

# Chapter 3

## Making Use of Multiple (Non-shared) First Languages: State of and Need for Research and Development in the European Language Context

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### 3.1 Introduction

As Durkin and Shire (1991) underline, “Mathematics begins and proceeds in language, it advances and stumbles because of language, and its outcomes are often assessed in language” (p. 3). The importance of language in mathematics learning illuminates the need for speaking a language that everyone is able to understand. Historically, the development of the European nations in the eighteenth and nineteenth centuries went along with the development of a monolingual self-concept of many of them, although some experienced a multilingual way of living, such as, for example, Switzerland and Belgium. But the monolingualistic *habitus* was strengthened during the Second World War and since then it has been one reason for the development of social and cultural communities. Today the role of language has changed. Emigration and immigration movements in Europe have produced a mixture of languages and cultural backgrounds (Gogolin, 2010). Vertovec (2007) uses the designation “Super-diversity” to describe the actual situation in the majority of European countries: a complete mixture of (first) languages and cultures.

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Although Europe is a multicultural and multilingual region, discussion and practices regarding multiple languages as resources for mathematics learning still seem to be underdeveloped. If the multilingual dimension of mathematics classrooms is neglected, this can under-privilege mathematics learning for those students whose first language does not correspond to the language of instruction. That is why making use of the multiple first languages<sup>1</sup> is often claimed as necessary. In her survey on different, mostly non-European, bilingual studies (not only in mathematics education), Gogolin (2011) emphasizes the relevance of asking about the effect of teaching in the first language on mastery of the second language. However, in this chapter we do not want to address this question. Instead, we handle the issue of whether (the use of) a plurality of languages can be useful in order to learn mathematics.

This chapter explores the *status quo*, actual tendencies and desiderata in research into the use of first languages for the European language contexts, which are often shaped by a large diversity of first languages in the same classroom. Thus, the chapter reviews these aspects and the development of classroom practices in the following steps: we briefly describe worldwide discussion and research on benefits of first language use, then consider European language contexts with their specificities, and finally report on current European developments, practices, and research on first language use in mathematics classrooms.

### 3.2 First Language as a Widely Accepted Resource for Giving Access to Mathematics

Many studies all over the world have highlighted the relevance of the first language for giving access to mathematics (e.g., Baker, 1996; Barwell, 2009; Clarkson, 1992). Various case studies have shown how the first language can provide wider options to participate in classroom interactions. Often this first language use is naturally interrupted by moments of code-switching, considered as a social practice of flexible use of languages (Moschkovich, 2007; Setati & Duma, 2009).

Other studies have emphasized the cognitive and meta-cognitive benefit of the first language while making sense of mathematical texts, for example, for mathematically successful bilinguals (Clarkson, 2007). Kern (1994) has specified the cognitive benefit of facilitating semantic processing, relieving the short-term memory and, especially, allowing concepts to become alive more easily, since first languages offer richer connections to students' networks of associations. This last aspect seems particularly important for conceptual understanding (Ellerton & Clarkson, 1996).

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<sup>1</sup>In this chapter, we use the term “first language” for the language that students usually speak at home, in which they think and feel comfortable using. Although acknowledging subtle differences, we use it as synonymous with “mother language,” since no single term can reflect the complexity of different individual language profiles with more than one language spoken in families. In the quoted data, different conceptualizations are used, mostly implicit, for example “language that is mostly spoken in the family” (OECD, 2007, p. 120) or “mother tongue” (Eurobarometer, 2006).

Additionally, Clarkson (2007) emphasized the meta-cognitive use of language switching, as bilinguals seem to self-correct themselves more frequently in their first language. These results are in line with general results on the relevance of individual languages for making sense of mathematical expressions and for developing conceptual understanding (Ellerton & Clarkson, 1996).

In the light of these international results, surveyed by Barwell (2009), it appears as a logical consequence that the Council of Europe nowadays advocates the inclusion of first languages into school subjects such as mathematics or chemistry, among others (cf. Little, 2010). However, these ideas spread very slowly and the majority of classroom practices do not match these ideals. This chapter explores why it seems particularly difficult in the European language context to activate first languages as a resource for mathematics learning.

Nevertheless, we have to mention other studies showing opposite results. On the basis of their meta-analysis of studies concerning bilingual education in the USA, Rossell and Kuder (2005) came to the result that a monolingual English education program has to be preferred to a mixture of English and Spanish education. Although their examinations did not focus on mathematics education, we have to be aware of an obstacle: in order to establish knowledge which can be taken as shared, children have to communicate their results to the other children. In a multilingual classroom this can often not be done by the use of the first language, if these children do not have the same first language.

As teaching and learning practices cannot simply be transferred from one language context to another, we felt the need to explore the opportunities and limits of using first languages in the specific language context of different countries and their mathematics classrooms. For this, we make a rough differentiation between those countries where the first languages are shared or non-shared between the different persons participating in the classroom interaction.

### **3.3 Cultural and Political Dimension: Presenting the European Language Context**

#### ***3.3.1 Multilingualism in Europe and European Schools: Demographic Facts and Emerging Consciousness***

In 2012, about 500 million people lived in the 27 countries of the European Union and there were 23 languages officially accepted in the European Union (Eurobarometer, 2006). Six countries had more than one official language, the majority of them regionally distributed. The main first language in Europe was German (with 90 million native speakers, 18 % of all Europeans), then English, French, and Italian (each about 60 million speakers, about 13 %) (European Commission, 2004; Eurobarometer, 2006). Although Europe as a whole is multilingual, however, many countries conceptualize themselves as monolingual societies.

Since the European Union was constructed, dealing with multiple languages has been a political aim. However, it is interesting to see how the European conceptualization of multilingualism has slowly changed from dealing with multiple foreign languages as a *learning goal* (for improving international relations between countries) to the awareness of multilingualism within each country as a *learning condition*, due to different first languages of minorities and immigrants: even as recently as 2004, the European Commission published a brochure titled *Many Tongues, One Family: Languages in the European Union* (European Commission, 2004) where the multilingualism within each country is marginalized to a brief reference to some regional languages such as Welsh or Sardinian, while immigrant languages are not even mentioned. Instead, it promotes the European campaign “mother tongue plus two” with its focus on the aim that each European citizen should speak two foreign languages (which is currently reached by about 30 % of all Europeans, most of them speaking English, German, and/or French). The rate of people mastering two foreign languages varies from the small countries (92 % in Luxembourg, 75 % in the Netherlands) to larger countries (Germany 27 %, UK 18 %) (European Commission, 2004). This indicates that speaking a second or third language is less common when your first language is one of the most spoken languages in Europe.

In contrast, a similar booklet published by the European Commission 2 years later (Eurobarometer, 2006) documents and acknowledges the variety of first languages, including not only regional languages but also immigrant languages (which have higher percentages in the majority of Western European countries). Typical immigrant languages are Arabic in Sweden, Turkish in Germany, and African languages in Portugal. Another important issue is that many people with immigrant backgrounds are born in European countries. Thus, they belong to a second (or even a later) generation. Table 3.1 shows an overview of the given percentages of people with a first language other than each country’s official languages. Note that the data comes from selective inquiries (Eurobarometer, 2006), since the variable “language backgrounds” has still not been captured in the countries’ official statistics (only nationality and migration status, see e.g., Mikrozensus, 2009). Other sources give higher rates of nonofficial first languages for the younger generation. For example, in Germany and Sweden, rates of 20 %<sup>2</sup> are estimated for students in compulsory education with first language other than German (Chlosta & Ostermann, 2008) or Swedish (Swedish National Agency of Education [Skolverket], 2013). The increasing rates of children with other first languages can be traced back to increasing immigration to European countries and to demographic factors (i.e., higher birth rates in some language communities) (Chlosta & Ostermann, 2008).

The development in the European Commission’s conceptualization of multilingualism (from a learning goal for European ideals to a multicultural learning condition within each country) reflects the slowly increasing awareness in some of the countries about their immigrant *status*. For example, Germany acknowledged only

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<sup>2</sup>In North Rhine-Westphalia (the most populous federal state of Germany) exactly 19.50 % of the primary school students have another first language (IT.NRW, 2012, p. 2).

**Table 3.1** Multiple first languages in European countries: “What is your mother tongue?”

Country	Percentages of indications of first language being ... (Multiple answers possible)		
	... a state language or other official language (%)	... another EU language (%)	... another language (%)
Austria	96	3	2
Belgium	Dutch 56, French 38, German 0.4	5	3
Bulgaria	90	0.4	11
Cyprus	98	2	1
Czech Republic	98	2	0.7
Denmark	97	2	2
Estland [Estonia]	82	1	18
Finland	Finnish 94, Swedish 5	0.8	0.4
France	93	6	3
Germany	90	3	8
Greece	98	1	0.8
Hungary	100	0.8	0.6
Ireland	English 94, Irish 11	2	0.2
Italy	95	5	1
Latvia	73	1	27
Lithuania	88	5	7
Luxembourg	Luxembourgish 77, French 6, German 4	14	0.8
Malta	Maltese 97, English 2	0.6	–
Netherlands	96	3	3
Poland	98	1	1
Portugal	100	0.6	0.1
Romania	95	6	0.7
Slovakia	88	12	2
Slovenia	95	1	5
Spain	Spanish 89, Catalan 9, Galician 5, Basque 1	1	2
Sweden	95	5	2
United Kingdom	92	3	5

Source: Eurobarometer (2006, p. 9)

in 2000 that it was an immigration country. These trends are also reflected in the political context of the schools.

These developments have been supported by the *Charter of Fundamental Rights of the European Union*, proclaimed by the European Parliament, Council, and Commission in 2000. The charter guarantees the rights of cultural, religious, and language diversity, including a non-discrimination law for languages (European Parliament, 2000, §21). These new perspectives on multilingualism are also reflected in changing language policies for schools.

### 3.3.2 *Multiple Languages in European Schools*

Notwithstanding the non-discrimination law for languages, nearly all European countries mainly organize their schools as monolingual institutions in which the official regional or state language is the only accepted language of instruction (except for some special private schools or Content and Language Integrated Learning (CLIL) classrooms with English or French as a foreign language of instruction). Within these similar policies, the school systems attain significantly different rates of success in allowing students with other first languages to achieve mathematics performance comparable to their native speaking classmates. As Table 3.2 shows, some countries (Germany, Denmark, Switzerland, Austria) particularly fail in mathematics achievement for students with other first languages.<sup>3</sup>

The comparative results of some less adequate school systems have raised discussions in these countries on how to give better access to mathematics for learners with different first languages. Among all national strategies for a better inclusion of second (or third or fourth language) speakers, most emphasis is given to language appropriation in the language of instruction (e.g., der Bundesrepublik Deutschland, 2010; Thürmann, Vollmer, & Pieper, 2010). In many countries, immigrant students' low performance in mathematics is attributed to their lack of competence in the language of instruction and a lack of cultural inclusion, such as, for example the so-called lack of "Swedishness" (Haglund, 2005; Runfors, 2003), and similarly for Denmark (Holmen, 2008). That is why, besides this focus on the language of schooling, many countries increasingly discuss a new language policy allowing the use of first languages, as reflected in the Council of Europe's goal of *plurilingualism starting from the individual resources*, which comprise also first languages (Beacco et al., 2010). Although these European first languages policies mainly refer to the early years of schooling (e.g., learning how to read), some documents also refer to learning in secondary schools and in subjects such as science and mathematics (e.g., Thürmann et al., 2010).

The use of first languages in education is not only supported by political demands, but also by educational reasons, for example, "Language is a tool for acquiring knowledge, one aspect of the development of the person, as both individual and social actor, a means of and factor in understanding and making sense of reality, and a vehicle for imaginative creativity" (Coste, Cavalli, Crisan, & van den Ven, 2009, p. 5). In spite of all the claims and reasons for first language use in European schools, its use in European mathematics classrooms is still quite rare. In order to understand this reluctance, it is important to consider that these types of practices are not equally easy to follow in all language contexts, since the existence of multiple languages among students in European schools has at least two significantly different facets.

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<sup>3</sup>Although this chapter focuses on language issues, we emphasize that not only the language background, but also other factors are crucial for school success of minority students, for example, their socioeconomic status, parents' literacy and educational background, and other individual circumstances (cf. Alrø et al., 2003; César, 2009; Heinze, Reiss, Rudolph-Albert, Herwartz-Emden, & Braun, 2009).

**Table 3.2** Different equity success of school systems—comparing European and non-European countries

Country	Students with first language ≠ language of instruction		Students with first language = language of instruction		Difference between both groups' mean scores
	Mean score	(S.E.)	Mean score	(S.E.)	
Austria	442	(12.8)	515	(3.5)	73
Belgium	515	(6.7)	530	(2.6)	15
Denmark	440	(7.0)	519	(2.6)	79
France	441	(10.1)	502	(3.2)	61
Germany	438	(8.4)	519	(3.3)	81
Greece	408	(9.7)	462	(2.7)	54
Luxembourg	494	(1.3)	525	(4.7)	30
Netherlands	472	(10.3)	536	(2.2)	64
United Kingdom	458	(9.8)	499	(2.0)	41
Norway	448	(7.5)	495	(2.4)	47
Portugal	445	(15.3)	468	(3.0)	23
Switzerland	473	(5.5)	546	(2.9)	73
Australia	523	(7.7)	521	(2.0)	-2
Canada	522	(4.3)	531	(1.8)	9
New Zealand	522	(6.7)	526	(2.3)	4
USA	440	(5.9)	480	(4.2)	40
OECD Average	467	(2.2)	504	(0.5)	37

Data from PISA, 2006. *Source:* OECD (2007, p. 120, Table 4.3b)

Some regions such as Catalonia (Catalan and Castilian, see Planas & Setati, 2009) or parts of Ireland (Irish and English, see Table 3.1) experience a more or less *shared bilingualism* where most people share the two main languages, at least if they are not immigrants (e.g., Catalonia has many immigrants from Africa). Here, the use of the other language is easy since usually many teachers and students understand each other. However, in those countries commonly designated as immigration countries in Europe, there are five to seven, and sometimes even more, different first languages spoken in multilingual classrooms and the only intersection is the language of instruction. In this language context, we talk about multiple (non-shared) languages. The teachers of these classes are usually speakers of the language of instruction. They might even speak other languages, but they normally do not speak the languages of (all of) the students (César & Oliveira, 2005; Gogolin, 1994).

Speaking another language having immigrated to another country often comes with participating in a group of lower socioeconomic status (Nusche, 2009). Concerning Germany, Prenzel and his associates (2005) analyzed the PISA results and found that a child from the second lowest socioeconomic background is four times less likely to join the highest track of secondary education compared with a child from the top quartile. Comparable results have been reported for black Caribbean people in the UK (Strand, 2007). Eckhardt (2008) points out that a migration

background is not the reason for systematic disadvantage of some children. Moreover, the analyses indicate that problems of appropriating the language of instruction can primarily be ascribed to the social backgrounds of the families and their communicative practices. Moreover, the disadvantages do not go along with the first languages of the children (comparable results in Gogolin, 2006). In her survey on research on mathematics teaching and learning of immigrant students in Europe, Civil (2010) points out that language issues are only a part of a culturally complex reality that needs further research. Of course, the policy and practices of allowing or forbidding first languages reflect the multicultural sensitivities of a country.

### **3.4 Practical Dimension: Teachers' Options for Including First Languages in European Classrooms**

#### ***3.4.1 Options for First Language Use Under Different Language Conditions***

In this section, we present a variety of ways in which first languages are used for increasing access to mathematics in European mathematics classrooms. We do not discuss bilingual or CLIL classrooms in which the official language of instruction is enriched or substituted by one of the major foreign languages, such as English, French, or German. Also we do not want to discuss those models of bilingual schools in immigrant minority languages, which are paid for and work under the supervision of the “home countries” and are not supported by mathematics education research. Instead, we refer to minority or immigrant first languages and focus on practical matters in the mathematics classroom.

The most far-reaching model for first language use refers to all language domains, namely oral and written language production and reception concerning all moments of mathematical lessons (see first column of Table 3.3). In such an “ideal maximum” model, the textbooks, the mathematical tasks, and the presented mathematical knowledge are translated into all languages spoken by the students in the specific classroom. Every utterance can be translated (by bilingual teaching assistants), thus guaranteeing full participation in the classroom communication. In such a classroom, students can switch between languages in all language domains (reading, writing, listening, speaking; see Table 3.3) and choose in each situation which language to use. A model close to this idealistic one has been tested in a Swedish project with students of different language and migration language backgrounds (Norén, 2007). Although the textbooks have not been translated, many teaching materials and blackboard writings were provided in Swedish and Somali/Arabic. The teachers were bilingual with migration backgrounds, and teachers and students could use these languages.

However, the language conditions can be more difficult when the teacher does not speak the students' first languages or when five to seven (or even more) first



**Table 3.3** Necessary language conditions for different options of first language use

	... not possible, if	... possible or... allowed, if	... encouraged, if	... obliged, if
Written language reception (READING)	Materials all in L2	Materials in L1 and L2	Materials in L1 and L2 and translation supported	Materials only in L1
Written language production (WRITING)	Readers understand only L2	One reader understands L1 and L2 and writing allowed	One reader understands L1 and L2 and writing in L1 valued	All readers understands only L1
Oral language reception (LISTENING)	Teacher and students only speak L2	Some other students speak L1	Some other students speak L1 and communication valued in L1	Spoken language input only in L1
Oral language production (SPEAKING)	Teacher and students only understand L2	Some other students understand L1 and speaking allowed	Some other students speak L1 and communication valued in L1	Some partners only understand L1

languages are in the classroom. Thus, in this case, as well as in many others, it is even harder to provide translated materials. For these language conditions, we can still find options for first language use, mainly in the informal communication between students of the same first language community (César & Kumpulainen, 2009; Elbers & de Haan, 2004, 2005; Favilli, César, & Oliveras, 2004).

Table 3.3 gives an overview of different options for first language use. All language domains (reading, writing, listening, speaking) are *obliged* in the first language if some materials or some communication partners only use the first language, for example, the teaching assistant or parent with migration background (last column in Table 3.3). The *obligation* to use the first language in selected moments helps to initiate first language use even when students are not accustomed to it (Meyer & Prediger, 2011). However, this mode cannot be applied constantly since L2 (being the official language of instruction) evidently must be appropriated too.

The usual model is to make first language use *possible* or *encourage* it (second and third column of the table), without *obligation*. For example, the UK's National Association for Language Development in the Curriculum (NALDIC) promotes (among other things) the method of defining words for all key concepts in first languages (see [www.naldic.org.uk](http://www.naldic.org.uk)). This method not only allows students to speak and write in their first language but also encourages them to really do so, even if the teacher does not speak these languages. The inclusion of parents as emphasized in the UK project "Our Languages" also encourages students to speak their first languages (see Melhuish, Sammons, Siraj-Blatchford, & Taggart, 2004).

Of course, providing texts in the first language cannot guarantee successful *encouragement* of first language use, as Meyer and Prediger (2011) show. In their design exper-

iments in Germany, the Turkish-speaking sixth graders often hesitated to use the tasks written in Turkish L1 because they did not change usual patterns of language that were established over years in the monolingual culture of mathematics classrooms. In contrast, a Belgian longitudinal study (van Avermaet, Slembrouck, and Verhelst, in Little, 2010) indicates that if students are confronted with texts in two languages from the beginning of their schooling, language production in L1 can be successfully encouraged. The part-time *obligation* to read in L1 might even catalyze this process towards part-time L1 language production. But one must not forget that the use of language(s) is also shaped by the number of official languages in a country and in central exams, as they are becoming more and more usual in many European countries.

A comparable situation is given in Greenland, an autonomous country within Denmark. According to Pedersen (2010), Greenlandic is spoken by less than 56,000 people. It has been used as a language of instruction for over a century, but from the 1950s there has been a growing tendency to use Danish as the language of instruction (Patrick & Shearwood, 1999). Today, Greenland has its own educational curricula, its own textbooks, including mathematics, and there are two languages used in schools: Greenlandic and Danish.

The first language option that is easiest to implement *allows* or *encourages* students to use their first language in oral situations, namely in oral communication. Especially for group-work settings, a multilingual teacher may not be needed. Sometimes it is sufficient if some other students understand the same first languages. This option happens in many classrooms, even when not intended by the teacher (Clarkson, 2009; Elbers & de Haan, 2004, 2005; Planas & Setati, 2009). Hence, especially for classrooms with many non-shared first languages, interesting options for enhancing first language production include building language-homogeneous small groups (Gorgorió & Planas, 2001) and consequently enhancing mathematical communication. However, some authors also stress possible risks of internal segregation by language-homogeneous small groups, which might attract lower teachers' expectations (and support), promoting a different type of segregation and discrimination (see César, 2009, 2013a, 2013b; Favilli et al., 2004).

To sum up, Table 3.3 offers a wide variety of options, their systematization according to the language modes (reading, writing, listening, speaking) and according to degrees of obligation helping to adapt suitable options purposefully, according to different local language conditions. However, so far, only a few empirical studies have investigated the effects and conditions of these different options of first language use under different European language conditions. In Sect. 3.5, we report on these studies and formulate further research needs.

### **3.4.2 Cultural Dimensions Beyond Language: Bridging Cultural Gaps**

Many researchers and practitioners have emphasized that first language use cannot be isolated from its social and cultural dimensions (see the survey in Civil, 2010). The switch from one language to another (e.g., from the first language to the

language of instruction) is not only a matter of one-to-one translation, but also of an interplay between different cultures which needs interpretation, because every attribution of meaning is culturally bound and embedded in a complex network of symbols, norms, sociocultural practices, and identities (de Abreu, Bishop, & Presmeg, 2002). As a consequence, many projects do not restrict themselves to languages in the narrow sense, but consider them as parts of different minority cultures that have to be included into school life in order to bridge the gap between school and everyday life (César, 2009, 2013a, 2013b; de Abreu et al., 2002). This happens, for example, by including parents in home-school partnerships, by collaboration between mainstream and complementary schools (e.g., Melhuish et al., 2004), by including handicrafts or other cultural artifacts from minority cultures to promote ethnomathematics in classrooms (e.g., Favilli et al., 2004; Jannok-Nutti, 2011), and by many other aspects that value minority cultures and help to bridge the sociocultural gap between schools and homes, particularly using regulatory dynamics between schools and families (César, 2013b) or other ways of empowering those who participate in vulnerable minorities which are socially undervalued (see César, 2013a; César & Kumpulainen, 2009). These considerations of the cultural embeddedness of languages make clear why including first languages can be difficult: it is not only a matter of concrete pedagogy, but also of multicultural attitudes that value and support cultural diversity (Alrø, Skovsmose, & Valero, 2003), something that can be particularly seen in sign languages used as first languages—an issue that is even more lacking in research and a robust and sustained discussion (César, 2010; Melro & César, 2010).

The cultural dimension also refers to some risks that have been stressed in the context of promoting first language use (César, 2009, 2013a, 2013b; Favilli et al., 2004; Melro & César, 2010). The aim of preparing students for universities or professional careers might be in conflict with promoting linguistic competencies in both languages. From this perspective, encouraging first languages might also include a certain risk of creating barriers to their access to the most socially valued universities, jobs, or social positions, such as representatives of the community. In short, research should include this dimension for understanding the conditions and challenges of first language use in different settings, scenarios, and situations.

### **3.5 Research Dimension: Three Examples of Empirical Studies on Effects on Students' Learning, Obstacles, and Conditions**

The evaluations of many practical development projects in Europe have provided empirical evidence that including first languages and other aspects of students' out-of-school cultures facilitate their access to mathematics achievement, even under more complicated language conditions of multiple non-shared languages (see, for instance, Elbers & de Haan, 2004, 2005; Favilli et al., 2004; Melhuish et al., 2004; César & Oliveira, 2005; Norén, 2010; Ventura, 2012; César, 2013a, 2013b; Machado, 2014).

Beyond these evaluations that illuminate the effects for mathematics learning, research studies aim to understand why and under what conditions teaching and learning practices have effects on students' mathematics achievement. For this deeper understanding, theoretical frameworks are needed to explain the connection between language, social interactions, and mathematics learning. The following three examples of studies show that not only the theoretical framework but also the concrete aims of the research shape its methods and results.

### ***3.5.1 Language and Agency***

Norén (2007, 2010) investigated practices in a model project in multilingual mathematics classrooms by means of the theoretical constructs discourse and agency, taking into account wider societal issues (Foucault, 2005). Two languages were used for teaching and learning mathematics: Swedish and Arabic, or Swedish and Somali. The findings indicate that bilingual communication in mathematics classrooms enhances students' identity construction as engaged mathematics learners. Language- and content-based mathematics instruction seemed to do the same, though monolingual instruction may jeopardize students' identities as bilinguals while the discourse may normalize Swedish and Swedishness exclusively. The focus on linguistic dimensions in mathematics, and students' first languages valued as resources for learning mathematics, allowed building up a communicative reform-oriented school mathematics discourse. The competing and intersecting discourses available in the multilingual mathematics classroom affected students' agency, foreground,<sup>4</sup> and identity formation as engaged mathematics learners. As a consequence, Norén concludes that each student's first language, in various ways, has to be acknowledged in the mathematics classroom, as it is crucial for a student's possibilities to act agentively, to become a participant in the classroom, and to engage in learning mathematics. This acknowledgment may vary from bilingual mathematics instruction (Norén, 2010) to peer-group work in the classroom, where students who use the same first language can work together (Planas & Setati, 2009).

A similar role for first languages in students' identity and agency was found in a study on Portuguese students' dialogical self and learning when they migrated from Portugal to the UK (de Abreu & Hale, 2009). Although not focusing on mathematics learning, this study illuminates the role of language in adapting into another country, society, and school system.

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<sup>4</sup>A foreground is formed through a person's interpretation of future possibilities regarding education and a "good" life, it is rebuilt and reconstructed in contexts of social interaction and learning processes (Skovsmose, 2012).

### 3.5.2 *Language and Cultures*

César (2009, 2013a, 2013b) analyzes the use of languages in mathematics learning in a theoretical framework based on Bakhtin (1929/1981), Vygotsky (1934/1962), and Wertsch (1991). Bakhtin's (1929/1981) distinction between meaning (as a social attribution for a particular word) and sense (as an individual attribution for a particular word in a specific situation, scenario, and context) leads to conceiving language, teaching, and learning as shaped by culture, time, and space. This approach gives great importance to the symbolic systems and cultural resources used by students and those needed to solve particular mathematical tasks, as exemplified in the analysis of social interactions taped while solving mathematics tasks (César, 2009, 2013a, 2013b; César & Santos, 2006). The mathematical tasks, the working instructions, the didactic contract, and even the evaluation system, shape the way social interactions are established among the different educational agents. Moreover, they also shape students' mathematical performances and their access to school achievement, or the various forms of segregation that emerge from teachers' practices, including the way they use language in learning situations within formal educational settings. But these studies also illuminate the need to promote regulatory dynamics that allow students' families, above all those participating in minority cultures that are often socially undervalued, to participate in more active ways in schools (César, 2013b).

In this approach, language is considered to be social before being internalized and becoming individual (Vygotsky, 1934/1962) and language only exists within a network of social interactions that allow participants to give sense to their talk (Bakhtin, 1929/1981). Thus, facilitating students' access to mathematics learning and achievement means creating conditions for those students whose voices are usually silenced to become legitimate participants in the learning community. This means not only changing teachers' practices within mathematics classes, but also changing the school organization and families' participation, such as, for example, when some teachers began learning Creole with parents so that the parents would also feel tempted to learn Portuguese, and also as a way of empowering those students and their families (César, 2013b).

César considers linguistic diversity to be directly connected to cultural diversity and has undertaken a long-term research project, *Interaction and Knowledge*, that lasted 12 years and had a 10-year follow-up to collect an empirical corpus of differences and similarities (for more details, see César, 2009, 2013a, 2013b; Ventura, 2012). The most striking empirical evidence referred to ideographic and mainly oral languages, and the mathematical reasoning and solving strategies students tended to prefer (César, 2009, 2013a; Machado, 2014; Ventura, 2012). For instance, some students from Cape Verde have Creole as their L1, a language that is mainly learnt and used in oral practices and which is an ideographic language. These students usually prefer a global approach rather than a step-by-step approach to problems and investigative tasks (Machado, 2013). They also prefer geometrical reasoning rather than the analytical reasoning that is usually prioritized in school mathematics (César,

2009, 2013a). These students and their families tend to highly value learning from older and experienced people, directly connected to daily life practices, rather than learning through books and in schools. Thus, it is not only their L1 language that shapes their engagement in mathematical school activities but also their culture and their expectations towards schooling and their future life trajectories of participation (César, 2013a, 2013b). Collaborative practices based on challenging mathematics tasks, higher teachers' expectations, and the use of very careful criteria to form dyads and groups, proved to be effective ways to promote intercultural and inclusive mathematics education (César, 2009, 2013a; Machado, 2014). Collaborative practices, developed within and outside classrooms, allow students to achieve in school and in mathematics. This applies particularly to those students participating in minority cultures and using other first languages (also in César & Oliveira, 2005; César 2013b), or presenting special educational needs (César & Santos, 2006). As a consequence of these observations, teaching practices should be guided by Vygotsky's claim for addressing each student's zone of proximal development (Vygotsky, 1934/1962; César, 2013a). Empirical evidence shows that these teaching practices favor students' learning and their mathematics and linguistic development (César, 2009, 2013a, 2013b; César & Santos, 2006; Machado, 2014; Ventura, 2012).

### ***3.5.3 Installing Teaching Strategies Against Established Monolingual Classroom Norms***

The empirical studies reported in Sect. 3.3 show that bilingual learners profit more from culturally comprehensive projects that encompass several years of schooling, all subjects, and not only language but also other cultural issues. However, classroom reality in many countries is much more restrictive, which is why it is also important to conduct research not only on maximum models but also on very modest attempts to make use of students' first languages. In this regard, Meyer and Prediger (2011) conducted a design research project with clinical interviews in which they investigated how to implement first language use in mostly monolingual classroom cultural contexts.

Starting from an interactionist theoretical background (Blumer, 1969; Goffman, 1959; Mead, 1934), the focus is not on the larger sociocultural context but on the classroom culture in itself. In the interactionist perspective, mathematical knowledge is established by elaborating shared meanings in the interactions between learners and between learners and teachers (Meyer, 2009; Voigt, 1998). Language is crucial as a mediator that gives the opportunity to elaborate meanings, but only for those students who can participate in the interaction. Hence, first language use is conceived as a vehicle to enlarge students' access to interaction and to the construction of meanings.

However, each classroom interaction is regulated by explicit and implicit norms (Voigt, 1995). If monolingualism is one of the implicit norms that students

have been socialized to, the inclusion of first languages cannot be started easily in later grades. In the interviews, different ways to promote written and oral first language production and reception were tested and some of them proved to be effective for developing conceptual understanding. For example, after observing the students' hesitations to use their first language (Turkish), the students were asked to teach the interviewing person counting in Turkish by making them believe that he/she would be going on holiday to Turkey. In this way, the Turkish language would be more valued in the situation. In a second experiment, it was found that talking to a monolingual Turkish-speaking interviewer resulted in a more frequent use of the first language. In many interviews this approach has been useful in order to elaborate mathematical concepts (e.g., fractions of fractions, Meyer & Prediger, 2011) or to identify defining characteristics of geometrical forms in the first language. Furthermore, the first language use seemed to influence social functions in the interaction, such as increasing verbal exchanges or clarifying the flow of work. This influence has not been observed before while using the language of instruction, and can be explained by moments of privacy established by the first language (detailed descriptions and examples are given in Meyer & Prediger, 2011; Krägeloh & Meyer, 2012).

Interpretative analysis of the design experiments provided some evidence that students can be invited to make use of their first languages in order to solve mathematical tasks even if they are used to a monolingual mathematics classroom and that they profit from it for constructing shared meanings of mathematical concepts and hence for developing their conceptual understanding. This way of allowing groups of students to use their first language in the phases of gaining and negotiating mathematical knowledge is not only a useful way but also an easy way to make use of the first languages of these students in mathematics. Nevertheless, interaction in the whole classroom necessitates a commonly shared language.

### 3.6 Final Remarks

There is a huge need for further research and development on multiple first languages in European mathematics classrooms.

In the *cultural and political dimension*, it can be shown that monolingualism has become an unrealistic fiction in many European classrooms due to increasing rates of migration and multiculturalism. However, the unequal distribution of achievement between native speakers and those students whose first language is not the language of instruction indicates that European school systems have not yet found adequate answers to this multilingual reality. Making more use of students' first languages is one of the approaches for dealing with the challenges of multilingual students, particularly in classes with few first languages among students.

In the *practical dimension*, different mathematics classroom practices are reported of how the inclusion of multiple first languages might be put into practice. The presented "ideal maximum model" and the systematization of different settings

for written versus oral language and language production versus reception might offer orientations for decision-making and application in classrooms. However, an enormous need for research in the European language context must be stated.

In the *research dimension*, brief excerpts from three empirical studies on effects of first language use and on the contributions of collaborative work are presented as examples to show different facets of the state of research, all being shaped by the different theoretical perspectives that were applied. The research deals not just with effects of first language use on students' learning and achievement but also with obstacles to and conditions of its realization.

Several studies give us hope that even under the more complex language conditions in Europe, with multiple non-shared first languages in each classroom, ways can be found to make use of students' important (cultural) resources, namely their first languages and the ways they shape their mathematical performances. Some research shows that obstacles can be overcome and benefits are illuminated in the learning processes that concern socio-cognitive and emotional aspects, as well as cultural identities and students' agency. These results suggest that the inclusion of the students' first languages and/or the valuing of social interaction and of different cultures can be useful in order to enable better access to the learning of mathematics.

However, many research questions have so far only been addressed as first attempts and need further exploration. Europe is in need of much research: learning conditions vary between the countries and sometimes even between one country's federal states. The migration backgrounds and the structures and proximities of the first languages differ. We also need to consider Deaf students, as although they are not migrants they do use a different first language—a sign language. Thus, we are not only in need of investigating useful ways of using selected first languages as resources in mathematics classrooms and their sociocultural implications but we also need to assume that learning is a situated process. Thus, for example, a method which has proven useful for Turkish immigrants in Germany, may not be useful for Italian immigrants, or even for Turkish students in another country or another community.

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