Chapter 7
Addressing Multi-language Diversity in Mathematics Teacher Education Programs

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7.1 Introduction

Try to answer the following questions from a numeracy test:

1.1 Bhala esi sivakalisi sibe linani. Amawaka angamashumi amabini anamakhulu amabini anesithandathu.
1.2 $102 - 36 =$
1.3 $1\ 048 + 21\ 376 =$
1.4 $23 \times 145 =$
1.5 $168 \div 12 =$
1.6 Dibanisa olu luhlu lwamanani lulandelayo.
213, 4 017, 1 273, 2 198, 21
(Webb, 2012)

Reflect on how you felt as you attempted to answer these questions in a language (isiXhosa) that is likely not the language of most readers of this chapter. Did you wonder if 1.1 was a set of directions for 1.2–1.5 or a separate problem? Did you wonder whether you were supposed to add the numbers in 1.6 or put them in sequence? What supports, if any, did the questions provide that enabled you to attempt the problems?

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The frustrations you likely felt are not unlike those experienced by primary and secondary students in mathematics classrooms in many countries. Increasingly, English is used as the language of mathematics instruction in many countries, regardless of the social or home language spoken by teachers and their students (ICMI Study 21 discussion document, this volume, pp. 297–308). What preparation do mathematics teachers need in order to address the language diversity of their students? What is the role of mathematics teacher educators (MTEs) in this preparation?

In this chapter, MTEs from two different environments join together to share insights on the role that MTEs might play in this preparation. Throughout, we assume that English is the language of learning and teaching (LoLT) in mathematics classes, even though many of the students may speak another language or languages, at home and in their community. The authors from South Africa work in an environment with 11 official languages, but in which English is the language of instruction because it is the academic language and is used as common language in multilingual contexts. In this setting, both teachers and students may have difficulty transitioning from informal use of mathematical language, often in the students' home language, to formal mathematical language in English (Webb, 2012). In contrast, the authors from the United States work in an environment where English is the primary academic and social language. However, US school systems face an increasingly diverse student population, with approximately 11 % being designated as English language learners (ELLs) (NCELA, 2011) who maintain the use of their mother tongue at home or in social/cultural settings. As a result, American classrooms may include students with varying levels of English proficiency; in fact, in some school districts, more than 100 different languages may be spoken by students.

We believe there are more similarities than differences in the issues and challenges we face as MTEs who prepare and support teachers.[1] Hence, we begin by raising awareness of some issues involved in helping students learn to read, write, speak, and listen to mathematics—a foreign language for most students, regardless of their English language proficiency. We then discuss issues related to orchestrating classroom discourse in such settings. We end by sharing strategies MTEs might use in teacher preparation programs to prepare teachers to teach students from linguistically diverse backgrounds.

7.2 Raising Awareness of Issues Related to Teaching the English Mathematics Register

MTEs must address a variety of issues when working with teachers, including mathematics content knowledge, content-specific pedagogical knowledge, and general aspects of pedagogy advocated as part of educational reform (e.g., inquiry learning, high expectations, and tasks with high levels of cognitive demand). Given these multiple areas of responsibility, it might be difficult to consider who might

[1] In this chapter, we use the word *teachers* to refer to both prospective teachers and practicing teachers enrolled in either undergraduate or graduate teacher education programs, respectively.

address additional needs, such as teaching students who do not speak the language of instruction. We take the position that mathematics is a sign system that includes language aspects unique to the mathematics register (e.g., words, symbols, sentences, graphs) (Halliday, 1978). Hence, we believe that MTEs must help teachers understand the features of this sign system that may influence student learning, including students who are learning the language of mathematics concurrently with English. Supporting such language development involves specific expertise of those who have knowledge of mathematics, as well as knowledge of language difficulties that students often face when studying mathematics.

Students engage in mathematical discourse through the language of instruction, in this case English. When the information to be conveyed is mathematical in nature, the context is complex because there is always an interplay of at least two languages—mathematics, thought of as a unique language (Usiskin, 1996), and the language of the classroom. Because the development of mathematics language occurs primarily within the confines of the classroom, all students, regardless of their home language, are mathematics language learners[2] (Thompson, Kersaint, Richards, Hunsader, & Rubenstein, 2008). This notion is supported by curriculum recommendations in many countries (e.g., Department of Basic Education, 2011; NCTM, 2000) that emphasize the importance of communication because learning and teaching mathematics is conducted largely through interaction, including talk, as well as the use of written symbols, diagrams, charts, and other texts. So, if individuals are to develop the ability to communicate mathematically, they need opportunities to communicate as a regular and ongoing part of their mathematics classes. This suggests that mathematics teachers must be prepared to help students learn and master mathematical language. To this end, MTEs need awareness of the issues that teachers face and about which teachers might need to be sensitized.

7.2.1 Understanding the Language Context in Which Teachers Work

Two different aspects of language learning should be considered by MTEs and teachers. Individuals first develop Basic Interpersonal Communication Skills, which are skills for personal and social communication. However, for mathematics they also need Cognitive/Academic Language Proficiency, that is, the academic language needed to communicate mathematically (Cummins, 1981). Individuals may be fluent in terms of Basic Interpersonal Communication Skills in either English or their home language, and yet lack the proficiency in the academic register needed to communicate mathematically.

Classroom demographics vary, both for the classrooms of MTEs and for the classrooms of their teachers. In some contexts, individuals are still learning English

[2] For simplicity in this argument, we ignore the fact that students' home language may also lead to a mathematics that may well be quite different from the school mathematics with which they engage.

(ELL students), while their peers are English mother tongue speakers. In such cases, teachers should help to develop the English of these students so they can effectively function on the same level as their native English-speaking peers. In this English-only context, teachers will introduce the academic language of mathematics in English to all students with appropriate support and scaffolds for those still learning English. Teachers consequently have to be sensitive to whether an ELL student is a "new arrival" with low expertise in English, has been born in the country but is more proficient in a community language other than English, or is a "high achieving" multilingual with high proficiency in English (Harris & Leung, 2007).

In other contexts, for example South Africa, most of the students in multilingual classrooms are English as Additional Language learners (EAL learners), adding English to a repertoire of various other languages. In this context, students need their home language as well as English to facilitate understanding while proficiency in English is developing as an ongoing process. Adler (2001) identified three different environments in South African multilingual classrooms that may be applicable elsewhere as well. First, the urban-suburban environment is basically the same as the English-only context described previously. Second, in the Urban/Township contexts there is a strong regional language that coexists with different other home languages and many students do not have high English proficiency. Third there is the rural context, where students hear English mainly at school and most of the students have the same home language. In the last two contexts, teachers often code-switch—in other words move back and forth between English and the students' home language (Vorster, 2009). MTEs should help teachers recognize the differences that can exist within these contexts so that teachers are able to choose language strategies and mathematical language teaching practices that are most conducive to students' success in each context.

7.2.2 Understanding Potential Difficulties with Mathematics Language

Despite the differences in English fluency that may exist, all students who learn mathematics in English must learn the mathematics register to communicate effectively in mathematics. The mathematics register includes "unique vocabulary, syntax (sentence structure), semantic properties (truth conditions), and discourse (oral and written text) features" (Kersaint, Thompson, & Petkova, 2013, p. 43). Because of this, teachers must become aware of features of the mathematics register that should be addressed explicitly as part of instruction, such as the following:

- Words are used differently in mathematics than in social English (e.g., *difference* between products vs. *difference* in mathematics).
- Words may have different meanings in various disciplines (e.g., *radical* in mathematics vs. in science vs. in social studies vs. in English).

- Words may have different meanings within mathematics (e.g., *base* of a triangle vs. *base* of a power, or *median* of a data set vs. *median* of a triangle).
- Phrases have meanings separate from the meanings of the individual words (e.g., *if-then*, *polygon* vs. *regular polygon*, *root* vs. *cube root*).
- Syntax can create issues, particularly when more complex sentence structures are used, such as passive voice or *if-then* constructions.
- Semantics is essential to draw meaning from language (e.g., 3 times a number is 5 more than the number meaning $3x = x + 5$ so that "a number" and "the number" are represented by the same variable).
- Cultural references are often embedded within word problems that may influence students' ability to comprehend (e.g., "in the red" to mean a deficit).
- Specific language groups have specific problems with English, which especially impact on mathematical language where every word has to be understood correctly (e.g., some South African indigenous language speakers specifically have a problem with connectives such as "or").

Symbols have their own issues but are crucial for understanding:

- Multiple words may be needed to verbalize a symbol (e.g., $\sqrt{\ }$ as *square root of*).
- Multiple verbalizations are possible for a single symbol (e.g., + as *plus*, *increased by*).
- In contrast to vocabulary words for which phonetic clues can be used to help verbalize the word, no clues are embodied within the symbol to help a reader verbalize it. Verbalizations, even for simple symbols, must be explicitly taught.

The goal is to prepare mathematics teachers to attend to language issues they themselves may not be aware of, but that can have a significant impact on how students make meaning in the classroom (Pimm, 1987). Once teachers have this basic knowledge, MTEs need to help them learn *how* to integrate mathematical literacy into their instruction to facilitate both mathematical language and mathematical understanding.

7.3 Facilitating Discourse in English as the LoLT in Mathematics Classrooms

In contrast to ordinary language that is used in many settings, individuals tend to use mathematics language primarily within the narrow setting of the mathematics classroom. Thus, teachers need to provide many opportunities for students to engage in the entire mathematics literacy spectrum (e.g., read, write, speak, listen to, interpret) if they are to become fluent. This section highlights a number of issues in preparing teachers to orchestrate discourse in classrooms with EAL students: using dialogic practices, means of questioning, and engaging in code-switching.

7.3.1 Dialogic Practices

Vygotsky (1978) maintained that learning is constituted through dialogic practices. An interpersonal dialogue is defined as:

> a discursive relationship between two or more participants characterized by thought-provoking activities such as questioning, interpreting, explaining and rethinking […] in an interaction, either teacher-student or student-student. (Gorsky, Caspi, & Trumper, 2006, p. 74)

Gorsky et al. (2006) maintained that learning is facilitated by interpersonal dialogue. They divide dialogue into two models: intrapersonal and interpersonal dialogue. *Intrapersonal dialogue* mediates learning and refers to the interaction between the student, individually, and the subject matter that the student is attempting to learn, in this case mathematics. The structural resources that enable intrapersonal dialogue are the materials from which the student is learning, either textbooks or previous examination papers written in English. *Interpersonal dialogue* facilitates learning and is enabled by the human resources of the teacher and fellow students in the mathematics class. The teacher often teaches mathematics in English or code-switches between English and the home language. Gorsky et al. noted that if students were faced with an insoluble problem, they first turned to intrapersonal dialogue, i.e. they relied on self-study mediated by texts, but if that failed they turned to student–student interpersonal dialogue, and seldom asked the teacher for help. In contexts where the structural resources are inaccessible because of language, students should be able to access interpersonal dialogue with fellow students in the class, in the form of exploratory talk in their home language.

The teacher plays a vital role in creating and maintaining this dialogue. In language diverse classrooms, the interpersonal dialogue between student and student should, as far as possible, be conducted in a language in which the students are proficient. However, as teachers aim to facilitate dialogue in classrooms with students whose English fluency may be at varying levels, they must confront difficulties caused by tensions between informal and formal language such as:

> how to encourage movement in their learners from the predominantly informal spoken language in which they are fluent [the home language], to the formal written language [mathematics in English] that is frequently perceived to be the landmark of mathematical activity. (Pimm, 1991, p. 21)

Pimm suggests three routes:

1. A direct route from informal spoken language to formal written language.
2. An indirect route from informal spoken language through more formal spoken language to formal written language.
3. An indirect route from informal spoken language through informal written language to formal written language.

Because of the added complexity of an additional language (in the South African case, English), Setati (2005, p. 84) adds steps along the way from informal spoken

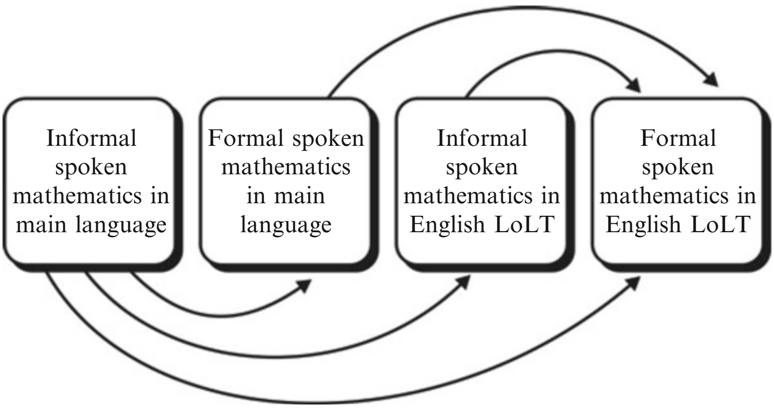

Fig. 7.1 Routes to formal spoken mathematics in English LoLT (adapted from Setati & Adler, 2000, p. 250)

mathematics in the students' home language to formal spoken mathematics in English. The route could be expanded to include: Informal spoken language in home language—formal spoken mathematics language in home language—informal spoken mathematics in English LoLT—formal spoken mathematics in English LoLT. These routes can be visualized as in Fig. 7.1.

The routes are varied and convoluted. Barwell and Kaiser (2005) argue that if students can be encouraged to talk informally about their mathematical reasoning in their home language, there is more chance that they will be able to develop formal mathematical discourse. In order to talk either formally or informally about mathematics, students have to acquire the mathematical words in the LoLT to use in sentences to develop a meaningful dialogue.

However, the mere presence of dialogue does not constitute meaningful talk and does not necessarily lead to understanding; rather, the quality and type of discourse are crucial in leading to conceptual understanding of mathematics. Mercer and Littleton (2007) analyzed talk and classified it into three types: *disputational talk* where participants agree to disagree, but where no reasons for decisions are given; *cumulative talk* when participants simply agree with each other's opinions without engaging with the issue; *exploratory talk* which is the most educationally sound method of communication. Mercer and Littleton structured dialogue as exploratory talk for primary school classes and provided teachers with specific guidelines for its implementation so teachers could negotiate with the class for the development of dialogue in groups. For example, students should share relevant ideas and help each other to understand the problems; they should listen to each other's contributions and respect their ideas, even if they disagree; they can challenge and counter-challenge arguments, but they should give reasons and substantiate their challenges with sentences such as, "I think … because …." If possible the groups should work towards an equitable consensus.

If the journey towards mathematical understanding can be smoothed by facilitating dialogue, in the form of exploratory talk, among and between students in the language in which they are most proficient, can the promotion of structured dialogue also facilitate the development of reasoning skills and language? If so, then what principles could teachers use to encourage their multilingual students to engage in dialogic practices?

Rojas-Drummond and Mercer (2004) studied interactions in Mexican classrooms and found that teachers whose pupils achieved the highest results either treated learning as a social communicative process or used judicious questioning. The teachers were observed organizing interchanges of ideas and mutual support amongst students and generally encouraging students to take a more active, vocal role in classroom events. They used question-and-answer sequences not just to test knowledge but also to guide the development of understanding. These teachers often used questions to discover the initial levels of students' understanding and adjusted their teaching accordingly, and used "why" questions to get students to reason and reflect about what they were doing (Mercer & Littleton, 2007). Thus, teachers play an active role in guiding their students in dialogic interactions.

7.3.2 The Role of Questioning

Although teachers use questions as a matter of course to monitor progress, the skill of asking higher-order questions that focus on communication and conceptual understanding is not a trivial one. MTEs need to help teachers develop skill at questioning techniques. Questions can serve many communicative roles: to test students' knowledge; to manage classroom activities; to assess students' understanding; or some combination of these roles (Mercer & Littleton, 2007). Teacher questioning can be used in the development of students' learning and their own use of language as a tool for reasoning. Teachers can encourage students to make explicit their thoughts, reasons and knowledge and share them with the class; teachers can model useful ways of using language that students can appropriate for themselves in peer group discussions; and teachers can provide opportunities for students to make longer contributions in which they express their current state of understanding, articulate ideas, and reveal problems they are encountering (Mercer & Littleton, 2007, p. 36). In many language diverse classroom settings, the discussion around the problem solving can be done in the students' home language; in the wrap-up phase, the teacher can rephrase and revoice the mathematical ideas in English, consolidating the learning process by writing the solution and the English terms on the board so that the spoken word in the home language is both heard and read in English.

During Socratic dialogue, Socrates took the part of a critical friend who questioned his students to develop their reasoning and argumentation skills. He continuously posed questions but did not provide answers or solutions. Although he did not openly disagree with his students, his questions were designed to help students arrive at their own conclusions (Frick, Albertyn, & Rutgers, 2010). A question is answered with a question in order to tease out the reasoning behind it. In multilingual mathematics classrooms, the teacher can force the student to defend his/her position by offering arguments against it. Very often there is no correct answer but the reasoning behind the stance is probed and critical thinking is engendered. It is not only the teacher who is responsible for judicious questioning in the classroom, but the role can be played by students among themselves in group interactions. To engender confidence in students, the answers (and the questions) can be posed and answered in students' home language. It is incumbent on the teacher to intersperse terms and phrases in English so that students are guided along the journey towards formal written mathematics in English. The practice of code-switching is widely used to facilitate this process, for example, Muke (2012) shows how the use of code-borrowing[3] within an explanatory indigenous sentence could empower learners to understand and use the English terminology.

7.3.3 The Practice of Using Code-Switching to Engage in Mathematical Discourse

Code-switching in sociolinguistics refers to the practice of using two or more linguistic varieties in a single communicative sequence (Moschkovich, 2007). Moschkovich (2007) views code-switching as a complex language practice which, while using the official LoLT, allows for more extensive use of the main language. She disagrees with the view that it stems from a deficit model where the speakers use code-switching when they are unable to recall suitable phrases in the language being spoken and sees it as the mark of fluency in two languages. In fact, Clarkson (2007) suggests that switching between languages is a distinct advantage as it gives students access to alternate meanings and relationships.

Code-switching in mathematics classrooms can be described as the intuitive use of both English and the students' home language to facilitate mathematical understanding. Although teachers may sometimes be unaware of students' use of code-switching, either overtly by talking with peers, or privately in their own thinking (Clarkson, 1996), teachers in different parts of the world actively use this language practice to try and ensure better communication with students during mathematics sessions (e.g., in Papua New Guinea, Muke & Clarkson, 2011; in Iran, Parvanehnezhad & Clarkson, 2008; in South Africa, Setati & Adler, 2000).

When teachers do actively encourage code-switching, this normally ensures that the percentage of main language usage increases and that an additive model is employed, with the resultant transfer of mathematical concepts from one language to the other. Students are usually allowed to communicate about mathematics in the language of their choice. Code-switching is therefore front staged to facilitate mathematics and not back staged only to give instructions and for disciplinary purposes

[3] In this chapter, code-borrowing refers to the use of English terminology in an indigenous sentence, and has to be distinguished from transliteration, discussed under Section 7.3.3.

(Heller & Martin-Jones, 2001). This presupposes that code-switching is a technique that comes naturally to multilingual teachers. However, there are two inhibiting factors. First, many educators feel guilty if they code-switch as they feel they are depriving their students of an opportunity to acquire English (Setati, 2005). Second, most teachers have been educated in English and they find the indigenous terminology difficult. This results in teachers tending to use either the English terminology or transliterated words while communicating in an indigenous language. Transliterated borrowed words may not facilitate understanding of a concept. In transliteration, the English sounds are directly transferred into the indigenous language, inflected to suit the structure of the language, but without relation to the meaning of the concept, e.g., "Square" becomes "sekwere"*: (**sq** - sêk, **ua** - wê, **re** – rê)* in Setswana. In contrast the original Setswana word "khutlonnetsepa" can be linked to the definition of a square *(khutlo-angle, nne-four, tsepa-straight up).* A new transliterated borrowed word therefore still has to be fully explained in the indigenous language as was the practice noted by Muke (2012) about English borrowed words.

Code-switching is only usable in contexts where the class's language profile allows a strong regional language to facilitate better understanding of mathematics, with the prerequisite that the teacher is also fluent in that language. In some schools, this may be possible in one mathematics class and not in the next. In rural contexts in South Africa, code-switching is often feasible and necessary because of students' low English proficiency. MTEs should sensitize teachers to the importance of determining the language profile of each mathematics class in order to consciously decide on the best language practice for a specific group.

Code-switching as a practice developed informally, with teachers practicing it in different ways according to their perceptions of when students need their home language for better understanding of the mathematics. Although limited research has been done on best practices in code-switching (Muke, 2012), there are some directives that MTEs can discuss with teachers.

Language is important as a facilitating medium of understanding. It is crucial that teachers facilitate opportunities for students to improve fluency in both English and their home language, and more specifically also in the mathematics register of their home language in so far as it is developed. Teachers have to take cognizance of the threshold theory of bilingualism that proposes in general that "there may be a threshold level of linguistic competence" that bilingual students have to attain in *both* languages "*to* influence cognitive functioning" positively (Cummins & Swain, 1986). Clarkson and Galbraith (1992) in Papua New Guinea and Clarkson (1996) in Australia found evidence in a mathematical environment that supports Cummins' threshold theory. This implies that if EAL learners' main language is allowed to lapse, it will influence their cognition negatively. Gaoshubelwe (2011) noted in his analysis of mathematics lessons that some teachers mixed languages in a way that does not model the correct sentence construction/grammar of either the English or the home language's mathematical register. Teachers have to facilitate grammatically and mathematically correct language both in English and the home language.

Although teaching mathematical language is essential, it is important to balance visibility and invisibility of mathematics language teaching (Adler, 1999). The visibility of mathematics language teaching can be illustrated by the use of the morphology of the indigenous term to explain a concept, for example "adjacent angles": dikhutlomabapi, di- many, kuthlo - angle, mabapi—sit beside each other (Setswana). The mathematical explanation is interrupted to teach mathematical language or explain terminology. This should not be so extensive that it interrupts the argument of the mathematical reasoning. Invisible language teaching occurs where language teaching techniques are used that do not interfere with the flow of the mathematical reasoning. For example, a teacher may be modeling correct mathematical language through re-voicing when reformulating a student's sentence in correct mathematical language (Herbel-Eisenmann, Drake, & Cirillo, 2009; Setati & Adler, 2000), recasting when using a word in different sentences and contexts (Khisty, 1995), or through the use of synonyms for the same word. In multilingual settings, it is important for students to hear different English synonyms so they can recognize concepts as similar, because their "word sense" in English (Vygotsky, 1962) is not well developed and they do not automatically link synonyms to each other. Using a term first in the students' main language and then saying the correct term in English can be considered an extension of recasting.

Bilingual written text in explanations, assignments, and class tests can enhance understanding because students can oscillate between the languages to negotiate meaning and they are able to revisit the texts again (Vorster, 2008; Vorster & Zerwick, 2011). Available bilingual mathematics dictionaries can help to provide definitions in indigenous languages. Such dictionaries or modified bilingual terminology lists can be made available during tests (analogous to adding formula sheets).

Teachers should be cognizant of debates on terminology: there is a difference of opinion on whether teachers should use English terminology when they code-switch to the home language, use transliterated words, or use the correct indigenous terminology. The question is whether the bilingual use of mathematical terminology would add to better understanding or add to the workload of students. Countries have chosen different paths in standardizing terminology. While Tanzania purposefully chose terminology that conveys meaning, Malawi chose to use transliterated terminology (Kazima, 2008), and teachers in South Africa have to make their own choice. Another debate is whether new terminology should be coined for terms in cases where the indigenous terminology does not exist (Schäfer, 2010; see also Meaney, Trinick, & Fairhall, 2011 for a discussion on this topic and the successful development of *te reo Māori* mathematical terminology).

Examples of negative practices include "ritualization" where students chant answers as a group (Heller & Martin-Jones, 2001, p. 13), providing "safe time" for students who cannot express themselves, and circumvention of language. Teachers sometimes use only one-word instructions, for example *solve*, *factorize*, etc., or ask mainly procedural or algorithmic problems to avoid language issues. These practices do not help students build mathematical literacy, which has become important in the current constructivist teaching and learning environment.

7.4 Incorporating Mathematics Language and Literacy into the Teacher Preparation Program

With the goal for cooperative learning and more discursive practices within mathematics classrooms, the need to communicate mathematically and to comprehend mathematics language (both verbal and written words and symbols) becomes essential. Thus, within teacher education programs, MTEs need to help teachers understand the influence of language in supporting students' ability to interpret information conveyed and communicated in the mathematics class. Once teachers have been sensitized to the issues and language practices identified in the preceding sections, many teachers might question how they can engage students in these literacy practices while still teaching "all the content they need to cover [as mandated by state or national curriculums]." Thus, our task as MTEs is to help them understand that "language is a tool, whereas discourse is an activity in which the tool is used or mediates" and that they need to "embrace the complex linguistic nature of mathematical activity" (Gutiérrez, Sengupta-Irving, & Dieckmann, 2010, p. 34). Put simply this is a way of teaching, not an extra topic that is to be added to the amount of content that is to be taught.

A challenge for MTEs is how to foster the knowledge and skills of prospective teachers regarding the effective teaching–learning of mathematics in multilingual classrooms (Graham & Phelps, 2003). Teachers need multiple opportunities to consider how to incorporate the development of mathematics language and literacy skills as part of their regular curriculum. It is one thing to provide teachers with information about language features that need to be considered and a range of practices to address them and to give teachers opportunities to experience these practices in their own learning. It is another thing to have teachers plan to implement these practices into their classroom in a way that becomes an integral part of their teaching and not considered a supplementary activity that can be ignored. In this section, we share strategies we have used in our teacher education programs to help teachers begin to consider implementing literacy into their own classrooms. MTEs can highlight and engage teachers in discussions about different instructional practices that can be used to emphasize language and concept knowledge development. Teachers can then be given opportunities to integrate these practices in lessons and practice implementing them with each other or with groups of linguistically diverse students in small group or whole class settings as part of practical teaching experiences in schools.

7.4.1 Developing Language Modules to Integrate into Methods Courses

MTEs might create modules dedicated to mathematical language and language practices in multilingual classrooms. Such modules should include experiential learning where dialogic practices, including exploratory talk and different mathematical language teaching aids and techniques, can be applied and practiced.

Alternatively, MTEs might choose one mathematical topic for discussion in the course and model how explicitly teaching the language of mathematics could be incorporated in the planning and teaching of that topic. Issues around teaching mathematical language would then be addressed during this time.

For instance, one module might focus on helping teachers learn to engage students in mathematics discourse as a means to address the entire mathematics literacy spectrum. By making direct connections to students' lived experiences, it might be possible to connect academic language to social language (e.g., an intersection of two roads can help provide meaning for the intersection of two lines). Through such connections, students can make meaning using insights from their social or home experiences. Another approach is to build common experiences as part of the mathematics class by engaging in brief conversations about unfamiliar contexts found in mathematics passages or word problems. When students engage in such discussion, teachers are ensuring that all students, regardless of class or social experiences, are interpreting the information in the same way.

A second module might focus on incorporating the use of visual representations and graphic organizers into mathematics instruction on a regular basis. The module could help teachers understand how the use of visual representations and graphic organizers can scaffold EAL students' learning of English, helping them make connections between and among concepts being studied. The use of visual representations to convey mathematics and English ideas allows students to examine similarities and differences between how mathematics language (words, symbols, and diagrams) is used to represent concepts and to explore different ways to convey mathematics ideas. For example, students can be encouraged to draw comparisons and contrasts between concepts (e.g., prisms and pyramids, rhombus and square) so that they see similarities and differences in order to develop a thorough understanding. Such discussions can be supported by the use of graphic organizers, such as Venn diagrams or concept maps, so students can visually see the connections and attend to ways to communicate these similarities or differences using the mathematics register. When such visual representations are used, teachers can include information to help students express mathematics ideas. For example, in addition to writing the symbol >, a teacher might say it, and then write the spoken language, such as "is greater than".

A third module could focus on helping teachers learn to adapt the use of regular English reading and language strategies to mathematics. For instance, many mathematics textbooks have headings within a lesson; students can learn to read the heading, convert it to a question, and then attempt to answer the question as they read the lesson (e.g., heading: Solving with a Table and a Graph; question: How do you solve an equation with a table? How is solving an equation with a table like solving with a graph?). Thus students learn how to use the textbook to support their own learning.

In addition, teachers can encourage students to develop personal dictionaries of mathematical terms, with definitions in students' own words, even in their home language, with diagrams and/or examples as appropriate. Tied to dictionaries can be the use of etymology and morphology. Etymology focuses on the origin of a word or symbol, e.g., the Greek symbol Σ for the capital letter S (used as the symbol for *sum* in series). Morphology is how a word is put together, e.g., trilateral = three sides. Morphology can be used with prefixes and suffixes to help students make sense of new words; if students know *tri* means three, they have a start on understanding *triangular*.

Another possible avenue for teachers is to set language aims for each lesson where applicable. This includes identifying any of the potential difficulties mentioned in Sect. 7.2.2; using different techniques to explain new terminology or linking it to the home language of the students; or practising correct grammar and sentence construction, for example, to formulate conjectures where concepts, relationships, and conditions have to be expressed.

7.4.2 Simultaneous Interpreting Between English and an Indigenous Language as a Tool in Teacher Education

In cases where teachers have been educated in English but will have to teach or code-switch to an indigenous language when teaching, as is the case in Malawi (Chitera, 2011) and South Africa, it can be advantageous if MTEs make use of simultaneous interpreting instead of teaching only through the medium of English. If the technique is used where the teachers listen to the interpreter in the indigenous language, using the headphone in only one ear while also listening to the lecturer, the teachers hear the correct mathematical terminology, as well as formulation of expressions in both English and the indigenous language. Simultaneous interpreting will benefit them when they themselves have to alternate between languages when code-switching, because they become better acquainted with the mathematics register in both English and the indigenous language. Furthermore, teachers become more aware of the necessity to teach mathematical language, both in the indigenous language and in English. They gain understanding of their students' problems to cope with the English mathematical register and to understand concepts when English is the LoLT. Furthermore, if teachers' study guides or workbooks are also bilingual, their expertise in writing mathematical language in both English and the indigenous language text is enhanced, and they are empowered to use written text in the indigenous language alongside English notes when teaching (Vorster & Zerwick, 2011).

7.4.3 Using Mathematics Educator Reflective Communities to Collaboratively Plan to Integrate Language in Mathematics Instruction

When teachers are empowered to determine for themselves those language practices they are able to integrate into their mathematics classroom, there is a greater likelihood such practices will be translated from planning into actual implementation. Thus, groups of teachers might work together to determine how they would incorporate mathematical literacy or other dialogic practices into the curriculum for a mathematics course of their choice. The goal is for teachers to consider how they will address the mathematics language issues (i.e., vocabulary, symbols) for a specific instructional segment, engage students in all aspects of the literacy spectrum (reading, writing, interpreting, speaking, and listening), determine the types of questions to use, and assess students so that insights about their mathematics language development can be ascertained. Engaging in such a project has the benefit of allowing teachers to consider instructional approaches that support mathematics and English language development without sacrificing a focus on rigorous content.

Rather than plan lessons to facilitate the knowledge and skills to teach in multilingual mathematics classrooms for an entire curriculum, an alternative model is *adapted lesson study* (see Fig. 7.2). Lesson study is a cyclical process used in Japan to professionally develop and focus the effectiveness of practicing teachers' teaching–learning experiences around students' learning (Lewis, Perry, & Murata, 2006). Internationally, teacher educators also use an adapted form of lesson study (Mathematics Educator Reflective Communities) for fostering/developing different aspects of mathematics education in their preservice mathematics teachers' classrooms (Fernandez, 2010; Murata & Pothen, 2011; Van der Walt, 2012). Lesson study has the potential to facilitate the knowledge, skills, and awareness of what multilingual classrooms require from teachers and to implement the various practices suggested in this chapter.

During the planning phase of the lesson (or unit of lessons), a group of teachers work collaboratively and cooperatively, taking into account the aims, including mathematical language aims, the school has set for multilingual students, focusing on multilingual students' learning and conceptual understanding. The lesson study group anticipates multilingual students' responses and reactions to the planned activities, problems, and exercises, including the activities, problems and exercises planned to accommodate and support students' language needs.

During the teaching of a lesson by one member of the group in one classroom of the school, the rest of the study group observes the lesson and collects data regarding students' thinking, understanding, and learning, with the aim to revise and refine

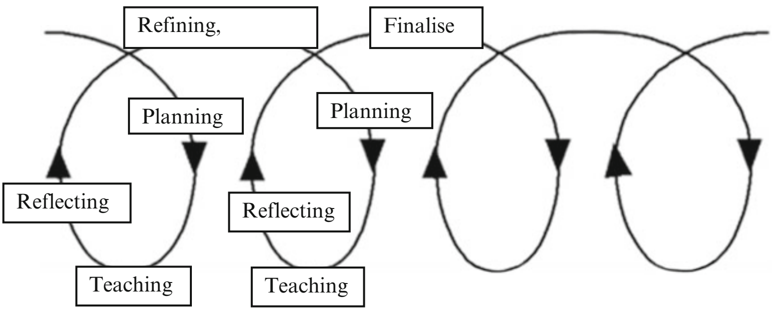

Fig. 7.2 Adapted lesson study (adapted from Van den Akker, Gravemeijer, McKenney, & Nieveen, 2006)

the lesson. The teachers facilitate discussions, also in multilingual classes to ensure students' engagement (Berliner, 2001), while the teacher models his/her own thinking to improve understanding and poses questions connecting students' developing mathematical ideas with mathematical language and symbols (Goos, 2004).

Lastly, the lesson study group comes together to reflect on and discuss the effectiveness of the lesson for students, using the data they collected and the experiences of the teacher who presented the lesson. Adaptations can be made to the lesson (or other lessons in the unit), and if necessary, the lesson can be taught again by another group member and observed again by the rest of the group. The lesson study cycle continues if necessary. To empower teachers to use Mathematics Educator Reflective Communities, MTEs can use this method in their course, for example with a group of teachers planning for a session of practical teaching.

7.5 Conclusion

Throughout this chapter, we have highlighted features of mathematical language that MTEs need to ensure their teachers know and we have shared approaches we have used in our teacher education programs to prepare mathematics teachers for addressing multilingual classrooms. However, we have little empirical data related to the effectiveness of these strategies, either from the perspective of the teacher and his/her willingness to implement the strategies in classrooms or from the perspective of the extent to which they help elementary and secondary students be successful with mathematics. Thus, there is a need to engage in studies that follow teachers from preparation programs in which practices for language diversity have been a focus into the field, in order to understand what practices are easily implementable and what effect those practices have. If different teacher preparation programs engage in different practices, we might begin to develop a body of research that suggests which practices work best with which teachers for which students in which contexts. The work described in this paper is appropriate for delivery by MTEs, not generalists, so that mathematics teachers have explicit instruction in applying these practices to support mathematics instruction. Thus, we advocate the need for MTEs to become more engaged with language issues as they prepare to support their teachers.

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