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Original Article

On completeness of interactive student networks ☆

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ABSTRACT

The completeness of interactive student networks are investigated in this paper, which can be used to improve middle school students' independent environment for their study outside class. This paper takes covering approximation spaces as mathematical models of interactive student networks and characterizes completeness of interactive student networks by connectivity of covering approximation spaces. Further, this paper gives a simpler and practical method to check completeness of interactive student networks. Consequently, a way to communication among students is explored and applied in education.

1. Introduction

In the world, middle school students' knowledge acceptance may be divided into three stages: preview before class, classroom learning and review after class. We observe the three stages, the problems arise more frequently in preview stage or review stage than in class learning. The most part of the problems can be solved in classes, but if they are solved outside class in previewing or reviewing, the effectiveness will be more beneficial. However, due to students' independent environments outside classes, it is difficult for students to solve these problems by themselves. In the past years, some more recent researches on using "social methods" in education has aroused more and more scholar's wide interests and many interesting results were obtained [1,5,14,19]. Having gained some enlightenments from these researches, we carry out the research for the following issue supported by a major project of Chinese postdoctoral scientific foundation.

1.1. Issue

How can we make use of modern educational technology to transform middle school students' environment in preview and review such that students can contact each other.

As an interesting exploration on the above issue, Z. Xia, the first author of this paper, designed a interactive student network for a group of twelve students in Suzhou No.3 High School (see the following 1.4). Students, who use the interactive student network, can send and receive information each other. This network created a new environment for these students and would promote the interaction among these students. We state the concept of interactive student network as follows, which is similar to wireless network system in [9].

1.2. The interactive student network $(V;\mathcal{B})$

A interactive student network is a pair $(V;\mathcal{B})$ satisfying the following conditions, where V is a collection of some information points and $\mathcal{B}$ is a collection of some base stations.

(1) Every information point in V can send information to some base stations in $\mathcal{B}$ and receive information from these base stations in $\mathcal{B}$.
(2) Every base station in $\mathcal{B}$ can send information to some information points in V and receive information from these information points in V.

1.3. Completeness of interactive student networks

Let $(V;\mathcal{B})$ be a interactive student network.

(1) An information point u in V and an base stations B in $\mathcal{B}$ is called to have a contact if u and B can send and receive information each other.
(1) Two information point u and v in V is called to have a contact if there are some information points $u_1, u_2, \ldots, u_n$ in V and

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some base stations $B_1, B_2, \ldots, B_{n-1} \in \mathcal{B}$ such that for each $i = 1, 2, \ldots, n-1$, not only u_i and B_i have a contact but also u_{i+1} and B_i have a contact, where $u_1 = u$ and $u_n = v$.

(2) $(V; \mathcal{B})$ is called complete if u and v have a contact for all pair u, v of information points in V.

Thus, any pair of students can send and receive information each other by using a complete interactive student network. In fact, let two students P_1 and P_2 use information points u and v in V respectively, where $(V; \mathcal{B})$ is a complete interactive student network. Then there are some information points $u_1, u_2, \ldots, u_n \in U$ and some base stations $B_1, B_2, \ldots, B_{n-1} \in \mathcal{B}$ such that for each $i = 1, 2, \ldots, n-1$, not only u_i and B_i have a contact but also u_{i+1} and B_i have a contact, where $u_1 = u$ and $u_n = v$. Since u_1 and B_1 have a contact, with the help of $u = u_1, P_1$ can send information to B_1. Also, since u_2 and B_1 have a contact, B_1 can send information to u_2. Thus, P_1 can send information to u_2. Successively, P_1 can send information to $u_n = v$. Moreover, with the help of $v = u_n, P_2$ can receive information sent by P_1.

1.4. Xia's interactive student network

Now we state Xia's interactive student network $(V; \mathcal{B})$, which is described in the following table (Table 1).

Here, $V = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}\}$ is the collection of twelve information points, $\mathcal{B} = \{B_1, B_2, B_3, B_4, B_5, B_6\}$ is the collection of six base stations, and the number, which lies in the cross of the row labeled by u $(u = u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12})$ and the column labeled by B $(B = B_1, B_2, B_3, B_4, B_5, B_6)$, is 1 or 0 by u and B can send and receive information each other or can not.

By the method of enumeration, it is not difficult to check that Xia's interactive student network $(V; \mathcal{B})$ is complete. Thus, any pair of students in Suzhou No.3 High School, who use information points in $(V; \mathcal{B})$, can send and receive information each other. As a further work, Xia tried to establish interactive student networks in a larger scale. However, it is necessary to design more information points and base stations in such an interactive student networks, which make it complicated and difficult to check the completeness of interactive student networks by the method of enumeration. This brings the following question.

1.5. The open question

For a interactive student network, how to check its completeness? More precisely, are there a simpler and practical method to check completeness of interactive student networks?.

To investigate the above question, it is necessary to analyze data collected from interactive student networks. In the research of modern sciences, in order to extract useful information hidden in data, many methods in addition to classical logic and classical mathematics have been proposed. Rough set theory, which was proposed by Pawlak [15], plays an important role in applications of these methods. Their usefulness has been demonstrated by many successful applications in information sciences and computer sciences [2,4,12,18,21,22]. In particular, rough set theory is also applied in education widely [3,7,17,20]. In the past years, with development of information sciences and computer sciences, applications of rough-set theory have been extended from Pawlak approximation spaces to covering approximation spaces in many research fields [10,11,13,16,23,24].

1.6. The purpose of this paper

This paper launches an investigation around Question 1.5 by covering approximation spaces. The purpose of this paper is to give a simpler and practical method for checking completeness of interactive student networks.

2. Covering approximation spaces

In this section, we state basic concepts and theories of covering approximation spaces.

Definition 2.1 24. Let U, the universe of discourse, be a finite set and $\mathcal{K}$ be a family of nonempty subsets of U.

(1) $\mathcal{K}$ is called a cover of U if $\bigcup\{K : K \in \mathcal{K}\} = U$.
(2) The pair $(U; \mathcal{K})$ is called a covering approximation space if $\mathcal{K}$ is a cover of U.

Remark 2.2. Let $(U; \mathcal{K})$ be a covering approximation space. If $\emptyset \in \mathcal{K}$ and $\mathcal{K}$ is closed with respect to both the union and the finite intersection of elements of $\mathcal{K}$, then $(U; \mathcal{K})$ is a topological space [6].

Now we state connectivity of covering approximation spaces (see [8,9], for example) and give its discussion again.

Definition 2.3. Let $(U; \mathcal{K})$ be a covering approximation space and $x, y \in U$.

(1) A subfamily $\{K_1, K_2, \ldots, K_n\}$ of $\mathcal{K}$ is called a chain between x and y if $x \in K_1, y \in K_n$ and $K_i \bigcap K_{i+1} \neq \emptyset$ for each $i = 1, 2, \ldots, n-1$.
(2) x is called to be chain connected to y if there is a chain between x and y.

Remark 2.4. Let $(U; \mathcal{K})$ be a covering approximation space. Then the relation for "chain connected" is an equivalent relation, i.e., the following hold for all $x, y, z \in U$.

(1) x is chain connected to x.
(2) x is chain connected to $y \Rightarrow y$ is chain connected to x.
(3) x is chain connected to y and y is chain connected to $z \Rightarrow x$ is chain connected to z.

Proof. Obviously, (1) and (2) hold. Let x be chain connected to y, and y be chain connected to z. Then there are $K_1, K_2, \ldots, K_n \in \mathcal{K}$ such that $x \in K_1, y \in K_n, K_i \bigcap K_{i+1} \neq \emptyset$ for each $i = 1, 2, \ldots, n-1$; and there are $K_{n+1}, K_{n+2}, \ldots, K_{n+m} \in \mathcal{K}$ such that $y \in K_{n+1}, z \in K_{n+m}, K_{n+i} \bigcap K_{n+i+1} \neq \emptyset$ for each $i = 1, 2, \ldots, m-1$.

Table 1
Xia's Interactive Student Network $(V; \mathcal{B})$.

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}	u_{11}	u_{12}
B_1	1	0	0	0	1	1	1	1	0	0	1	0
B_2	0	1	1	0	1	0	0	1	0	1	0	1
B_3	1	1	0	0	0	0	1	0	1	0	1	1
B_4	1	0	1	1	0	0	0	0	1	1	0	1
B_5	0	0	1	1	0	1	1	1	1	0	0	0
B_6	0	1	0	1	1	1	0	0	0	1	1	0

Consequently, there are $K_1, K_2, \ldots, K_n, K_{n+1}, K_{n+2}, \ldots, K_{n+m} \in \mathcal{K}$ such that $x \in K_1, z \in K_{n+m}$ and $K_i \bigcap K_{i+1} \neq \emptyset$ for each $i = 1, 2, \ldots, n+m-1$. This proves that x is chain connected to z. So (3) holds. □

Definition 2.5. A covering approximation space $(U;\mathcal{K})$ is called connected if for each pair $x, y \in U$, x is chain connected to y, i.e., there is a chain between x and y.

As a classical result in topology, a topological space $(X, \mathcal{T})$ is connected if and only if $(X, \mathcal{T})$ has no non-empty clopen proper subset. Note that there is not concept for clopen subset for covering approximation spaces. So we need to introduced a class of new subsets of covering approximation spaces, which has relation with the following covering upper approximation operator.

Definition 2.6. Let $(U;\mathcal{K})$ be a covering approximation space. For each $X \subseteq U$, Put

$$SH(X) = \bigcup\{K : K \in \mathcal{K} \text{ and } K \bigcap X \neq \emptyset\}.$$

Then $SH : 2^U \longrightarrow 2^U$ is called a covering upper approximation operator, and $SH(X)$ is called a covering upper approximation of X.

The above covering upper approximation operator SH is called the second type of covering upper approximation operation in [24].

Definition 2.7. Let $(U;\mathcal{K})$ be a covering approximation space and $X \subseteq U$. X is called an SH-subset of $(U;\mathcal{K})$ if $SH(X) = X$.

The following theorem is an important result of this paper, which characterizes connectivity of covering approximation spaces by their SH-subsets.

Lemma 2.8. *Let $(U;\mathcal{K})$ be a covering approximation space and $x \in U$. Put $X = \{u \in U : x \text{ is chain connected to } u\}$. If $X = U$, then $(U;\mathcal{K})$ is connected.*

Proof. Assume that $X = U$. Let $u, v \in U = X$. Then x is chain connected to both u and v. By Remark 2.4(2), (3), u is chain connected to v. So $(U;\mathcal{K})$ is connected. □

Theorem 2.9. *Let $(U;\mathcal{K})$ be a covering approximation space. Then the following are equivalent.*

(1) $(U;\mathcal{K})$ is connected.
(2) $(U;\mathcal{K})$ has no non-empty proper SH-subset.

Proof.

(1) ⇒ (2). Suppose that $(U;\mathcal{K})$ is connected. Let X be a non-empty SH-subset of $(U;\mathcal{K})$. Then $SH(X) = X \neq \emptyset$. We only need to prove that X is not a proper subset of U, i.e. $X = U$. Let $x \in U$. Pick $y \in X$, then y is chain connected to x, i.e., there are $K_1, K_2, \ldots, K_n \in \mathcal{K}$ such that $y \in K_1, x \in K_n$ and $K_i \bigcap K_{i+1} \neq \emptyset$ for each $i = 1, 2, \ldots, n-1$. Since $y \in K_1 \bigcap X \neq \emptyset, K_1 \subseteq SH(X) = X$. Furthermore, $K_2 \bigcap X \supset K_2 \bigcap K_1 \neq \emptyset$, so $K_2 \subseteq SH(X) = X$. In the same way, we can obtain that $K_n \subseteq SH(X) = X$. Thus, $x \in K_n \subseteq X$. This proves that $U \subseteq X$. It follows that $X = U$.

(2) ⇒ (1). Suppose that $(U;\mathcal{K})$ has no non-empty proper SH-subset. Let $x \in U$. Put $X = \{u \in U : x \text{ is chain connected to } u\}$. Then $x \in X \neq \emptyset$ by Remark 2.4(1). Let $y \in SH(X)$. Then there is $K \in \mathcal{K}$ such that $y \in K$ and $K \bigcap X \neq \emptyset$. Pick $z \in K \bigcap X$. Then x is chain connected to z, and z is chain connected to y. So x is chain connected to y by Remark 2.4(3). It follows that $y \in X$. This proves that $SH(X) \subseteq X$. On the other hand, It is clear that $X \subseteq SH(X)$, and hence $SH(X) = X$. Thus, X is a SH-subset of $(U;\mathcal{K})$, it follows that $X = U$. By Lemma 2.8, $(U;\mathcal{K})$ is connected. □

3. The method of checking completeness of interactive student networks

In this section, we establish a relation between completeness of interactive student networks and connectivity of covering approximation spaces. By this relation we give a method to check completeness of interactive student networks.

Proposition 3.1. *Let $(V;\mathcal{B})$ be a interactive student network. For every base station B in $\mathcal{B}$, let K_B be a set of some information points in V such that $u \in V$ is an information point in K_B if and only if u and B have a contact. Put $U = V$ and $\mathcal{K} = \{K_B : B \in \mathcal{B}\}$. Then $(U;\mathcal{K})$ is a covering approximation space.*

Proof. It suffices to prove that $\mathcal{K}$ is a cover of U. Let $u \in U$, i.e., u is an information point in V. By 1.2(1), there is a base station B in $\mathcal{B}$ such that u and B have a contact. So u is an information point in K_B, i.e., $u \in K_B$. This proves that $\mathcal{K}$ is a cover of U. □

By the above proposition, we can take covering approximation spaces as mathematical models of interactive student networks. Here, the covering approximation space $(U;\mathcal{K})$ is called to be induced by the interactive student network $(V;\mathcal{B})$.

Lemma 3.2. *Let $(V;\mathcal{B})$ be a interactive student network, and let $(U;\mathcal{K})$ be a covering approximation space induced by $(V;\mathcal{B})$. Then the following are equivalent for all $u, v \in V = U$.*

(1) *u and v have a contact in $(V;\mathcal{B})$.*
(2) *There is a chain between u and v in $(U;\mathcal{K})$.*

Proof.

(1) ⇒ (2): Let u and v have a contact. Then there are some information points $u_1, u_2, \ldots, u_n \in U$ and some base stations $B_1, B_2, \ldots, B_{n-1} \in \mathcal{B}$ such that for each $i = 1, 2, \ldots, n-1$, not only u_i and B_i have a contact but also u_{i+1} and B_i have a contact, where $u_1 = u$ and $u_n = v$. Note that $(U;\mathcal{K})$ is induced by $(V;\mathcal{B})$. For each $i = 1, 2, \ldots, n-1, K_{B_i} \in \mathcal{K}$, we put $K_i = K_{B_i}$. Then $K_1, K_2, \ldots, K_{n-1} \in \mathcal{K}$, and for each $i = 1, 2, \ldots, n-1$, $u_i, u_{i+1} \in K_i$. It follows that $u = u_1 \in K_1$, $v = u_n \in K_{n-1}$, and for each $i = 1, 2, \ldots, n-2$, $u_{i+1} \in K_i \bigcap K_{i+1} \neq \emptyset$. This shows that $K_1, K_2, \ldots, K_{n-1}$ is a chain between u and v.

(2) ⇒ (1): Let $K_1, K_2, \ldots, K_n$ is a chain between u and v, i.e., $u \in K_1$, $v \in K_n$ and $K_i \bigcap K_{i+1} \neq \emptyset$ for each $i = 1, 2, \ldots, n-1$. Put $u_1 = u$, $u_{n+1} = v$, and for each $i = 1, 2, \ldots, n-1$, choose $u_{i+1} \in K_i \bigcap K_{i+1}$. It follows that $u_i, u_{i+1} \in K_i$ for each $i = 1, 2, \ldots, n$. Since $(U;\mathcal{K})$ is induced by $(V;\mathcal{B})$, there are base stations $B_1, B_2, \ldots, B_n \in \mathcal{B}$ such that $K_i = K_{B_i}$ for each $i = 1, 2, \ldots, n$. Thus, for each $i = 1, 2, \ldots, n$, $u_i, u_{i+1} \in K_{B_i}$, i.e., not only u_i and B_i have a contact but also u_{i+1} and B_i have a contact. This proves that u and v have a contact. □

By Lemma 3.2, we obtain the following theorem immediately, which shows that the completeness of interactive student networks and the connectivity of covering approximation spaces are equivalent.

Theorem 3.3. *Let $(V;\mathcal{B})$ be a interactive student network, and let $(U;\mathcal{K})$ be a covering approximation space induced by $(V;\mathcal{B})$. Then the following are equivalent.*

(1) $(V;\mathcal{B})$ is complete.
(2) $(U;\mathcal{K})$ is connected.

Now we give a simpler and practical method for checking completeness of interactive student networks.

Method 3.4. Let $(V;\mathcal{B})$ be a interactive student network. The method for checking completeness of $(V;\mathcal{B})$ consists of the following five steps.

(1) By Proposition 3.1, we convert $(V;\mathcal{B})$ to a covering approximation space $(U,\mathcal{K})$, where $(U,\mathcal{K})$ is induced by $(V;\mathcal{B})$.
(2) By Definition 2.6, we endow the covering upper approximation operator SH on $(U,\mathcal{K})$.
(3) By Definition 2.7, we obtain all SH-subsets of $(U,\mathcal{K})$ by a simple algorithm.
(4) By the above (3) and Theorem 2.9, it is known that whether $(U,\mathcal{K})$ is connected.
(5) By the above (4) and Theorem 3.3, it is known that whether $(V,\mathcal{B})$ is complete.

4. Some applications

In this section, we give some applications to show that our approach does work. This work is to assess completeness of interactive student networks.

4.1. Check of Xia's interactive student network

(1) Description of Xia's interactive student network $(V;\mathcal{B})$ (see 1.4 in Section 1).
(1.1) $V=\{u_1,u_2,u_3,u_4,u_5,u_6,u_7,u_8,u_9,u_{10},u_{11},u_{12}\}$ is the collection of twelve information points.
(1.2) $\mathcal{B}=\{B_1,B_2,B_3,B_4,B_5,B_6\}$ is the collection of six base stations.
(1.3) Contacts between information points and base stations.
(1.3.1) B_1 and u_i have a contact for $i\in\{1,5,6,7,8,11\}$.
(1.3.2) B_2 and u_i have a contact for $i\in\{2,3,5,8,10,12\}$.
(1.3.3) B_3 and u_i have a contact for $i\in\{1,2,7,9,11,12\}$.
(1.3.4) B_4 and u_i have a contact for $i\in\{1,3,4,9,10,12\}$.
(1.3.5) B_5 and u_i have a contact for $i\in\{3,4,6,7,8,9\}$.
(1.3.6) B_6 and u_i have a contact for $i\in\{2,4,5,6,10,11\}$.
(2) The covering approximation space $(U;\mathcal{K})$ induced by $(V;\mathcal{B})$.
(2.1) Put $U=V$.
(2.2) For each $i\in\{1,2,3,4,5,6\}$, let K_i be a set of some information points in V such that $u\in V$ is an information point in K_i if and only if u and B_i have a contact.
(2.2.1) $K_1=\{u_1,u_5,u_6,u_7,u_8,u_{11}\}$.
(2.2.2) $K_2=\{u_2,u_3,u_5,u_8,u_{10},u_{12}\}$.
(2.2.3) $K_3=\{u_1,u_2,u_7,u_9,u_{11},u_{12}\}$.
(2.2.4) $K_4=\{u_1,u_3,u_4,u_9,u_{10},u_{12}\}$.
(2.2.5) $K_5=\{u_3,u_4,u_6,u_7,u_8,u_9\}$.
(2.2.6) $K_6=\{u_2,u_4,u_5,u_6,u_{10},u_{11}\}$.
(2.3) Put $\mathcal{K}=\{K_1,K_2,K_3,K_4,K_5,K_6\}$.
(2.4) It is clear that $(U;\mathcal{K})$ is a covering approximation space induced by $(V;\mathcal{B})$.
(3) The connectivity of $(U;\mathcal{K})$.
By a simple algorithm, it can be obtained that if X is a non-empty SH-subset of $(U;\mathcal{K})$, then $X=U$. In fact, let X be an SH-subset of $(U;\mathcal{K})$ and $X\neq\emptyset$. Then there is $u_i\in X$ for some $i\in\{1,2,\dots,12\}$. If $u_1\in X$, then $K_i\bigcap X\neq\emptyset$ for $i=1,3,4$. Thus, $X=SH(X)=\bigcup\{K:K\in\mathcal{K}\text{ and }K\bigcap X\neq\emptyset\}\supseteq K_1\bigcup K_3\bigcup K_4=\{u_1,u_2,u_3,u_4,u_5,u_6,u_7,u_8,u_9,u_{10},u_{11},u_{12}\}=U$. It follows that $X=U$. By the same method, we can obtain that if $u_i\in X$ for $i\in\{2,3,\dots,12\}$, then $X=U$. This shows that $(U;\mathcal{K})$ has no non-empty proper SH-subset. It follows that $(U;\mathcal{K})$ is connected from Theorem 2.9.
(4) The completeness of $(V;\mathcal{B})$.
By Theorem 3.3, $(V;\mathcal{B})$ is complete.

4.2. A simulative interactive student network

As a re-examine of our method, the following gives a simulative interactive student network.

(1) Description of the simulative interactive student network $(V;\mathcal{B})$.
This network is described as follows.
(1.1) $V=\{u_1,u_2,u_3,u_4,u_5,u_6,u_7,u_8,u_9\}$ is the collection of nine information points.
(1.2) $\mathcal{B}=\{B_1,B_2,B_3,B_4,B_5,B_6\}$ is the collection of six base stations.
(1.3) Contacts between information points and base stations.
(1.3.1) B_1 and u_i have a contact for $i\in\{1,3,8,9\}$.
(1.3.2) B_2 and u_i have a contact for $i\in\{1,4,9\}$.
(1.3.3) B_3 and u_i have a contact for $i\in\{3,4,8\}$.
(1.3.4) B_4 and u_i have a contact for $i\in\{2,6,7\}$.
(1.3.5) B_5 and u_i have a contact for $i\in\{2,5,7\}$.
(1.3.6) B_6 and u_i have a contact for $i\in\{5,6,7\}$.

Similar to Xia's interactive student network, we can describe the simulative interactive student network by the following table (Table 2).

(2) The covering approximation space $(U;\mathcal{K})$ induced by $(V;\mathcal{B})$.
(2.1) Put $U=V$.
(2.2) For each $i\in\{1,2,3,4,5,6\}$, let K_i be a set of some information points in V such that $u\in V$ is an information point in K_i if and only if u and B_i have a contact.
(2.2.1) $K_1=\{u_1,u_3,u_8,u_9\}$.

Table 2
A simulative interactive student network $(V;\mathcal{B})$.

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9
B_1	1	0	1	0	0	0	0	1	1
B_2	1	0	0	1	0	0	0	0	1
B_3	0	0	1	1	0	0	0	1	0
B_4	0	1	0	0	0	1	1	0	0
B_5	0	1	0	0	1	0	1	0	0
B_6	0	0	0	0	1	1	1	0	0

(2.2.2) $K_2 = \{u_1, u_4, u_9\}$.
(2.2.3) $K_3 = \{u_3, u_4, u_8\}$.
(2.2.4) $K_4 = \{u_2, u_6, u_7\}$.
(2.2.5) $K_5 = \{u_2, u_5, u_7\}$.
(2.2.6) $K_6 = \{u_5, u_6, u_7\}$.
(2.3) Put $\mathcal{K} = \{K_1, K_2, K_3, K_4, K_5, K_6\}$.
(2.4) It is clear that $(U; \mathcal{K})$ is a covering approximation space induced by $(V; \mathcal{B})$.
(3) SH-subsets of the covering approximation space $(U, \mathcal{K})$.

By a simple algorithm, we obtain all SH-subsets of $(U, \mathcal{K})$. They are $\emptyset, U, \{1, 3, 4, 8, 9\}$ and $\{2, 5, 6, 7\}$.

(4) The connectivity of $(U; \mathcal{K})$.
By the above (3), $(U; \mathcal{K})$ has two non-empty proper SH-subset. So $(U; \mathcal{K})$ is not connected from Theorem 2.9.
(5) The completeness of $(V; \mathcal{B})$.
By Theorem 3.3, $(V; \mathcal{B})$ is not complete.

Remark 4.1. In Xia's interactive student network and the simulative interactive student network, fewer information points and base stations are designed. However, if we establish interactive student network in large-scale, then voluminous information points and base stations should be designed in general. It is noteworthy that the method for checking their completeness is the same, which can be achieved by some simple algorithms under the help of computer technology.

5. Conclusion

Conclusion 5.1. The main results of this paper are to investigate completeness of interactive student networks. Here, interactive student networks are used to transform middle school students' independent environment in preview and review such that students can contact each other by using the complete interactive student networks. In order to investigate completeness of interactive student networks by mathematical methods, this paper takes covering approximation spaces as mathematical models of interactive student networks and establish a relation between completeness of interactive student networks and connectivity of covering approximation spaces. Further, this paper introduces SH-subsets of covering approximation spaces to characterize their connectivity. By converting completeness of interactive student network to connectivity of covering approximation spaces, this paper gives a simpler and practical method for checking completeness of interactive student network. In the end of this paper, two examples are given to demonstrate applications of this method.

Remark 5.2. This paper focuses on check of completeness of interactive student networks, without dealing with the method of establishing them. How do we establish a complete interactive student network? This is an interesting question, which deals with either mobile communication project or virtual organization framework. We will discuss this question in our future work.

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