Chapter 14
Language Diversity in Research on Language Diversity in Mathematics Education

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14.1 Identifying the *Problématique*: Language(s) of Education, and of Research

In research on mathematics education, as in many other fields, "international" has become synonymous with "English." It is no surprise that the announcement (ICMI Study 21 discussion document, this volume, p. 307) for the present ICMI study conference stipulated that papers must be in *English*. This can be justified by the fact that, presumably, no other language is mastered (to some degree) by more researchers who are likely to be able to participate in a study of this kind. But it also raises a number of concerns, which this chapter aims to point out, together with a tentative approach to address them. We feel it is particularly appropriate to raise these concerns in connection with an international study whose subject—language diversity in mathematics education—seems, to us, is in need of a special sensitivity to this issue. Indeed, one of the sub-themes of the study being "Researching mathematics teaching and learning in multilingual contexts" (see Appendix), one could not help noticing that "contexts" in which English is not at all present (as a language of instruction) might indeed be somewhat underrepresented in a study where

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English is expected on the part of researchers. For instance, a considerable body of research literature, including international journals, appears in other "old world" languages such as French, German, Russian, Spanish, and Portuguese, which, like English, became "dominant" in large regions beyond Europe, or have become common channels of publication, even where they are not dominant. One could also add to this list the vibrant body of mathematics education research literatures, which are formulated and published in non-English languages in India, Japan, and China.

The authors of this chapter share one commonly used research language, French, while our mother tongues are Arabic, Vietnamese, and Danish. In the heading of this first section, we have put the word *problématique* in italics to show that it has been appropriated into the English language.[1] We do so because there does not seem to be a genuine English word to designate what *problématique* means in French, an issue that occurs many times over in translations between languages. With some approximation we could explain the meaning of it as "a collection of related questions, phrased within a certain theoretical framework and, in particular, based on some fundamental assumptions related to this framework" (see also Wedege, 2006, for a thorough discussion of this point).

The reader will notice two things here: this supposed translation raises more questions than it answers, in the sense that we have not explained the more numerous terms used to explain the term *problématique*, in particular, what is (or could be) a "question" and a "theoretical framework." Moreover, our discussion of the subheading has taken us into a meta-discussion of terminology which, in itself, may seem to have rather little bearing on any question related to the teaching and learning of mathematics.

But we strongly argue that despite the clear reasons for communicating in "big" languages, one should not be naïve about theoretical frameworks—and the language which carry them—when formulating research questions. This, of course, is particularly true in a volume that bears on language diversity. Unlike the kinds of question we pose everyday, such as "How do you do?" or "What time is it?," we cannot allow research questions to grow on of what Chevallard (1999a, p. 7) calls "teashop English": certainly not when we address, and belong to, an international audience for which English of some sort may be the only available common ground, but in which "teashop English" remains the private business of a minority of so-called "native speakers."

The *problématique* we are trying to develop, bearing on research on mathematics teaching and learning, clearly requires a theoretical framework that allows us to talk *about research* and in particular *about* the language(s) used in such research (which we call the R-language(s)). It must be able to do so *in distinction* from language(s) appearing in the phenomena which the research bears on, i.e., the language(s) that are used by teachers and learners of mathematics in a particular context (these, we call the P-language(s)). The theoretical framework itself will employ a language, which we could call the M-language (with M for *meta*): in this paper, English.

[1] Indeed, the online service *Google translate*, http://translate.google.com, does recognize it to the level of providing reasonable translations of it into other languages.

We have so far mentioned, even if not entirely explained, some first terms of our M-language: besides *P-languages*, *R-languages* and *M-language*, also the term *problématique* and the terms used to explain it. We notice that except for the three abbreviations and one term (perhaps) imported from French, we have so far done with more or less standard terms from the English language, in accordance with the study call.

There is little chance, however, that a reasonable P-language can do with standard terms of any natural language, when the phenomena involve mathematics teaching and learning. While the R-language does in practice often remain naïve with respect to what is meant by "mathematics," mathematical practices in teaching and learning contexts will rarely fail to display crucial language-like items which are foreign to any natural language: number symbols, diagrams and, so on; the R-language may or may not contain systematic ways to refer to such items. The fact that these items are often similar in contexts where the natural languages appearing in the P-language differ greatly, may lure the researcher into the illusion that the P-languages are somehow equivalent or at least that the differences remain superficial. For instance, the solution of quadratic equations in symbolic form may be easy to recognize and follow on the blackboard of a classroom, even for an observer who understands nothing of what is said and written in natural language. In a very local sense, this impression of "following" may of course not be illusory at all. But the R-language can rarely, for a *problématique* bearing on teaching and learning, ignore the conditions and constraints which surround (often determine) these more or less transparent items. Thus, the R-language would need to be sensitive to the P-language in a wider sense, which almost invariably involves specific features of the P-language, such as (what we could, at this point, loosely term) *cultural connotations*; for instance, number words are often specific to and deeply rooted in cultural practices, and the differences may cause challenges for second language learners (cf. "Personal Case 2" below). We notice that "sensitivity" is, in general, not just a question of language, but also of the ways in which it is used, including a number of assumptions, specific rules and assertions. For example, in P-languages related to elementary mathematics, the English word *triangle* focuses on different defining aspects of a class of geometrical figures than the Danish word *trekant* (meaning, literally, "three-edge"). We shall term this "way of language use" a *theory* (so that we can speak of P-theory, R-theory, and M-theory).

The M-language, and in fact the M-*theory* which we are about to develop, must thus identify crucial features of the R-language and its capacity to deal with specifics of the P-language. Moreover, the M-language must enable us to ask relevant research questions about the R-theory itself. We note in passing the parallel between this preliminary discussion of M-language and a debate, more than a century old, on the foundations of mathematics as a science, in fact, as a theory. Here, logic—which, at least from a certain point of view, is part and parcel of mathematics—has been proposed as a framework for, at least in part, how to inquire into the basis of mathematics itself. While this approach is of course not free from problems, we notice that the particular problem of self-reference—taking a model from within a theory to model the whole theory—is not substantially more questionable than

exhibiting an entirely foreign model, since the pains of infinite regress arise as soon as we ask how this "foreign" model would somehow equip us with safer grounds than the object we are about to model. We mention this because we will, in what follows, propose an M-theory for framing our problématique which is taken, almost wholesale, from a particular R-theory.

14.2 The *Problématique* and Its Meta-Language

Whether we talk of P-language, R-language, or indeed M-language, we refer to ways to talk about certain human *practices*. The R-theory on which this paper is based is the anthropological theory of didactics (ATD) takes as a basic assumption that P-languages serve to *talk about* certain practices related to *doing mathematics* or, more specifically, *solving tasks of a mathematical kind* using corresponding *techniques* (Chevallard, 1999b). The "mathematical kind" is, according to another fundamental assumption of ATD, determined by institutions.

As ATD is extensively developed in the literature (an excellent overview is given by Bosch and Gascón, 2006), we do not give a separate account here. We note, however, that the use of ATD as an M-theory is well established through a number of European projects aiming to compare and relate the variety of R-theories on mathematical education (Artigue & Bosch, 2014). This means that research is modeled as *research praxeologies* $[T/\tau/\theta/\Theta]$. Here T denotes a type of problems addressed in a research activity, τ the technique used, θ the technology (i.e., the discourse framing, explaining and justifying the techniques, based on what we have called the R-language), and Θ is a wider theoretical framework in which this discourse is defined (or at least makes sense), that is, the R-theory. We notice that the ATD model insists that R-practice $[T/\tau]$ and R-theory $[\theta/\Theta]$ (including scientific discourse) define each other mutually, as is the case for any praxeology: not only is theory and discourse created and adapted to explain and justify the challenges and methods used in research, but researchers' tasks and methods are themselves profoundly shaped by their scientific language and theories. This, in particular, situates the question of R-language(s) in a precise model of the research activity, at the level of technology: any R-technology is based on a R-language.

We can now formulate our *problématique* as precise research questions which one can undertake in order to design and assess a given research study with an emphasis on language sensitivity (particularly relevant, of course, for research on teaching in multilingual contexts!):

Q1. Are phenomena related to P-language(s) explicit in the types of problems addressed? If so, what techniques are used to tackle these phenomena? (even if P-language(s) are not explicit in the problems raised, we could ask the question about techniques, e.g., if the R-language and the P-language(s) are different).
Q2. At the level of technology, what means are used, if any, to justify the ways in which P-language phenomena are handled in the study?

Q3. Does the theoretical framework of the study provide means to situate and assess the study's assertions related to P-language, and how (if at all) are these means mobilized in the study?
Q4. Are institutional, societal or cultural specifics related to the P-language addressed by the technology? Could this be supported by the R-theory?
Q5. Does the technology consider the relation between P-language and R-language, especially as regards institutional, societal or cultural specifics? How?

We will address these questions for the research studies which were proposed and accepted for the ICMI Study conference, as well as in some further examples from our own research. But first, we illustrate the significance of our problématique by a case which spurred our initial interest in the questions above. We believe it demonstrates how the R-language issue, if ignored, may appear surprisingly (and inadequately) as results instead of questions.

14.3 Motivating Case: "Researching Mathematics Teacher Education"

The ICMI survey team on "researching mathematics teacher education," carried out by Adler, Ball, Krainer, Lin, and Novotna (2008) surveyed a certain corpus of research literature in order to provide a picture of "research that focuses on mathematics teacher education," and the team "saw as its responsibility to describe 'where we are' globally, in the field" (p. 127). The formulation of basic questions in the study (p. 128) does not explicitly mention P-language. However, the introductory remarks include the contention that "more and more learners should learn mathematics in English, a language that is not their main spoken language" (p. 124).

What is more surprising is the way in which R-language was handled:

> We selected from multiple outlets for this work, including peer reviewed journals, international handbooks and key conference proceedings. We looked across international journals as well as a handful of journals in Asia, Europe, i.e., published in languages other than English where it was possible to access these. In general, however, we did not have time and resources to investigate thoroughly journals written in e.g., French, German, Russian or Spanish. (p. 130)

So, what the team ultimately considered amounted to recent issues of three journals (ESM, JMTE, JMRE) and six congress proceedings (PME 1999–2003, ICME-9), where about 180 papers were considered relevant and "leading" relative to the theme.

As an outcome of scrutinizing these papers, the team then formulated four claims. One of them is on P-language: "Research in countries where English is the national language dominates the literature" (p. 135). This was demonstrated with a table showing, for instance, how papers from South and Central America make up only 3 % of the considered papers in PME, and are completely absent in JMTE. The authors admit that "these disparities are not surprising. The prevalence and increasing hegemony of English was referred to in the opening ceremony of the

Congress" (p. 135; the reference is to a Minister's opening address, where the importance of mastering English was underlined). As it appears, the "result" or "claim" is not in the least linked with the initial choice of considering only research produced in (select) English language journals. A lot of issues remain implicit here, like, apparently, the contention that "international" means "accessible to an audience who knows no other language than English." The possibility that a substantial literature on the theme could exist with other R-languages (in fact, it does!), no doubt with a higher share of contexts with other P-languages, has been ruled out from the beginning. It follows that almost all of our questions Q1–Q5 must be answered in the negative.

To be fair, we should mention the final "commentary" by Ball which shows some degree of alertness to this potential relation between P-language and R-language, at least when it comes to training new researchers in English-speaking countries: "It is important to develop a stance that avoids confusion between the local and the global. And so it is important to be able to work (read and speak) in more than one language" (p. 136).

14.4 Contributions to the ICMI 21 Study Conference: Overall Analysis

The proceedings of the study conference (Setati, Nkambule, & Goosen, 2011) contain a total of 54 papers, all written in English as required. In 52 of the papers, we can identify a "dominant" P-language in the context they study, in the sense that the papers study mathematical instruction, which occurs either solely or principally in this language. The dominant P-language turns out to be English in 34 of these papers, while a few others appear as well (Table 14.1; we notice that two of the 52 papers each treated two different contexts with different dominant P-languages, which is why the sum of the numbers in the table is 54, not 52). Relative to our questions, the ease with which the P-languages are identified implies that phenomena related to P-languages are indeed dealt with explicitly in virtually all papers, and most of them develop an explicit technology related to their treatment. The answers to Q1–Q3 are, therefore, positive for most of the contributions to this study.

The fact that 65 % of all papers described language diversity in contexts where English is the dominant P-language of instruction, is striking, because it certainly does not reflect an international situation. While it seems difficult to determine the percentage of the world's school children who are taught in English, less that 65 % of them are taught in English. In fact, only about 13 % of the world's population has English as their first or second language (according to Graddol, 1997, p. 10 the figure is about 750 million; here, having English as a second language is defined as the situation where English belongs "in a repertoire of languages where each is used in different contexts"). Adding to this a roughly similar number of people speaking English as a foreign language, we end up with roughly a quarter of the world popu-

Table 14.1 Dominant P-languages in ICMI Study 21 conference contributions and the corresponding average number of languages represented in references (as an estimate of R-language numbers)

Dominant P-language	English	Portuguese	Spanish	French	German	Catalan	Other[a]
# Papers	34	5	3	2	2	2	4
# R-languages	1.16	2.2	1.5	2	2	2	1.25

[a]Other dominant P-languages, each represented in the context of one paper, were: Swedish, Farsi, Chinese, and Russian

lation mastering English to some degree (and most certainly, much less than a quarter of the world's school children). While there are more first or second language speakers of Chinese and almost as many speakers of Hindi and Spanish (Graddol, 1997, p. 8), one might contend that English is of specific importance in multilingual settings, used as a language of instruction for children with another first language, particularly in some parts of Africa, North America, and Asia; indeed many of the papers address this situation. On the other hand, Spanish, French, and Portuguese have a similar role both in other parts of these same continents, and there is no doubt that these contexts are proportionally underrepresented in the contributions as listed in Table 14.1.

One might reasonably think that the choice of R-language for the study is linked to this bias. Naturally, in contexts where English is neither a first or second language, but where another language (like Spanish or Portuguese) is a dominant P-language, one is likely to find this other language also in higher education and, indeed, as an R-language. We think that indications of this could be seen in the references used, even in papers, which are, like the ICMI Study 21 conference papers, written in English. If R-language is, roughly, estimated according to the references cited by authors (as the basis for their presentation), it is interesting to note that 30 papers cite only English language references, while one paper cites only references in Spanish; the rest of the papers have references in more than one language (18 of these in two, and most of them with almost all references still being in English). Almost all (26) of the papers which have English as the only R-language (judging from references) deal at the same time, with contexts where English is the dominant P-language. On the other hand, English is the dominant P-language in only 5 of the 18 papers that have more than one R-language (in terms of referenced texts). In the last row of Table 14.1, the tendency also appears clearly: for papers dealing with contexts where English is the dominant P-language, it is also very often the only R-language, while in contexts where other languages are a dominant P-language, this other language typically appears in the references (except for the last case of "other languages") as evidence of its importance for the authors as an R-language.

It is, perhaps, surprising that while many papers discuss the institutional and cultural implications of a dominant P-language in contexts where this language is not the first language of students, virtually no papers specifically reflect on the significance of the R-language, particularly in the case where both are English (well over half of all contributions). This means that Q5 is hardly ever touched and that

Q4 may only be implicitly addressed (for instance, through discussions of more or less problematic translation issues). This is perhaps not surprising, given the dominance of English as an R-language in the countries of most of the participants. However, Q5 must be asked especially when the R-language is identical to the dominant P-language, to avoid the implicit naturalization of dominances of cultural and institutional norms in the latter by their dominance in the former.

At the same time, most of the ICMI Study, 21 conference contributions (43 of 54) study teaching contexts where two or more P-languages occur explicitly, typically because students use a second language in some way even when the dominant language is the only official language of instruction (as one might expect in a context of language diversity). We recall that in 34 papers, the dominant P-language was English, and that 30 of these 34 papers cite *only* literature in English.

When different P-languages occur, the use of English as an R-language presents some challenges, at least for exposing and analyzing exactly what children say, what their difficulties are etc.; however, this would be the case for any choice of R-language. More importantly, we contend that specific cultural and scientific perspectives tend to be imposed along with the R-language. For instance, a theoretical or methodological approach which is dominant in research contexts where English is the dominant (or only) R-language, tend to impose themselves with that language even where those perspectives are not otherwise dominant or even widely known.

To complement this picture, we now discuss, through close-up case studies, how these challenges appear in some of our personal work and experiences as researchers. We have found it most natural to use the first person in presenting these cases. We focus in particular on the more problematic questions Q4 and Q5.

14.5 Personal Case 1 (Faïza)

I am a Tunisian mathematics educator, with a Ph.D. in didactics of mathematics (joint degree between University of Tunis and University of Lyon). The roles of Arabic and French language in Tunisian mathematics teaching have changed over time but have remained relatively stable since 1988. The common daily language of Tunisians is a "dialectical" Arabic, while in school, only classical Arabic is taught. French is taught from third year in elementary school and on to the end of high school. For the first 6 years of elementary school, mathematics is also taught in Arabic. In the 3 years of lower secondary school, most oral parts of teaching continue to be in Arabic while the symbolic parts are written and read out in French (in particular, these parts are written from left to right, contrary to Arabic). This creates a certain divide in the mind of students. And from high school onwards, mathematics is only taught in French.

My research bears mainly on logical analysis of mathematical statements as they appear at different levels of teaching, and on the interaction between natural and symbolic (or more generally, "mathematical") language at the different stages mentioned above. In particular, for my thesis, I carried out an experiment with six pairs

of students in their first year of university studies on mathematics and informatics (Chellougui, 2009). I want to discuss this case further as an illustration of the problems linked to the P-language (French) at this level, and then the challenges of treating the "implicit" P-language (Arabic) when the R-language is also French.

My experiment had two main phases: solving exercises (topic: basic features of lower upper bounds of subsets of R) and an interview about the solutions. For each pair, the students worked together to solve the exercises and to respond to the interview questions. The language used for both the solving phase and interviews was French. Among the specific and general phenomena identified, and what is most interesting in the context of this chapter, was the instability of the choice of vocabulary and symbols in the work of the students. As an example, consider the following dialogue from an interview with two students, J and T:

Interviewer: OK, we'll try to understand the proof together. A small remark, if you want to show that a statement is true and you say you want to use indirect proof, what do you mean?
T: It means that it is the inverse.
J: No, on the contrary.
(Silence)
Interviewer: OK, if you want, we go on to the next question [*one of the exercises, to show that* $B = \{x \in \mathbb{Q}_+ : x^2 > 2\}$ *is the set of upper bounds in* $\mathbb{Q}$ *of* $A = \{x \in \mathbb{Q}_+ : x^2 > 2\}$.
J: A and B is the negation. A is the negation of B.
T: A is the contrary of B.
J: They are different.
Int: Can you determine the intersection of the two sets A and B?
T: The empty set, A is the opposite of B or B is the opposite of A.

In this short excerpt, the vocabulary of oppositions (see Durand-Guerrier & Ben Kilani, 2004) appeared in disturbing ways; there was an inappropriate use of expressions like negation, contrary, different, and opposite. The terms "contrary" and "negation" apply normally to statements, but the students also used them when dealing with sets. Evidently, this and other mathematical distinctions in the P-language presented particular difficulties when it was a second language. In other points in the interviews it was noted that some students had a tendency to privilege (read aloud) symbolic language in their interchanges. This was possibly a way to circumvent language difficulties and gain precision. On the other hand, for developing a more familiar relationship to logical structures, it could easily be problematic for the students to be relying almost entirely on symbolic logic and algebra as a tool of conceptual clarification and support of reasoning. Without the support of informal "familiar" expressions, students in fact seem to have failed to deepen their own understanding of the mathematical processes in which they were involved.

The more general situation for Tunisia is not likely to change. There are no clear global solutions in sight. The constraints of our educational language policies are not likely or easy to change. As well the mathematical culture and literature in Tunisian universities has for many years been entirely based on French. For the didactician, therefore, the situation must be studied and possibly amended more locally.

At the level of the R-language, to address difficulties of the type mentioned, we need technology and theoretical tools that take into account the implicit interaction between the "official P-language" and the Arabic language in which the students live outside of mathematics classroom (their implicit P-language in this context). While the use of formal language (logical expressions, symbolisms, etc.) is a challenge also for students whose mother tongue is French, the research theories and technologies developed in French do not envisage the presence of an "implicit" mother tongue. Hence the spontaneous and supposed easy transition to the more formal language for French students in France cannot be interpreted in the same way for Tunisian students. In fact the asymmetry between French and Arabic in Tunisian mathematics education seems to reinforce the gap, noted in other contexts, between students' informal and informal reasoning in mathematics.

14.6 Personal Case 2 (Hien)

I am a Vietnamese Ph.D. student, studying in Belgium (French-speaking community). Even though the French language is not the dominant language used for scientific research and everyday life in Vietnam, the "French mathematical didactic" way of thinking is well diffused throughout Vietnamese society and plays an important role in the mathematical education field. The consequence is that references in the mathematical education field are mainly in the French language and from a French-speaking community. As I began my studies in Vietnam and then graduated for a Master degree in France, I used the French-speaking references more often for my research work. I am now studying in a Belgian French-speaking University where English dominates as an R-language. Consequently, English references have become dominant in my work environment. These new references are not just studies presented in another language, but they brought me new theories and more importantly, a new way of thinking, a new way of approaching other studies, quite different from the French ones.

My present work examines the influence of language and other factors such as the mathematical program or the learning context on the mathematical performances and competences of children. It is a comparison study between the Vietnamese and French languages. The research idea comes from the particular characteristics of Vietnamese language as a P-language for verbal number denomination. The verbal numbers in Vietnamese have a perfect correspondence with their decimal form, like in Chinese, Korean, or Japanese (e.g., 13 is said "ten-three"). The Vietnamese language also has peculiarities when the digit in the tens or hundreds position is a zero. This is not found in other Asian languages cited above. For example, the Arabic number 3 024 is named "three thousand zero hundred two ten four." There is also an exception when the zero is in the tens place where it is replaced by a word meaning "remainder." For example, the Arabic number 309 is named "three hundred remainder nine" in Vietnamese. Here, we can understand the word "remainder" as the remainder of the division of 309 by 100.

Our[2] research undertakes a comparison of Vietnamese and French P-languages. For my contribution to the ICMI Study 21 conference, we had to present in English, which in effect became the R-language. Even though French and English languages are in many respects similar, there are crucial differences such as in the verbal number structure. Producing a study in a third language cannot be reduced to a simple text translation. Obviously, we had to pay attention to explain the meaning of key terms so they could represent the exact meaning of the P-language; for example, terms such as "remainder" and "zero hundred" in Vietnamese (the first P-language) had to be clearly referenced in English. We also had to explain the original context in which those key terms were used. The study reported on also focused on the cultural impact of the use of large numbers. Therefore, precision was needed to describe the special context in which the P-language was used, in order for the reader to fully understand problems and situations related to language and culture that appeared in this context.

In our contribution to the ICMI Study 21 conference, even if the P-languages of the research task was Vietnamese and French, the R-theory is mainly based on literature using English as the R-language. I struggled to "transcode" between the P-languages and the R-language to produce a contribution. In the case where the P-language and the R-language are different, this "transcoding" process contains more challenges because it also must to keep the characteristics specifically related to P-languages, in order to comprehensively report the results of research.

Previous studies undertaken in Europe (Censabelle, 2000), have showed that the syntactic zero (see Granà, Lochy, Girellid, Seron, & Semenza, 2003) is a source of difficulty for children when they transcode a verbal number into Arabic code. This difficulty can be explained with reference to the masked character of the syntactic zero in its verbal form. In our study, we showed that the syntactic zero is, in contrast to what was found in European P-languages, an advantage for Vietnamese children during the same transcoding task. This contrast in the ways students come to understand just this small aspect of mathematics is hard to explain in English. One could consider taking the option of using Vietnamese as an R-language. But if research is conducted with a R-language that is not dominant, it will not be diffused and discussed on a wide scale. So both options limit the opportunities in different ways to examine and understand better some questions in a more general way.

14.7 Personal Case 3 (Carl)

My mother tongue is Danish and in most of my research, the P-language is Danish while the R-language is English or French. For several years, I have been somewhat split between the last two. Most of my colleagues and students do not read French and so I put a lot of effort into writing papers in English and finding good references

[2] "Our" is used here and in the following paragraphs to reference the research group I work in, not the authors of this chapter.

in English, even if the papers and many of these references are strongly linked to a research literature primarily written in French, in particular within the paradigms founded by scholars such as Artigue, Brousseau, and Chevallard. This is also evident in my contribution to the ICMI Study 21 conference. My experience during the conference in fact contributed to confirming a main point in my conference paper: that the use of an R-language is much more than the use of a medium, as it entails also a number of cultural and scientific biases, or at least implications. This impression was shared by a number of participants who, like me, regularly or mostly use other R-languages than English. Among the main biases are theoretical perspectives, and of course these will be found also in an international context where the shared R-language is, for instance French, German, or Spanish. In an international study on multilingual classrooms it is certainly natural to reflect on how these biases affect our work and what we can do to address them explicitly.

I now turn to a concrete example of how a delicate mixture of R-languages and P-languages can be handled explicitly (to take care of the questions Q4 and Q5 in an explicit, albeit not symmetric way). The case bears on certain practices related to Japanese mathematics teaching and hence the P-language is Japanese. I am lucky to understand this language reasonably well as I did my Ph.D. in Tokyo. But of course, this adds to the language complexity, even if I have never been able to familiarize myself thoroughly with—and much less draw on—the extensive research paradigms in mathematics education which are based on Japanese as a R-language.

In 2001, I met a Japanese colleague, T. Miyakawa, at the summer school of the French Association for Research in Didactics of Mathematics. Miyakawa was at the time writing his doctoral dissertation in Grenoble, and our common R-language is and remains French (even if we can communicate in English and Japanese, for other purposes). However, working with Japanese colleagues clearly entailed some influence and inspiration also from these sources, as will be illustrated by my case.

At the summer school in 2007, we were invited to present a Japanese format for teacher collaborative lesson planning, known in English-speaking countries as "Lesson Study." This led us to examine the parallels and differences of this format, and the "open-approach theory" by Nohda (1991), with the French research tradition in didactics, strongly linked to the theory of situations (Brousseau, 1997) and more specifically, a case of lesson study on proportions and the famous "puzzle situation" by Brousseau (1997, Chap. 4). Our exposition (Miyakawa & Winsløw, 2009) of this comparison in English was based on material in French and Japanese, and of course we had to explain very carefully how we had handled the corresponding methodic and methodological challenges related to the two P-languages. Indeed, we had to examine explicitly and critically the meanings of basic terms such as "research" and "question," as it could be rendered slightly differently, but potentially importantly, in the three R-languages involved, as well as the central terms from the P-languages. It appeared to be a considerable advantage to share our comparison of the two settings of "lesson research" in a third R-language, since no term, theoretical assumption, or indeed, cultural and societal specific, could be taken for granted (in the sense of being naturalized among readers). While my intention is not

to claim that we achieved any kind of perfect neutral and transparent perspective, I do think the effort of taking the two contexts, and in particular the two R-languages, into a third one, was both rare and, as it transpired, rewarding for us as researchers as well as for our audience.

14.8 Conclusions

The main point of the ATD model of research praxeologies is to insist on the mutual, or co-constructive, relation between *practice blocks* (tasks, techniques) and *theory blocks* (technology, theory). Research tasks, or more generally problématiques, do not exist independently from technology and theory. Both are imbued with language: our research practice deals with contexts with particular P-languages, and our learned technologies and theories are formed in and by R-languages. Researchers focusing on the effects of language diversity in the mathematical praxeology of teachers and students should also be sensitive to the effects of their own choices and constraints when it comes to R-language. The five questions proposed in our framework can be used both to evaluate how a given study displays this kind of reflective sensitivity, and to shape future research in the area.

We have outlined some cases to demonstrate the multiple ways in which these questions may appear and be tackled, or in fact be ignored when we take our own technological and theoretical equipment to be somehow a natural standard. It is inevitable that we will always ignore *some* potential approach to research questions, simply because we do not know them. But in the context of studies of multilingual or cross-cultural contexts, and indeed in studies that claim to be "international," it is certainly not defensible to ignore the role of R-languages.

In fact, the absence of explicit reflection on the impact of an R-language seems to be particularly common, and unreasonable, in the case where the R-language coincides with a "dominant" P-language, as in the vast majority of studies surveyed in Table 14.1, and in Personal Case 1 of this chapter. The notion of asymmetry, introduced in that case, can be made more precise by pointing out the usual meaning of an R-language and more precisely an R-theory. If the R-task is to study some specific challenges raised by the multilingual contexts (even if only one dominant language is "allowed" or "practicable") then the R-theory can contribute to naturalize or even enforce the ignorance with respect to crucial challenges and opportunities as regards the use of P-language.

It is also evident that the study of phenomena linked to P-languages different from the R-language may cause problems in terms of communicating and transitioning between these languages, precisely because of these phenomena (see Personal Cases 2 and 3). What is perhaps less evident is that such difficulties may cause "international" studies to become somewhat regional, as illustrated by our motivating case and in fact to some extent the present study (see Table 14.1). This is because no language is a P-language to more than a small fraction of the world's population.

A researcher must master the P-language of the students and teachers in the context she investigates. A more subtle challenge, which is often hidden by formal restrictions to the use of just one R-language, is that no language is a functional R-language to more than a fraction of the world's mathematics educators. Any monolingual research community will therefore be faced with severe constraints in an attempt to address the role of P-languages in mathematics education at any level other than in her own language. This is especially so at an international level. Such limitations should be explicitly acknowledged in our research endeavors.

The role played by natural languages in this unjustified naturalization of scientific and cultural (including mathematical) perspectives is only the tip of an iceberg which is loaded with societal and political implications; but it is an important "tip" because it can be exhibited and analyzed in fairly obvious and objective forms when we are explicit about the *problématique* introduced and exemplified in this chapter.

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