Chapter 1
Introduction: An ICMI Study on Language Diversity in Mathematics Education

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1.1 History of the Study

ICMI's concern about the development of research on mathematics education and language diversity dates back to 1972, and the second International Congress on Mathematical Education (ICME-2) in Exeter, United Kingdom. As explained by Howson (1973) in the introduction of the congress proceedings, the inclusion of a working group on language was an important novelty. Moreover, during the ICMI general assembly at ICME-2 a decision was made to host an international symposium on 'Interactions between Linguistics and Mathematical Education' as a response to the need for fundamental research on the relationship between the learning of basic mathematical structures and the language through which they are learnt. The Symposium was held in 1974 in Nairobi, Kenya, with sponsorship by UNESCO in cooperation with ICMI and the Centre for Educational Development Overseas. There had not been international conferences focusing exclusively on the relationship between mathematics and language before. This early event actually focused

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on the relationship between *linguistics* and mathematics education, which is different from a focus on *language* and mathematics education. Linguistics and mathematics education are two disciplines while the role, use and effect of language on mathematics education implies all happening in one discipline.

In the final report of the symposium (UNESCO, 1974), the lack of research on the relationship between language and mathematics was highlighted. The report concluded that 'difficulties in mathematics learning depend on the language of learning' (p. 8). It was further affirmed that all languages include linguistic features of benefit for the acquisition of mathematical concepts and thus can be used for mathematics teaching and learning. It was claimed that the problems of learning mathematics in an additional or foreign language are not peculiar to learning in a world language such as English or French because there are many countries such as Tanzania and India, where many learners have to learn mathematics in a national language (e.g. Kiswahili, Hindi) which is not their home language. This situation still continues. Meanwhile, European countries like Sweden or Greece experience the pressure to ensure that their learners are fluent in at least one of the world languages.

Both ICME-2 and the Nairobi Symposium gave the impetus for the first paper to appear in a mathematics education journal focusing on mathematics and language diversity; it was authored by Austin and Howson (1979) and published in *Educational Studies of Mathematics*. Austin and Howson conclude that the challenge of language and mathematics learning and teaching is not just an issue for developing countries but for the whole world. In most developing countries, the challenge is that of learners learning mathematics in a language that is not their mother tongue; in developed countries such as Belgium and Canada there are communities with well-established 'minority' languages; and in some countries learners and teachers have to face the non-standard nature of the local vernacular (e.g. Jamaica, England

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and the United States). Austin and Howson acknowledged the fact that bilingualism is a political matter and thus change in society may lead to policy change. Indeed much has changed since 1979: the world has become more multilingual and some countries have changed their language policies and practices, which makes this ICMI Study timely and relevant.

1.1.1 What Do We Mean by Language Diversity and Why Does It Matter for Mathematics Education?

A first challenge for the International Program Committee (IPC) for this study was to decide on the title of the study. We elected to use the expression 'language diversity' rather than specify particular situations (e.g. multilingual, bilingual, etc.). In making our choice, we drew on ideas from recent work in sociolinguistics (e.g. Pennycook & Makoni, 2005; Makoni & Pennycook, 2007). This work challenges three sets of ideas about language in contemporary society in the context of increasing diversity (Blommaert & Rampton, 2011).

First, *languages* should not be understood as monolithic, static, rule-governed forms of communication. While the idea of languages is socially useful, in research terms it is more difficult to sustain in this form. In practice, for example, it is difficult to objectively identify boundaries between many languages. Indeed, the ideology of distinct languages arguably has its roots in nineteenth century Europe where a rationality of classification and creation of distinctions was not only part of the development of social sciences such as linguistics, but also underpinned the ideological formation of nation states. This view of discrete languages belonging to separate peoples (German for the Germans, French for the French) was subsequently exported to much of the rest of the world through European colonisation. In parts of Africa, for example, colonisation resulted in a language spectrum being carved up into discrete languages, which then defined the people who spoke them. Where before there had existed a language continuum, with colonisation there were distinct, named languages (Makoni & Pennycook, 2007). This ideology of languages informs how classrooms are understood today. For example, students who do not speak according to a pre-conceived standard form of a language may be seen as less educated.

Second, in the same way that languages are not monolithic, so *language speakers* are not monolithic either. This point leads to a critique of notions like 'native speaker', 'fully bilingual speaker' or designations like 'first language', 'second language', etc. From this perspective, terms like 'English as a second language' or 'English as an additional language' are open to question, since they imply a neat sequence and a separation between 'English' and the 'second language'. In many settings, however, learners of English as a second language may in fact draw on several languages for different aspects of their lives and combine aspects of these languages for different purposes. It then becomes difficult to identify what is

'English', what is 'second language' and what is 'other languages'. Similarly, a 'bilingual' student may also speak other languages, or different varieties of the official languages.

These terms are problematic politically because they privilege particular idealised or preferred forms of language use and hence privilege learners who conform to these preferences. For example, to describe a student as a learner of English as a second language in a context in which English is the official language of schooling is to erase their first language (or possibly first language*s*) and to privilege English. Similarly, to describe a classroom as bilingual can privilege two languages and a particular ideal of the bilingual speaker. The more one looks carefully at almost any mathematics classroom, the more difficult it becomes to adequately characterise language ecology in a few simple words. Each participant in a mathematics classroom, including the teacher, brings their own particular combination of languages, varieties and ways of talking.

Finally, the nature of *communication* itself has been increasingly rethought. In the context of contemporary super-diversity (Vertovec, 2007), it is apparent that speakers draw on multiple semiotic means in order to communicate. These semiotic means include parts of recognisable languages (e.g. some Hindi combined with some English). But they also draw on additional semiotic means, such as accent, style, register or genre, as well as different varieties within any particular language. In mathematics, for example, these means include gesture, diagrams, and symbols.

Rather than specify particular forms of classroom or setting, then, we elected to frame this study in terms of language diversity. The final title for the study, 'Mathematics Education and Language Diversity', ensured that the study was broad enough to encompass any conceivable language setting. There is always diversity of some kind. Moreover, the form of language diversity may vary from one classroom to another. This variety starts with the different languages and other resources on which students may draw. These resources may not be common to all students. It also includes the different languages and resources on which the teacher may draw. These languages and resources may not be the same as those used by the teacher's students. There is also variety in the different policy environments in which mathematics education takes place, for example, in terms of which languages are recognised, mandated, required or ignored. These policies themselves reflect the long-standing assumptions about the nature of language and of speakers that are questioned in this section. In this volume, we have succeeded in ensuring that a wide variety of forms of language diversity are represented.

These ideas matter for mathematics education for the simple reason that mathematics education is always happening in the context of language diversity. In our superdiverse societies, shaped by migration, mobile technologies, social media and global trade, learners and teachers of mathematics operate in a complex linguistic landscape. This landscape forms the backdrop to their participation in mathematics. This book is testament to this reality. In every specific example of mathematics classroom interaction in this book, participants are drawing on multiple resources of some kind, including multiple languages, gestures, registers, or genres.

1.1.2 Outline of the Chapter

The rest of this chapter begins with a brief discussion of the background and scope of the study. We then outline the process followed, including the discussions of the appointed IPC, the preparations of the discussion document and the organisation of the study conference, which not only aimed to deliberate on what relevant research had already been undertaken but envisaged what was needed to be done to push the boundaries further. The chapter concludes with some reflections on crucial themes within this volume for both the wider communities of curriculum developers, teachers, politicians and members of non-dominant communities, as well as pointers to some underlying issues that should inform continuing research in the nexus of language and mathematics education.

1.2 Changing Perspectives on Mathematics Education and Language Diversity

The beginnings of research on language and learning started with a focus on bilingualism and bilingual learners. This location of the problem in the learner was based on an underlying assumption that there is something wrong with the bilingual learner. A great majority of studies undertaken before 1979, when the paper by Austin and Howson was published, concluded that capacity in more than one language had negative effects on learners' linguistic, cognitive and educational development (Reynold, 1928 and Saer, 1923, both cited in Grosjean, 1982). Bilingualism was seen as unnatural and it was argued that a bilingual child hardly learns either of the two languages as perfectly as would be the case if only one language were learnt. There was also a widespread view that the effort required by the brain to master two languages instead of one diminished the child's power of learning other content. Weisgerber (1933, cited in Saunders, 1988) argued that bilingualism could impair the intelligence of a whole ethnic group, while Reynold (1928, cited in Saunders, 1988) was concerned about the fact that bilingualism could lead to language mixing and language confusion which in turn resulted in a reduction of the ability to think. From his study of Welsh–English bilingual children in rural areas, Saer (1923, cited in Saunders, 1988) concluded that bilingual learners had lower IQ scores than monolingual children. Commenting on this result, Saunders (1988) warned that caution must be exercised when comparing monolinguals and bilinguals on tests of intelligence, particularly on tests of verbal intelligence, and especially if, as so often happens, the bilinguals are tested in only one of their languages, often their second language.

Another stream of research, emerging in the 1960s, began to argue for a more nuanced understanding of this literature. These authors argued that under certain conditions competence in more than one language can have positive effects on the learning process (Ianco-Worrall, 1973; Ben-Zeef, 1977; Bialystok, 1987; Pearl &

Lambert, 1962, all cited in De Klerk, 1995). In 1962, Peal and Lambert conducted a seminal study that suggested that bilingualism could be an asset to the learning of the child. They studied the effects of bilingualism on the intellectual functioning of 10-year-old children from six Montreal schools. They found that instead of suffering from 'mental confusion', bilinguals were benefiting from a language asset that monolingual children did not have. While these results were criticised on the grounds that only the intellectually brighter children were chosen for the bilingual group (e.g. by Macnamara, 1966, cited in Saunders, 1988), the empirical studies that followed reinforced the idea that bilingualism can indeed be an asset (e.g. Ianco-Worrall, 1973; Been-Zeef, 1977, both cited in De Klerk, 1995). Knowing what we know now, it is strange that these arguments based on a deficit-model were accepted as plausible for so many years, and despite the fact that asset-based arguments were being documented. Perhaps this shows the power of simple, even naive, ways of understanding how the brain works and the implications this has for learning.

In 1979, Swain and Cummins compared the deficit-based arguments and the asset-based arguments in the context of the different studies and concluded that the asset-based findings were usually associated with majority language groups in immersion programmes. In other words, for these children the second language is added at no cost to the first. However they also noted that the parents of the children in these immersion programs were of relatively high socio-economic status and the parents placed a high value on knowing two languages. Deficit-based findings, on the other hand, were found with immersion students who are surrounded by negative attitudes. They were forced to learn the majority language and were not encouraged to retain their first language. Further, they did not live in a social environment that was conducive to learning. Nevertheless, Swain and Cummins (1979) argued that while there were a variety of factors impacting children's intellectual development, bilingualism was one of the significant factors that could have a positive impact, and was not automatically negative for children's learning. Cummins's (1979, 1981) theory of the relationship between language and cognition elaborates on the conceptualisation of the learner as someone who brings all the acquired strengths from the first language to use them in the process of learning a second language. In this regard, the learner's first language leads to the acquisition of fluency in the second language so that bilingualism becomes a 'cognitive advantage' rather than a factor that impedes learning. While research in this area of study at this stage did not foreground the role of the social dimension of language (although Cummins did anticipate this in some of his earlier work), it is clear that there was an acceptance that it is possible that bilingualism per se might not necessarily have any effects (either negative or positive) on the cognitive and intellectual development of children. That is, what may account for the contradictory results reported in the literature during this period are the psychosocial differences between bilinguals and monolinguals, and not bilingualism per se.

While the research described above did not specifically focus on mathematics education, its findings influenced the research that followed in mathematics education. Interestingly, the studies of language and mathematics education reviewed by

Austin and Howson (1979) were largely dominated by a deficit perspective of the learner. In the bibliography that they presented at the end of their chapter, it can be observed that the majority of the works focused either on linguistics or on issues relating to the linguistic aspects of mathematics, rather than issues relating to the challenges of teaching and learning mathematics in the context of language diversity. Nowadays the deficit perspective is being more and more overcome, particularly due to the impact of results from several studies in different research contexts all around the world. It cannot be said, however, that this perspective has been totally overcome, either in research or in educational policy, where compensatory responses and remedial approaches are common in the interpretation of the needs of the learners whose home languages are different from the language of instruction.

Using Cummins's work as a basis, in the early 1980s, Clarkson working in Papua New Guinea and, separately, Dawe working with immigrant children in England showed that Cummins's threshold language theory (Cummins, 1979) had good explanatory power for mathematics performance (Clarkson, 1991, 1992; Dawe, 1983). Their results suggested that bilingual students who were competent in their home language and the language of teaching outperformed other students in mathematics, even when other factors such as socioeconomic status and parental education were accounted for. Later, working together, similar results concerning performance on mathematics were obtained for bilingual immigrant students in Australia (Clarkson, 1996; Clarkson & Dawe, 1997). These studies, although suffering from some of the pitfalls outlined above, showed that the notion of bilingualism had to be far more nuanced than had been the case in earlier research, and global applications of deficit models were just not appropriate. The impact of bilingualism on children's learning of mathematics was neither simple nor unitary. They also showed some of the ranges of individual and social factors impacting children's learning of mathematics, including language, and showed that many of these factors, such as the socio-economic status of families, could not be influenced by schooling. But these studies also suggested that the way children used their languages was a learnt behaviour and hence could perhaps be influenced by schooling. Hence, the ways teachers and schools dealt with students' multiple languages in relation to mathematics learning was for the most part an unrecognised but critical issue in both research and practice.

More than a decade after Austin and Howson's paper, Secada (1992) provided an extensive overview of research on bilingual education and mathematics achievement (with a North-American-orientation), and pointed to findings of a significant relationship between the development of language and achievement in mathematics. This publication is not a mere update of the former review by Austin and Howson. It recognises that the social dimension of language needs to be integrated in theory and research, in order to understand and explain differences in the mathematics achievement of bilingual learners from different racial, ethnic and social class contexts. In particular, he concluded that the studies he reviewed indicated that oral proficiency in English in the absence of mother tongue instruction is negatively related to achievement in mathematics. Nevertheless, he found that correlations between language proficiency and mathematics achievement were highly variable

and stated that there was much variance to be explained. He pointed to this finding from his review along with another important consistent finding: that middle and upper social class learners entered school with higher achievement levels in mathematics than lower social class learners. Social class was thus highlighted as a relevant variable in his analysis of the learning of mathematics by bilingual learners.

While questions relating to the relationship between language and mathematics had been the focus of study for over 30 years, it was only in the mid-1990s that the focus shifted to multilingual mathematics classrooms. This was mainly due to Adler's work, which was carried out in multilingual South Africa, exploring mathematics teaching and learning in classrooms where the teacher and learners did not share a home language or one in which there was an imposed language of learning and teaching which may not have been the home language for either the teacher or learners (Adler, 1995, 1997, 1998). Through her exploration, Adler (2001) captured three interrelated dilemmas that lie at the heart of teaching mathematics in multilingual classrooms: code-switching, mediation, and transparency. She provided a sharp analysis and strong theoretical grounding, pulling together research related to the relationship between language and mathematics, communicating mathematics, and mathematics in multilingual settings and offered a direct challenge to dominant research on communication in mathematics classrooms in which the normalised setting was, and sadly still is in many studies, taken to be the monolingual mathematics classroom. The shift from a focus on bilingualism to multilingualism occurred during the same period as the shift from a conceptualisation of language as a problem in mathematics teaching and learning, to language as a resource (Adler, 1995, 1997, 2001). It was through this ground-breaking work that the multilingual mathematics classroom has become the 'new normal' in research on mathematics and language diversity published after 2000 (see, for example, Setati, 2005; Moschkovich, 2008; Barwell, 2009; Clarkson, 2009).

1.3 Establishing the Scope of the Study: Preparing the Discussion Document

ICMI Study 21 was announced in July 2008 during the 11th International Congress on Mathematical Education (ICME-11) in Monterrey, Mexico, with Mamokgethi Setati Phakeng (South Africa) and Mario do Carmo Domite (Brazil) as co-chairs, and an IPC of nine further academics from around the world (Richard Barwell, Philip Clarkson, Anjum Halai, Mercy Kazima, Sinfree Makoni, Judit Moschkovich, Nuria Planas, Paola Valero and Martha Villavicencio). The preparation of the discussion document started immediately thereafter with Mamokgethi Setati Phakeng preparing the first draft, initiating and leading the interaction between members of the IPC via email and incorporating their ideas into the draft document. This early discussion, mostly via email, was robust and engaging. Key strands in the discussion concerned: how to describe and conceptualise multilingualism and language diversity; the possible effects of our choices about how to describe and

conceptualise multilingualism and language diversity; and the implications of these conceptualisations for the study and for mathematics education.

The debates were not surprising given our diverse theoretical backgrounds. After 5 months of interaction on the draft discussion document, the IPC held a successful meeting on 17–21 February 2009 in Pretoria, South Africa, organised and hosted by Co-Chair Mamokgethi Setati Phakeng. She also organised travel funding for four of the IPC members, as well as accommodation for all members of the IPC. The programme for the meeting included inputs by Bernard Hodgson, ICMI Secretary General, Jill Adler, ICMI Vice President, as well as sessions for IPC members to work on aspects of the discussion document in groups. IPC members also visited schools in the west of Johannesburg to have some shared experience of mathematics education and language diversity in South Africa.

At the end of the meeting it was agreed that a small team comprising of Mamokgethi Setati Phakeng, Maria do Carmo Domite, Judit Moschkovich, Nuria Planas and Richard Barwell should work on finalising the discussion document. Thereafter the final draft of the discussion document would be distributed to all the IPC members for final comment. The completed discussion document was released in September 2009 after a year of rigorous interaction between IPC members and became the basis for the study conference and the development of this volume.

The discussion document (this volume, pp. 297–308) set out five themes, which served to organise preparations for the study conference:

1. *Teaching mathematics in diverse language contexts*. This theme focused on language issues in the teaching of mathematics in different language contexts. The assumption here is that language issues that emerge in different contexts are not only shaped by the complexities of the language of mathematics, but are also shaped by the linguistic contexts in which mathematics is taught and learned.
2. *Teacher education for diverse language contexts*. The focus here was on issues in and for teacher education in diverse language contexts. An assumption is that teacher education principles and practices are rooted in the real world of the classrooms, and therefore must take into account the different language contexts in which mathematics is taught.
3. *Researching mathematics teaching and learning in multilingual contexts*. This theme focused on the theories and methods for doing research in multilingual contexts and includes theoretical, ethical, methodological and philosophical issues.
4. *Mathematics, language diversity and society*. Diverse language settings reflect broader social, cultural and political issues. When considering classroom dynamics, an assumption is that they are shaped by and go on to shape the broader social and political settings. Language diversity issues intersect with multicultural, policy, and wider social issues.
5. *Student mathematics learning and experiences in multilingual classrooms*. This theme focused on students' learning and students' experiences of learning mathematics in different language contexts. An underlying assumption is that it is important to focus on learners in order to support them to learn mathematics

effectively. This theme includes work on classroom discussion, in different mathematical domains, and in different age groups.

Some issues were not explicitly mentioned in these themes. For example, code-switching is one of the most widespread topics of research in this area but is not referred to directly in the themes. The IPC felt that given the extent to which this issue had been written about in the mathematics education literature, contributors would be well aware of its importance and implications for our work and would address it in the context of several of the themes. This assumption proved to be the case, as can be seen from several of the chapters in this volume.

Similarly, some crucial sociological factors such as gender and poverty are not explicitly referred to. The IPC were aware that such factors have a profound impact on the way teachers teach and how students learn mathematics whether in a multilingual context or not. The IPC also understood that these factors are intertwined with issues of language learning and language use and with how language and learning impact mathematics learning in school settings. Again, the IPC was confident that contributors would foreground these issues in relation to all of the themes.

On the other hand, the IPC wanted to foreground ideas from applied linguistics. Research on language diversity in mathematics education must be informed by theory, methods and research in relevant disciplines such as applied linguistics and linguistic anthropology. There are many empirical findings and theoretical ideas in these disciplines that should be broadly disseminated within the mathematics education research and practitioner communities. To this end, Sinfree Makoni, an internationally known applied linguist was included in the IPC. In addition, two applied linguists were invited to address the study conference (see below).

1.4 The Study Conference

The study conference was held 16–20 September 2011 in Águas de Lindóia, São Paulo State, Brazil with Co-Chair Maria Do Carmo Domite as the lead organiser. The conference was planned as a working event and had two main goals: to enable discussion of the latest research of relevance to the study theme; and to generate writing teams and proposals for the study volume. To meet these goals, and following a call for papers, a total of 54 papers, authored or co-authored by 91 individuals from 27 countries, were accepted for presentation at the conference. The papers covered all of the five themes of the study as follows:

- Theme 1: Teaching mathematics in diverse language contexts: 18 papers.
- Theme 2: Teacher education for diverse language contexts: 14 papers.
- Theme 3: Researching mathematics teaching and learning in multilingual contexts: 8 papers.
- Theme 4: Mathematics, language diversity and society: 4 papers.
- Theme 5: Student mathematics learning and experiences in multilingual classrooms: 10 papers.

Table 1.1 Number of papers given per country of authors

Country	No. of papers	Country	No. of papers
Australia	2	Mexico	1
Belgium	1	Mozambique	1
Brazil	5	New Zealand	1
Cameroon	1	Pacific Region	1
Canada	5	Pakistan	3
China	1	Papua New Guinea	1
Denmark	1	Peru	1
Germany	3	South Africa	11
India	1	Spain	2
Iran	1	Swaziland	1
Kenya	1	Sweden	1
Malawi	2	Russia	1
Malaysia	1	USA	6
		Vietnam	1

Table 1.1 shows the number of papers per country. These themes were a starting point for the Conference.

That South Africa had the highest number of papers is not surprising given the nature of work focusing on teaching and learning mathematics in multilingual classrooms conducted in South Africa. Of interest are the countries that were not represented at the study conference despite how multilingual the world (and thus mathematics classrooms) has become in the last 20 years. The papers focused on different levels of schooling (primary, secondary and tertiary). A large majority of papers used a qualitative case study methodology.

The conference proceedings (Setati, Nkambule, & Goosen, 2011) containing all the accepted papers were published well ahead of the conference and delegates were asked to read papers of interest beforehand, since conference time would be devoted to discussing the papers and presentations would only be summaries of what had been written. Given the number of papers received, the program was organised into three groups focusing on the following themes:

GROUP 1: Theme 3 and 5: Focus on students' mathematics learning and experiences in multilingual classroom and on researching mathematics teaching and learning in multilingual contexts.
GROUP 2: Theme 1: Focus on teaching.
GROUP 3: Theme 2 and 4: Focus on teacher education and on mathematics, language diversity and society.

During the conference delegates were asked to join one of the three groups and to continue in that group for all sessions until the end of the conference. The activities of the groups included short presentations and follow-up discussions of

important developments in research and/or practice related to the theme under consideration. Each group was co-ordinated by a team of four theme leaders drawn from the IPC.[1] Two members were appointed co-chairs for each session with the other two acting as rapporteurs for the session, with these roles changing for each session. The purpose of the groups was to provide an opportunity for presentations and discussions in order to share key findings and ideas, moving to a point when collaborations between group members could form, and possibilities for book chapters of the study volume could be discussed. This outcome was discussed at the first meeting of the groups and time was deliberately set aside each day for these extra discussions.

Two linguists (Marilyn Martin-Jones, UK, and Marco Barone, Brazil) were invited to present plenary lectures and workshops. The plenary lecture by Marilyn Martin-Jones was entitled 'New times, new dimensions of linguistic diversity: Rethinking research and practice' and the plenary lecture by Marco Barone was entitled, 'An overview of native languages in Brazil, the scientific importance of preserving linguistic diversity and how mathematics can contribute in this regard'. Marilyn Martin-Jones presented a workshop with the same title as her plenary lecture, while Marco Barone's workshop was entitled 'Mathematical methods for linguistics: How to build an Atlas'. Both speakers achieved the fine art in their presentations and workshops of drawing issues from the participants' papers and reflecting back potentially important nuances in their thinking.

At the beginning of each day, two members of the IPC working with two different thematic groups were tasked with presenting a short 'provocation' to the conference. They were asked to reflect on the preceding day's activities and raise ideas but mainly questions: in particular, questions that they found had challenged their own thinking. In this way, there was a running commentary of what was happening across the conference and an air of deep questioning and inquiry was foregrounded for all participants.

1.5 Preparation of This Volume

Although the conference and the published conference proceedings were important in their own right, the IPC had always regarded the conference as a step towards the development of the study volume. The development of chapters for the study volume had an organic and self-organising nature so that contributors from different regions who attended the conference coalesced around common topics of interest. By the second day of the conference, with the encouragement of the IPC members leading the groups, a number of delegates coalesced into writing teams. The IPC encouraged chapter proposals to include multiple authors and geographic settings. By the end of the conference most of these teams had decided they were prepared to

[1] All members of the IPC attended the conference, with the exception of Sinfree Makoni, who was not able to participate and who took no further part in the work of the IPC.

continue their thinking together and had developed written proposals for chapters for the study volume. The IPC established some criteria for reviewing the proposals:

- Quality of content and clarity of proposal
- Relevance to the study themes and questions
- Not longer than 8,000 words
- At least two authors from different settings or countries
- Representation of different mathematical/linguistic settings

In reviewing the proposals received, the IPC also sought to ensure that the study volume would reflect diversity of geography, mathematics, educational level, languages and theoretical perspectives.

Most of the 91 delegates were included in at least one chapter writing team. As had been planned, the teams of authors were composed of colleagues from different countries thus bringing to their writing a confluence of different multilingual contexts, and in many cases different traditions of educational research and very different experiences of educational praxis. Of the submitted proposals, 19 were accepted by the IPC, with one subsequently withdrawn. Of course, not every proposal developed into a completed chapter.

The 18 proposals were organised into five subgroups, to be guided and edited by one or two members of the IPC. The editors worked with their authors to prepare their manuscripts and organised blind peer reviews. Each chapter was reviewed by at least one other member of the IPC, as well as by an expert external to the IPC.

During September 2013, a meeting of most of the IPC[2] was hosted in Lima by Martha Villavicencio, sponsored by the Ministry of Education of Peru. Other members of the IPC participated by email from time-to-time during this 2-day meeting. During this time, each chapter was reviewed again by at least two IPC members who had not previously seen the chapter. After this review, a number of chapters were accepted as ready for publication, while others were accepted for inclusion in the study volume subject to additional revisions or, in one or two cases, acceptance after completion of another round of peer review. Hence each chapter that has been included in this study volume has undergone a number of reviews.

Additional meetings of multiple IPC members were also held at the ICME-12 conference in Seoul, South Korea, in July 2012, and at the PME conference in Vancouver, Canada, in July 2014. In 2012, Mamokgethi Setati Phakeng was not able to continue in her role of co-chair, although she continued to participate as a member of the IPC. At the meeting in Seoul, Richard Barwell agreed to co-ordinate the final stages of the preparation of the volume, including the preparation of the book proposal, collection and preparation of the chapter manuscripts, and preparation and final editing of the completed manuscript. The production of the volume has been in a very real sense the product of a collective process involving the nine members of the IPC whose names are listed as editors.

[2] After the study conference, Maria do Carmo Domite withdrew from the IPC.

1.6 Summary of Research Ideas in This Volume

The chapters in the volume as a whole present a variety of results from existing research and, at the same time, pose a number of challenges and questions for the future of research in this area, as well as having implications for school practices in multilingual settings (these latter two points are dealt with more extensively in the following sections). Rather than summarising all chapters, we present a synthesis of some key issues that are addressed across different chapters, and which constitute important points of insight in current research.

What is mathematics? Authors in this volume recurrently come back to the ongoing discussion in mathematics education about how mathematics is conceived and thought of in educational settings. Beyond the philosophical discussions about the nature of mathematics as a discipline or as a field of practice for professional mathematicians, a debate that rightly continues among our mathematics colleagues, the issue of what is 'mathematics' when configured in educational practices is also far from resolved. In this volume, there is clearly not one unified answer to this question. More importantly, however, the chapters point to the fact that commonly held assumptions about the nature of mathematics in the realm of educational practice are challenged when the language diversity of learners and teachers alike result in tensions in generating unified meanings. The mediation of natural languages in forming what is mathematics and mathematical in a particular educational situation cannot be ignored. As a consequence, there emerges the recognition of the multiplicity of mathematics that may be present simultaneously in one multilingual, often also multicultural, educational setting. Mathematics then becomes the sum of all the varied 'mathematics', as well as the possible relations and points of divergence among them.

How are mathematical and natural languages related? Some chapters implicitly or explicitly discuss the notion that in educational practice, as well as in existing mathematics education research, there are strong assumptions about the universality of mathematical language. Such universality is implicitly assumed in the apparent transparency of meaning of numerical and symbolic language in mathematics registers. The dominance of such assumptions overshadows the fact that the natural language(s) of teachers and learners alike are fundamental in teaching and learning, not only as mediators but also as constitutive elements of culturally bounded forms of thinking. In other words, there still seems to be an understanding that the more 'mathematical language' (of symbolic type) learners can acquire, the less learners and teachers are dependent on and hence can rely less on the resources of their natural languages. This assumption runs through the different levels of education, and takes different forms at each level. Such an assumption would mean that the problems of learners' natural language diversity are pressing and evident as learners are young and in the primary levels of schooling. Authors in this volume clearly agree with this. But as learners grow and develop more symbolic mathematical language, the diversity of possibilities of meaning created by the diversity of natural languages may be thought to decrease. From this perspective, the problems of multilingualism

in mathematics education would not seem to be so relevant or pressing at the higher levels of mathematics education. Although some authors seem to agree with this, nevertheless others challenge the second part of this assumption and suggest that students are still accessing their natural languages as resources in their learning of mathematics through secondary schooling and into university level. However, the impact of natural language is also canvassed beyond the teaching and learning context by some authors. The impact of researchers' natural languages on the way *they* formulate ideas and the hegemony of English in the research literature is challenged. This seems to have a narrowing effect on what in the end is acceptable to the research community at large. Hence evidence is presented in this volume that the impact of students' natural languages on learning and teaching mathematics needs to be an ongoing research theme for mathematics education. As well, there also needs to be more detailed investigation of the impact of researchers' own natural languages on their work, and whether ideas and notions not easily expressed in English are being lost to the world-wide community of researchers.

Which are the multiple languages in multilingualism? In most chapters in this volume, language diversity refers to the variety of natural languages spoken by the learners and teachers. However, some chapters raise interesting points that move the discussion further. The growing presence of different media in teaching and learning situations makes it evident that other languages, registers and modes contribute to the linguistic complexity of today. Furthermore, international trends for the inclusion of blind and deaf students in schools also raise the issue of how other types and forms of language are dealt with. Various contributors have written about these challenges arising in such (to many) novel contexts of language diversity and about how these issues affect teachers and students and their meeting with mathematics.

Is mathematics education in contexts of language diversity setting political? Culturally dominant groups, their languages and worldviews establish the norms to follow, strive for and classify people accordingly. In any society, the mathematics in educational curricula embeds the values, worldviews and languages of the culturally dominant group(s) in society. This is a condition of the historical and social organisation of schooling and education in all societies. Thus, learners who do not share or comply with the norm are seen as deficient. This deficiency perspective is part of the form of operation of schooling as an institution in society. In settings of language diversity, many learners who are not part of the 'norm' will be deemed as deficient in relation to both the language of instruction, and in relation to many of the values and forms of being that school promotes. This means that even though some mathematics education research has tried to evidence and criticise the pervasiveness of deficit perspectives of learners (see above for a very short summary), it is quite difficult to eliminate. This terrain is highly political, as highlighted in several chapters and the mathematics education community needs to find ways to build constructive dialogue with those wielding political power. As well, new options for pedagogies and forms of understanding educational practices continue to be desirable. A number of the chapters in this volume suggest interesting paths to follow.

1.7 Implications for Policy and Practice

Many of the questions that were originally included in the discussion document remain open to further research, but even what has been accomplished so far can offer some guidance to policy-makers, curriculum developers and teachers in classrooms or lecture halls. In this section, we first offer some points of guidance. These points are a brief distillation of what we regard as important issues raised in various chapters of this volume. Following these points, we then note a number of questions that impinge on practice, which authors have suggested still need investigation.

We begin each point with a question, since the whole tenor of this volume is to open up multiple discussions and dialogue rather than promote 'one size fits all' results, and then offer some comments in note form.

What is our basic advice?

- Accept and acknowledge the reality and diversity of multilinguistic contexts.
- Language diversity is a reality for mathematics learners and their teachers and is a complex phenomenon. Learners in mathematics classrooms live in communities that use one or more languages besides the language of instruction. Language diversity also exists in language varieties, including dialects as well as regional, social and economic variation and sign languages.

How can policy and practitioners deal with this diversity and complexity?

- Know the local context, setting, and details of language history, policies, and especially the experiences of disenfranchised language groups.
- Shift from focusing on deficiencies to noticing and building on competencies.
- Although deficit models of mathematics learners or their communities are pervasive, they do not work: they do not provide an accurate picture of learners' potential for progress, they do not describe any resources, strengths, or competencies that instruction can build on, and they may condemn students to endless cycles of remedial instruction that simply do not work.

How can policy and practitioners avoid deficit models?

- Balance a focus on challenges that learners may face with an equal focus on deep and detailed knowledge of the resources, competencies, and strengths that these learners bring to the classroom. Again, know the students and their communities.
- Accept and acknowledge the complexity of language issues in mathematics classrooms.
- Language issues in mathematics classrooms are complex. It is not possible to say that using home languages is always the right thing to do. The question to ask is: when and how should home language(s) and the language of instruction be explicitly used and encouraged by teachers, depending on the goals of mathematics instruction, and in the full knowledge that multilingual students will, for a variety of reasons, code-switch whether the teacher encourages, or even

recognises this practice, or not. As well it should be acknowledged that language and mathematical discourse is much more than vocabulary, number names, or the logical structure of sentences. One important aspect of mathematical discourse is that it is multimodal and involves a variety of modes such as listening, talking, writing, reading, drawing, graphing, etc. Another aspect is that it is multisemiotic; it involves a variety of sign systems such as mathematical symbols, written text, etc. This adds up to acknowledging that language and culture are closely and intimately related and cannot be separated.

How can policy and practitioners avoid simplifying issues of language?

- Avoid reducing language issues to vocabulary, provide opportunities for students to participate in multiple modes (oral, written, concrete objects, drawings, etc.) and use multiple sign systems.
- Consider the systemic and political nature of language issues in classrooms.
- Language diversity issues function in a system that includes not only teachers and classrooms but also schools, families, communities, and language attitudes as well as classism, racism, and other systems of institutionalised oppression. Language diversity is a political issue: the hegemony of the language of dominant cultural groups shows up in the classroom, in policies, curriculum, and language choices, and learner identities with regard to mathematical competence. Language policies reflect who counts and who does not count in society. This is so even in mathematics classrooms.

How can policy and practitioners address systemic issues?

- Include research that addresses the systemic and political nature of language issues in education in policy and practice discussions.

We now list a series of questions that have emerged for us in participating in this study, and many of which have been asked by authors in this volume. They are organised according to the themes in the discussion document. It is a deliberate choice to leave them as questions, rather than giving summary statements as to what research says about these questions, since we regard the field as one that still is emerging and diverging in many constructive paths. This is not to say that research does not articulate some specific notions regarding some of these questions, as can be seen in this volume and elsewhere. Nevertheless the articulation of these questions taken together suggests our thinking is deepening, since we are no longer asking the type of questions that were foregrounded by researchers some 30 years ago. We also invite our colleagues in policy and curriculum development, as well as teachers, to think along with us, since they too are agents, along with representatives from non-dominant communities, who need to be involved in discussions of future research.

Teaching mathematics in diverse language contexts:

- What strategies and resources can educators in multilingual classrooms use to teach mathematics effectively to learners who are in the process of learning the language of instruction?

- How do assessment and curriculum systems relate to mathematics teaching and language policies in diverse language contexts?
- Which current teaching practices are sensitive to the relationship between multilingualism and mathematics learning?
- What are the relationships between teaching language and teaching mathematics?

Teacher education for diverse language contexts:

- Which current practices in teacher education are sensitive to the relationships between multilingualism and mathematics teaching and learning?
- What can be done to prepare teachers to teach mathematics effectively in multilingual classrooms?
- What kind of data from multilingual classrooms would be useful in designing teacher education programs?
- What knowledge and skills do teachers need to teach in multilingual classrooms and what are teachers' perspectives on this question?

Researching mathematics teaching and learning in multilingual contexts:

- What types of theories and methods enable the development of research in this area?
- What ethical issues arise in pursuing this kind of research and how can researchers address them?
- On what basis can researchers interpret the mathematical worlds of students who come from linguistic backgrounds with which they are not familiar?
- To what extent is mathematics education research sensitive to linguistic diversity?

Mathematics, language diversity and society:

- To what extent and for what purposes do research in multilingual contexts need to address multicultural issues?
- What role does teaching mathematics in diverse language settings play in reproducing or challenging prevailing social patterns?
- How can researchers engage productively with policy-makers involved in mathematics education to address language diversity?
- What is the relationship between the teaching and learning of mathematics in multilingual settings, and wider social discourses?

Student mathematics learning and experiences in multilingual classrooms:

- What are the characteristics of students' mathematical discussions and explanations in different languages, in multiple classroom contexts, and in multiple mathematical domains?
- What are the demands on multilingual students learning mathematics in different mathematical domains (i.e. algebra, geometry, etc.) and at different ages?
- How do students themselves see and describe their experiences in multilingual mathematics classrooms?

- What are students' strengths and resources, what can we learn from successful students, and how can instruction build on these resources, strengths, and successes in linguistically diverse settings?

1.8 Some Issues for Future Research

In this study, research is reviewed that investigates the teaching and learning of mathematics at different levels including in schools, in university undergraduate courses, and in teacher education. The work has also covered teaching and learning using ICT and digital media. Furthermore, the concept of the language of teaching and learning has gone beyond the spoken language to that of teaching and learning mathematics to deaf and blind learners. Thus this volume draws on a wide research and knowledge base. However our research needs to be taken further.

Mathematics education researchers, mathematics teacher educators, graduate students in mathematics education, mathematics teachers and student teachers, among others, can progress the work of this book further with future research. Within various chapters and indeed in this introduction, there are explicit calls for future research in specific areas. For example the call for more research in undergraduate mathematics, where often language is assumed not to be an issue of concern, and the call for more research in European classrooms, where there are many languages that are not shared, are two that are quite explicit. There are also less explicit calls, such as the work on blind and deaf learners that brings an awareness that language diversity includes non-verbal communication.

Equally, while some situations of language diversity have been extensively examined, other aspects have not. For example, most research to date has focused on what can approximately be termed bilingual and multilingual contexts. There is little that examines the specificity of trilingual contexts where learners are exposed to a home language, a national language and an official language of instruction. The study of such contexts in mathematics teaching and learning remains a gap in our field. Its specificity lies in the fact that unlike in multilingual contexts where there are multiple languages, but only two languages (home language and language of instruction) that are in competition, learners in trilingual contexts have to deal with three languages, each of which has its own power and influence: one as a home language, the second as a national language and the third as a world language.

Research in mathematics education in general, and the linkage between mathematics education and language specifically, can appear to be somewhat inward looking, in the sense that it is often mainly concerned with the issues of teaching, learning and curriculum in mathematics classrooms. These foci may, in turn, be reduced to strategies and methods to promote teaching and learning. However, the broader issue of what is termed education is not necessarily considered as an issue that needs to be deconstructed. Assumptions and values that underpin the current system of formal education appear to be taken as normative and are often not questioned. This is an issue particularly pertinent to this volume, since mainly European/

Western perspectives and values inform current education systems in many countries. As a result, approaches to education that might be beyond the 'normative' frame of reference are considered as lacking and deficient. These certainly impinge on our more focused territory in mathematics education.

A related issue is the understanding of 'what counts as mathematics?' While historical development shows mathematics as fallible, as a work in progress, and undertaken from a particular cultural and epistemological tradition (normally Eurocentric-Western), it is almost always taught in schools as a culture-free subject that involves learning supposedly universally accepted facts, concepts and contents.

In addition, mathematics enjoys a position of prestige, such that proficiency in the subject can potentially open the gates to opportunity and is, therefore, often uncritically assumed to be a worthwhile goal for teachers and learners. However, it is not any mathematics that performs this function of gate-keeping: it is only the 'valorised' mathematics of the few that performs this function. Hence, it is important to understand the nature of the subject as an educational goal.

Several issues in mathematics education go beyond the disciplinary boundaries, as does the issue of language diversity and mathematics education. There are many cases of marginalisation due to language and culture, which raise issues and concerns that require instruments of analysis from linguistics and sociology. Therefore, research in mathematics education needs to shift its 'inward' stance and look across disciplines and undertake cross-disciplinary research to understand these issues from a more multidisciplinary and nuanced perspective.

Finally, much research in mathematics education, including studies on the interplay of language and mathematics education, focuses exclusively on praxis; how can teachers teach more effectively and how do students learn what is taught. We have no problem with these outcomes but question the exclusive focus. We call for more studies that run parallel to these but also incorporate and take seriously the role of theory so that the theoretical basis of our work can be enriched and in turn give deeper insight into our praxis.

1.9 Summary Statement

This volume brings together the combined thinking of over 5 years of work by colleagues from many parts of the world working in contexts of language diversity. There is huge diversity within these contexts, and yet there is also commonality for us. There are chapters that not only summarise what research has been done, but use that work as a basis for envisaging what can be researched for the benefit of future learners, teachers, educators, policy-makers and others. Other contributions, some tentatively and others more robustly, declare ways forward that suggest options for our colleagues to re-envisage their praxis. We believe there is much here for colleagues to consider that will help push forward this area of research and practice.

Nevertheless this volume also repeatedly highlights that after more than 40 years of research that suggests that for multilingual students, well-thought out use of all

their languages can be beneficial in their learning, including mathematical learning, the default setting for many politicians and bureaucrats is that students must only use the official language of teaching. In many countries the official language of teaching is a world language deemed to be important for the economy of the country as a whole. Of course there is some truth in this kind of assertion. However educational research suggests there are better options that need to be considered rather than this simple default position. Hence one of the outcomes of this volume should be that we as researchers should find ways to engage with politicians and bureaucrats productively, so that they are aware of what research does say, and work with them to ensure such dialogue continues.

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