, offering opportunities to see beauty in mathematical ideas and representations, as well as musical performances. Boorstin (1990) adds that movie soundtracks offer visceral sensations to the audience.

13.2.2 Case B: Brazilian Students Explore Sequences and Series of Numbers

The overall conclusion that emerges from research is that the teaching of algebra is typically instrumental rather than relational, with a dominance of symbolic algebra over other representations (Borba & Confrey, 1996; Kieran, 1992; Kieran & Guzmán, 2009; Kieran & Sfard, 1999). Teachers seem to "hold a symbol precedence view of student mathematical development" and they seem to "overestimate the accessibility of symbol-based representations and procedures for students' learning introductory algebra" (Nathan & Koedinger, 2000, p. 209). Consequently, though they learn to manipulate algebraic expressions, students do not seem to be able to use them as tools for meaningful mathematical communication (Kieran & Sfard, 1999). The majority of students do not acquire any real sense of algebra and, early on in their learning of algebra, give up trying to understand algebra and resort to memorizing rules and procedures (Kieran, 1992). Such learning of algebra

Fig. 13.5 Odd numbers

creates a weak foundation for relating good math stories in response to the question. "What did you do in math today?"

In the case described below, from a Grade 7 classroom in Rio Claro, Brazil, students started exploring the problem of finding the sum of the first N odd numbers with concrete materials and noticed that they fitted together to form a square, as shown for the first four odd numbers in Fig. 13.5. Then they explored the sums of even numbers as well as combinations of the two. In doing so they did two things differently from the typical classrooms described above. First, they explored mathematics problems that are complex. Finding a formula for the sum of the first N odd or even numbers is typically a senior secondary school topic. Second, they used concrete and visual representations (as well as others, as we will soon see) to explore the problems, and develop and communicate their understanding.

Grade 7 students investigated odd numbers as L patterns (see Fig. 13.6) using several materials, such as songs, lyrics, video clips, manipulative blocks, and online applets. The L patterns approach to the study of sequences and series of odd and even numbers has the potential to offer surprise in generalizing patterns. Working with manipulatives and applets, students can construct sequences of numbers and they can connect blocks (or simulate the connection) to have a geometrical representation for the series which works as a visual proof. Playing with L patterns becomes a surprising and visceral experience when students connect Ls forming squares (odd numbers) and $N \times (N+1)$ rectangles (even numbers).

Noss et al. (2009) argue that the difficulties that mathematical generalization and algebraic expression pose for students have been thoroughly studied. For these authors, the difficulties students face when dealing with generalization activities are in some measure due to the way in which they are presented and the constraints of the teaching approaches used. Usually, teachers tend to teach the techniques isolated from all context to help their students find the rule. There is a need to introduce students to different approaches involving generalization of patterns (Noss et al., 2009). We do see the exploration of the L patterns in a digital-artistic environment as one more pedagogical approach to address this need.

From the humans-with-media perspective (Borba & Villarreal, 2005), experimentation-with-technologies and visualization play fundamental roles in mathematical thinking. Students–teacher-media were involved in figuring out a

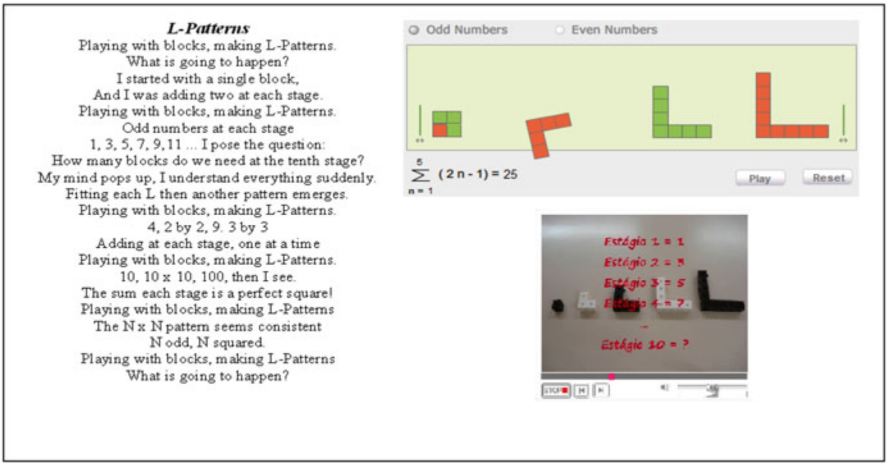


Fig. 13.6 L patterns

generalization for $S_n = 1 + 3 + 5 + \cdots + (2n - 1)$. By investigating the algebraic sum at each stage (writing, singing, and watching a video) and articulating it to the connections of Ls forming a square (constructing with blocks and simulating with an applet), the students–teachers-media identified a new pattern together, as thinking collectives. Then, one student came up with an interesting conjecture asking: “But, if we think about it, there are even numbers in rectangles, are there not?” The teacher said: “Perfect. What is the shape related to the series of even numbers?” The student then argued: “A rectangle, right? I can see it with the applet.”

We suggest the student did an important articulation involving several representations and media in investigating the L patterns. Thinking-with-Applet-song-lyric-video-blocks-and-other-media, *the student* was able to conjecture a visual representation for both the sequences and series of odd and even numbers. The group of students had not developed an investigation about even numbers until that moment, but the student was able to visualize and manipulate the applet and the blocks, relate it to the lyrics, images, and sounds, recognize an approach relating to the sum of even numbers, connect it to the investigation of odd numbers, and communicate that the series of even numbers can be geometrically represented by rectangles. This moment revealed a significant role for technology and multimodal representation in shaping students’ thinking and learning.

Based on the investigation of the L patterns, another pair of students decided to create a new sequence of numbers. These students created a first F using six blocks. They discussed the growth of the sequence and they decided to add three blocks at each stage, forming the sequence (6, 9, 12, 15,…) (see Fig. 13.7). The teacher brought up this discussion of F pattern to all students. He praised students’ conjecture and imagination in creating a new pattern. He proposed that they develop a similar investigation on the F pattern as they had made to the L pattern. “What could be a generalization for the sequence? What would be the series? Would it be possible

Fig. 13.7 F patterns

to connect the blocks and create a visual proof to the series? Could we create a performance about F pattern?" One of the students said they were trying to figure out the series, but they did not find a regular shape (as a rectangle) to express an algebraic formula for the series. Figure 13.7 shows students' tentative start of creating a visual proof for the *F pattern* series. The teacher asked them if they had already found a formula to the sequence. After 15 min of dialogue between the pair of students, where they used blocks, and paper and pencil, one of them said: "I found it! It is n times 3 plus 3." The teacher said the generalization $3n+3$ seemed to be a good candidate. The teacher proposed to all students in the class to test it for several stages, and they confirmed the students' conjecture. The pair of students celebrated the confirmation of their conjectures greeting each other. This moment revealed students' pleasure in thinking mathematically. It reveals mathematics activity as an emotional, visceral, and surprising experience in learning (Gadanidis, Hughes, & Borba, 2008).

The pair of students came up with the idea of creating a new performance based on the theme of TV News. The idea was to present an interview with a mathematician who had discovered the F pattern. This was accepted by the group and students made suggestions such as: provide humor, use specific songs and sound effects, the kind of images they should portray about reporters and mathematicians, and how to explain the series.

The teacher emphasized students could explore a formula for the series of the F patterns, but students had difficulty in developing it. The teacher described what he called the "Gauss method" to figure out an algebraic formula for the series, which involves adding the first and last terms, then the second and second last terms, and so on, to see if a pattern emerges. After that, students started to write the script of the roles they would play in the skit. They collaborated and negotiated the meanings, reorganizing or rewriting their scripts. This negotiation and rewriting also happened during the performance. Students also negotiated their use of the manipulative blocks, the whiteboard, what should be written on the board, angles of video recordings and close-ups, moments of surprises and emotions, etc.

Students–teacher-with-media recorded the scenes in the same order they wanted to present it. The process of playing and recording involved students' speech repetition and improvisation through many takes of scenes. These actions revealed a significant aspect of improving communication and imagination in learning mathematics through creating a digital skit performance. After the recordings, based on other students' suggestions, the teacher edited the video called *The F Pattern*

Fig. 13.8 Making the F patterns video

News (see Fig. 13.8) with the participation of the students. The teacher worked on the Portuguese/English translations to create subtitles and some students were interested in learning how to say in English the words they said in the video (another aspect of language diversity). Students agreed that the mathematical idea was communicated clearly in the digital text. Then, the thinking collective created a final draft at the school's Lab and one of the administrators of the school submitted the video to the Math Performance Festival.[1] Below we present a transcription of the video:

Reporter 1: We are interrupting the TV show to tell you terrific news! One just discovered that beyond the L patterns, there is an F pattern!
Reporter 2: F pattern? What are you talking about?
Reporter 1: Our reporter has more information about it.
Reporter 2: Hey, can you hear us?
Reporter 3: Yes. I am here with Hypotenuse. She knows everything about math.
Reporter 2: What is this story about F pattern?
Hypotenuse: The F pattern is here represented by blocks.
Reporter 1: Could you please ask her what would be the sequence?
Reporter 3: Sure. What would be the sequence for the F pattern?
Hypotenuse: As we can see, the sequence increases three blocks each stage.
Reporter 3: Look, at the first stage you used six blocks. At the second stage you used nine blocks. At the third, 12 blocks. And so on. What about the 100th stage? How many blocks do you need to construct the 100th stage?
Hypotenuse: [using the whiteboard and the blocks] We have a simple formula to figure it out. You can notice the result each stage is equal to the index of the stage times three plus three. That is, $3N+3$. At the 100th stage we have 100 times three plus three. It is equal to 303.
Reporter 2: [with a surprising sound on the background] I am getting the information that the series was discovered. What would it be?

[1] Available at www.edu.uwo.ca/mpc/mpf2010/mpf2010-134.html.

Reporter 1: In other words, what would be six plus nine plus twelve and so on?

Reporter 3: Hypotenuse, what would be the series?

Hypotenuse: [in fast motion and using the whiteboard] I am going to use Gauss's ideas. The series would be the sum of these numbers. Gauss did it twice. Once from the beginning to the end, that is, $S=6+9+12+\cdots+3n-3+3n+3n+3$. And once from the end to the beginning, that is, $S=3n+3+3n+3n-3+\cdots+12+9+6$. Then, we can add these two expressions. So, we have $2S=(3n+9)+(3n+9)+(3n+9)+\cdots+(3n+9)$. Therefore, we have $2S=n$ times $(3n+9)$. It gives us the following formula… [Showing it on the board]. With this formula we can calculate the total number of blocks we need until the stage we wish.

Reporter 1: Last time we talked about the L patterns. Today, we talked about the F pattern. What about you? Which one is your favourite pattern?

Reporter 2: Which one will be our next pattern?

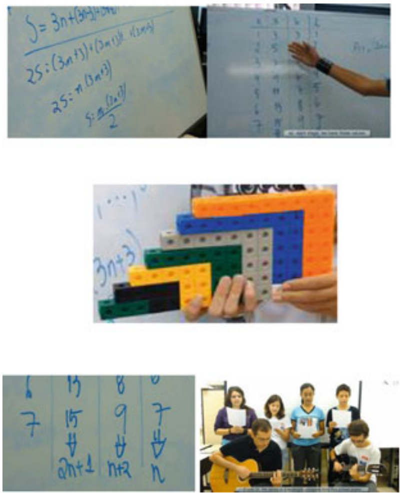
Reporter 1: Your local show is coming up next.

Reporter 2: Good night … have a good day … good afternoon.

Later, when these students were in Grade 8, we invited them to share their digital performance with a class of Grade 7 students in their school. Groups of students were engaged in exploring and discussing both the L and F patterns, in a context we potentially designed as a multimodal learning environment. Some students of these groups were thus exploring the ideas for the second time (Grade 8), and some of them for the first time (Grade 7).

After watching the digital performance, one of the Grade 7 students was manipulating the blocks, playing with the blocks, and making Fs using paper and pencil. The student said the group could explore the sequence 3, 6, 9, 12, 15,… instead of 6, 9, 12, 15,… There were interesting reasons that the student justified: (1) the new sequence with general term $3n$ is very similar to the original sequence with general term $6n-3$, and (2) the Fs made with blocks to represent the new sequence can be reorganized, rearranged, or reshaped in order *to fit*, forming a trapezoid. We see this event as a *visceral* moment in students' mathematical thinking, in terms of the esthetic pleasure of mathematical fit (Sinclair, 2006), as thinking about the problem was reorganized using manipulatives and other media. Another student noticed that the original F pattern could be seen as a combination of two patterns: the natural numbers 3, 4, 5, 6,… and the odd numbers 3, 5, 7, 9,…

The scenario is interesting in terms of the role of new media and the arts in students' mathematical investigation, meaning, and knowledge production. After watching a digital performance produced by students, a solution could be redesigned based on the use of several media. Students-with-blocks-video-and-other-media, dialogically and visually, conjectured that the series $3+6+9+12+\cdots+3n$ could be geometrically represented as a trapezoid. The (visceral) idea was further explored since students were engaged in producing a new digital mathematical performance. The use of new media (e.g., video camera and software to edit videos) and the arts (drama and music) to produce a digital performance offered ways for students to enhance their meaning production, because they were seeking to refine and improve their communication as they were producing the performance



F Patterns

I will sing about what I've learned,

I will talk about what I've felt,

about Ls and Fs.

With Ls we have even and numbers,

many emotions.

With Fs we learned, more than once,

multiple of three.

Even numbers, 2n,

the series is a rectangle.

Fig. 13.9 A new performance: the series of F patterns represented as a trapezoid

(creating the story and the lyrics, practising and recording them, editing the video to share it online to the world). The process of expanding language diversity based on the use of new media and arts offered ways to better understand the conjecture and justifications, (re)organize thinking, connect representations, elaborate new conjectures, etc. It was a rich moment for mathematical meaning production, which is a significant aspect of learning.

The student performance explored the L and F patterns and introduced a geometric and algebraic investigation of the series related to the F pattern. Scucuglia and Gadanidis (2013) analyze aspects of this performance highlighting that the engagement with digital mathematical performance may open windows into mathematical thinking. The engagement in a multimodal learning environment offered ways for collective dialogues, new mathematical investigations and insights, and the production of a digital performance in which students communicated and represented aspects of their mathematical ideas learning. The new performance has the potential to open new windows for other groups of students, teachers, or classes. Figure 13.9 shows scenes of the performance and the lyrics.

The activities described above incorporated a variety of multimodal communication features:

- Concrete materials: blocks were used to construct odd, even, and F patterns.
- Drawings: students used drawings to record the concrete patterns they explored.
- Tables: students used tables to record sequences of numbers and investigate patterns.
- Interactive simulations: students explored a simulation of odd numbers fitting in a square.
- Songs: a song was used by the teacher to introduce the problem of finding the sum of the first *N* odd numbers.

- Drama: students authored and performed a skit about the F patterns problem.
- Videos: students explored a video about noticing patterns that might help to find the sum of odd numbers; students created a video of their skit about F patterns. They used gestures and movement to embody and communicate mathematical ideas.

The activities also incorporated a variety of performative communication forms, as elaborated above:

- The following performative modes of communication were used by the teacher:
  - Song
  - Video
  - Interactive simulation
- The following performative modes of communication were used by the students:
  - Drama
  - Video performance

It is interesting how easily the students took up alternative forms of mathematical communication, displaying great language diversity in an interconnected fashion. They used a variety of texts, materials, resources, and digital technologies: lyrics, paper, and pencil, manipulative blocks, computers, videos, Internet, and applets. It is also interesting that in this environment students were drawn to create and perform a dramatic skit to present their learning about F patterns. When students share their mathematical ideas by playing roles as actors they think about the audience. The production of a dramatic event (a skit) involving mathematics helps students on the (re)organization of their mathematical ideas to suit a wider audience. The development and creation of a mathematical narrative deals with sense-making in a way that may better appeal to "just plain folks," by using forms of communication that are commonly used in the media and narrative structures that draw attention to discrepancy and surprise. The process of recording a performance also involves speech repetition and helps students improve their mathematical oral/verbal communication skills and understanding. In addition, the production of drama-based narratives involves ways of imagining and reflecting about the "self" and its relation to doing and learning mathematics. When students perform mathematics they construct identities as performance mathematicians (Gadanidis & Borba, 2008).

13.3 Audience, Multimodal Communication, and Language Diversity

In this chapter, we have critiqued the issue of language diversity in terms of the traditional and persistently narrow mathematics education focus on textual/symbolic communication suggesting there are ways to expand the boundaries for this notion. We have contrasted this phenomenon with the multimodal and performative out-of-school environment that students experience due to the pervasiveness of digital media. We have also highlighted the new media affordances of multimodality and performance and illustrated what these might look like in the mathematics classroom through two case studies: one from Canada and one from Brazil. In this section, we discuss the potential interplay between performance, multimodality, and language diversity.

Reflecting on the two cases we have presented in this chapter, and keeping in mind our similar work in other classrooms in Canada and in Brazil, we note two important themes. First, students engage with multimodal forms of communication with little hesitation. Although we did not collect data to shed light on the reasons for this, we speculate that it might be the case because multimodal communication is part of students' digital environment, especially in out-of-school experiences, or because students find multimodal forms of communication interesting and engaging, and indeed for many students compelling. Second, what is different in these classrooms is more than simply the modes used to communicate information: there is also a relationship difference or shift with respect to mathematics learning, such as, from passive to engaged, from consuming to producing, and from thinking to thinking and feeling. Although our digital age is often labeled as an information revolution (as contrasted with the industrial revolution), Schrage (2001) suggests that this label misses the essence of the paradigm shift:

> *The so-called* "*information revolution*" *itself is actually*, *and more accurately*, *a* "*relationship revolution*." Anyone trying to get a handle on the dazzling technologies of today and the impact they'll have tomorrow, would be well advised to re-orient their worldview around relationships […] *When it comes to the impact of new media*, *the importance of information is subordinate to the importance of community*. The real value of a medium lies less in the information that it carries than in the communities it creates. (pp. 1–2, original emphasis)

Lankshear and Knobel (2006) suggest that the relatively recent "development and mass uptake of digital electronic technologies" represent changes on an "historical scale," which "have been accompanied by the emergence of different (new) ways of thinking about the world and responding to it" (pp. 29–30). These new ways of thinking can be characterized as "more "participatory," "collaborative," and "distributed" and less "published," "individuated," and "author-centric" … also less "expert-dominated" (Lankshear & Knobel, 2007, p. 9).

We suggest that the case studies we have shared offer glimpses into "what might be" when students are encouraged and supported in taking up the multimodal and performative affordances of new media. We further suggest that "what might be" is about a change in relationships between students and the world around them, such as taking up roles as producers of mathematical knowledge for audiences beyond their classroom. Change in how students communicate with a wider audience and how their use of language positions them in relation to the world around them involve change in their literacy identity. This change may be seen as increased language diversity, as students use new forms of communication and for new purposes (we mean "new" in relation to traditional mathematics teaching and learning contexts). Weber and Mitchell's (2008) notion of identity as "personal and social bricolage" views identity construction as "an evolving active construction that constantly sheds bits and adds bits, changing through dialectical interactions with the digital and non-digital world, involving physical, psychological, social, and cultural agents" (p. 43). New literacies, as defined by Lankshear and Knobel (2007), are not characterized solely by their digital or technical features. They also involve a new mindset or a new ethos that focuses on participation, collaboration, and distribution (Lankshear & Knobel, 2007). Traditional forms of literacy that focus solely on reading and writing in the dominant language typically fail to recognize the impact of digital media in students' lives or take into account students' multilingual and multicultural backgrounds and experiences.

We also suggest that once students focus their communication (at least in part) for an audience beyond the walls of their classroom and their school, the criteria by which "good" communication is determined changes, as the traditional "breach between learning and use" is bridged (Brown et al., 1989, p. 32). Students (and teachers) aim their communication not for their classroom, where the focus is traditionally on "precise, well-defined problems, formal definitions, and symbol manipulation" but for the "just plain folks" outside the classroom walls who reason with "causal stories" rather than "laws," act on "situations" rather than "symbols," resolve "emergent problems and dilemmas" rather than "well-defined problems," and produce "negotiated meaning and socially constructed understanding" rather than "fixed meaning and immutable concepts" (Brown et al., 1989, p. 35). Creating a math performance that outlines the common algorithm for adding two or three digit number or the rule of inverting and multiplying when dividing fractions does not make for a "good" story to share with the outside world.

Although not typically evident in mathematics classrooms, human cognition is story based and we naturally seek to experience and to relate good stories (Schank, 1990). We think in terms of stories, we understand the world in terms of stories that we have already understood, we learn by living and accommodating new stories, and we define ourselves through the stories we tell ourselves (Schank, 1990). Our lives make sense when shaped into narrative form (McIntyre, 1984). Story is a human symbol system used to comprehend events and entertain questions, and represent those events and questions in a sequence that offers a new perspective, surprise, emotional moments, and visceral sensations in a way that makes sense (Boorstin, 1990): that is, the story must provide its own justification, or prove itself.

Thus, as students and teachers use new media to share aspects of classroom learning with the wider world, they naturally seek to engage their audience through surprise, emotional moments, and visceral sensations, as did the students and teachers in our two cases. The simplest (and most common) way to do this is to keep the mathematics unchanged while couching it in a nonmathematical story context, a sugar-coating, that offers the story pleasures of surprise, emotional moments, and visceral sensations. But there is also the potential to create and relate truly mathematical stories that engage a wider audience. For this to take place, school mathematics needs to become more complex and less focused on procedural learning. As Gadanidis, Hughes, and Cordy (2011) note:

> We do not believe that artistic *mathematical* expression is possible in a mathematics program which focuses on procedural rather than conceptual knowledge. Students can add artwork to "decorate" procedural knowledge, thus adding a layer of "sugar-coating" to otherwise dry mathematical ideas, but mathematical art, like art in general, requires a deeper engagement and understanding. Thus, for us, challenging mathematics is a co-requisite for artistic mathematical expression. (pp. 423–424)

In the two cases we shared, the "challenging" mathematics took the form of a focus on optimization as a context for learning about area and perimeter and a focus on sequences and series of odd and even numbers as a context for learning about algebraic representation. These contexts offered Grades 2 and 4 students the mathematical surprise that squares maximize rectangular area and minimize rectangular perimeter and Grade 7 students the mathematical surprise that the series of odd numbers "hides" in squares (and in $N \times (N+1)$ rectangles for even numbers). The multimodal representations of these relationships also offer the visceral pleasure of mathematical "fit," as students slide the odd numbers to form a square, for example. The emotional mathematical moments experienced by the students are evident in their performances, through song and drama.

13.4 Concluding Remarks

When we expand the scope of what we mean by language diversity to include digital literacy we realize that new media not only offers students and teachers new ways of communicating about mathematics learning but it also offers the potential to shift literacy identities in ways that require new roles and new ways of thinking about what constitutes a "good" mathematics story or experience. Borba (2009) discusses some scenarios of the Internet being fully accepted in the classroom. Performance was one of the possibilities. It could be the case that mathematical performance could be a means that students who cannot speak, or students who do not speak the official language well, can express themselves. It could also be the case that students create performances that disrupt traditional school mathematics. In this manner, mathematical performance could gain a political dimension, which has not yet been explored in a theoretical manner nor in empirical research. For example, imagine exploring *Mathematics of the Oppressed* as a way of drawing a parallel between Boal's (1985) political work *Theatre of the Oppressed*, which introduces the term *spect-actor* to bring focus on the need for personal and collective agency, and the pervasiveness of mathematics education structures that minimize students' cognitive and affective agency. We believe such issues could be part of the research agenda for mathematics educators interested in political dimensions of mathematics education and mathematical performance.

The cases we shared in this chapter from Brazil and from Canada are early examples or first drafts of "what might be" when new media and its affordances are taken up by students and teachers. We believe that these examples, by exploring the new possibilities of digital media, combine mathematics and the arts, show the connection between media, emotions, and thinking, and show how language diversity may be expanded to include issues of multimodality and audience. We have argued that such changes reorganize thinking, generating changes in the knowledge produced by this collective of humans-with-media. We hope that the cases we have presented and discussed serve as objects for reflection and critique, so that better and more varied examples of "what might be" in mathematics teaching and learning may be developed.

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