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**ORIGINAL ARTICLE**

# Self-similarity and stationarity of increments in VBR video

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Traffic spread;
Stationary increments

**Abstract** As self-similarity trail is being detected in many types of traffic, and the Markovian models failing to represent some statistical behaviors, the tools being used for traffic testing are still complex. Our study here is related to VBR video. Its self-similarity and long-range dependence aspects will be tested using a wavelet-based tool. As the test tool requires stationarity of the increments of the traces, a novel testing technique will be suggested for this aim. Then, the degree of self-similarity will be related to both the traces time scale and its statistical measures of spreading.


## 1. Introduction

The pervasive ubiquitous support of multimedia traffic over communication networks, including the Internet and wireless mobile networks, imposed very stringent requirements on the coding schemes used to generate these types of traffic and led to many developments in related areas. Consequently, very complex features and characteristics of the generated traffic. Thus, many advanced techniques in motion estimation, such as multiple reference frames, variable block sizes, and quarter pixel resolution have been implemented in the new H.264/MPEG-4 AVC standard. A very high computational complexity has been engendered, which led to many research works to optimize it. The use of the stationarity characteristic has been the key element in most of them. Using spatial homogeneity and temporal stationarity of video objects, a decision algorithm has been proposed in Wu and Al (2005), while a selective multiple reference frame motion estimation that uses the stationary property has been suggested in Tsui et al. (2010).

In many instances, however, stationarity is not necessary, only stationarity of increments is sufficient. For instance, the authors in Wendt and Abry (2007) propose a new bootstrap-based technique to differentiate between non-Gaussian finite variance self-similar processes with stationary increments and multi-fractal processes. The goal of our study here goes along these lines. We will propose a correlation-based test for the existence of stationarity of increments in video traffic characterized by some form of self-similarity.

Accurately modeling such traffic has become a crucial part of many research groups. Initially, it was concluded that the simple Markov Modulated Poisson Process (MMPP) models inadequate for the representation of such traffic (Garret and Willinger, 1994; Ryu and Elwalid, 1996; Pruthi et al., 1999) were far from adequate. Even relatively more complex models, that assumed short-term correlation, have failed as well. The repercussions of such findings on the performance and analysis of current data networks have been very drastic. For instance, it was reported very early (Park and Willinger, 2000), that the existence of self-similarity and ubiquitous heavy-tailed

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phenomena in networked systems will have a drastic effect on traffic modeling, queueing-based performance analysis, and traffic control.

In this paper, a tool proposed by Leland et al. (1994) will be used to test a couple of representative VBR video traces for self-similarity trail. Since the tool requires the stationarity of increments in the trace frame sequences, a very efficient tool has been proposed, implemented, and tested on the used traces. All results were conclusive and in harmony with the preset assumptions.

The rest of the paper is organized as follows. In Section 2, the characteristics of VBR video and the models that have been used will be given along with a description of the video traces to be used in this study. In Section 3, some conventional testing techniques will be presented, along with the wavelet-based technique to be used in this work. Then, in Section 4, some novel feasiblity tests will be developed to check the stationarity of the increments of the considered traffic sequences. In Section 5, the stationarity test will be first applied, followed by tests on the effects of the used number of vanishing moments in the test tool, and lastly, by the results obtained when applying the test tool to estimate the self-similarity trail in the video traces, along with a look at any possible correlations with the spreading parameters of the traces. The conclusions will be included in Section 6.

## 2. VBR video traffic

### 2.1. Characteristics

The main factors affecting the complexity of multimedia traffic may be classified into three main categories:

- *Inherent aspects*: This takes care of the fact that the movements that may be recorded, for instance in a video sequence, have in general some common aspects that may be translated into some sort of short and long-term correlation between successive frames.
- *Compression and coding*: The techniques being used take advantage of the previously listed inherent aspects. They take advantage of both the spatial and temporal correlations in the scene and the frames.
- *Traffic shapers*: In some networks, some quality of service (QoS) is provided. The user has to abide by certain traffic constraints which will be enforced through special traffic shapers. Although, this will not add to the correlation aspects of the traffic, it will add to its complexity.

### 2.2. Modeling

Due to the cumulative effects of these technological advances, modeling video traffic has become a tedious and complicated task. Mathematical models of video sources tried to keep pace with these advances. The developed models may be classified into two categories. Markovian (or Embedded Morkovian) models, which is based on the memory-less property, and takes into consideration only the short-term correlation properties. Examples include Markov Modulated Poisson Process model (Heffes and Lucantoni, 1986), Markov Modulated Fluid model (Anick et al., 1982), Versatile Markovian Arrival Process model (Neuts, 1989), Memory Markov Chain model (Rose, 1999), and D-BIND model (Knightly and Zhang, 1997).

The second category includes long-range correlation models, which tries to capture a peculiar behavior found in certain processes where the autocorrelation function decays to zero at a rate slower than the exponential function. Examples include F-ARIMA (fractional autoregressive integrated moving average) (Beran et al., 1995), FBM (fractional brownian motion) (Taqqu and Levy, 1986; Norros, 1995), FGN (fractional Gaussian noise), highly variable ON-OFF sources (Likhanov et al., 1995; Willinger et al., 1997), and $M/G/\infty$ model (Krunz and Makowski, 1998).

As the major new character in the current video traffic may be related to its long-range dependence, some models based on *self-similarity* have been proposed lately. The notion of *self-similarity* is based on fractals, where zooming in or out keeps the general object shape the same. In the case of network traffic, this notion is known in the stochastic sense not the topological sense. In the stochastic sense, *self-similarity* is applied to the statistical aspects as the time scale is varied. The self-similarity property has been identified, although with various degrees of conclusiveness, in Ethernet-LAN traffic (Willinger et al., 1997), in the World Wide Web traffic (Crovella and Bestavros, 1997), in ATM queues (Tsybakov and Georganas, 1997), and recently in multimedia networks (Sahinoglu and Tekinay, 1999).

**Table 1** Basic statistics of the traces.

| Trace name | GOP frames | | | I-frames | | |
|---|---|---|---|---|---|---|
| | Mean | Covar | Peak/mean | Mean | Covar | Peak/mean |
| Terminator | 10,905 | 1.03 | 7.30 | 37,388 | 0.22 | 2.13 |
| Soccer | 27,129 | 0.96 | 6.90 | 79,143 | 0.32 | 2.37 |
| Starwars | 9313 | 1.39 | 13.40 | 44,012 | 0.32 | 2.84 |
| Talk | 14,537 | 1.14 | 7.35 | 64,734 | 0.16 | 1.65 |
| News | 20,664 | 1.26 | 9.41 | 85,420 | 0.30 | 2.28 |
| Race | 30,749 | 0.69 | 6.58 | 79,241 | 0.26 | 2.35 |
| Lambs | 7311 | 1.53 | 18.36 | 38,024 | 0.34 | 3.53 |
| Dino | 13,078 | 1.13 | 9.15 | 55,076 | 0.21 | 2.17 |

### 2.3. Traces

The work to be undertaken will be based on video traces used in Rose (1999). The traces were extracted from MPEG-1 sequences which have been encoded with the Berkeley MPEG-encoder. The frame sizes are in bits, and were generated with a capture rate of 25 frames per second (for more information consult Rose (1995)). The basic statistics of the traces (variations of these were reported in Rose (1997)) are shown in Table 1.

## 3. Self-similarity trail

### 3.1. Conventional tests

Before applying the considered tool, some well established test will be first applied to the considered video traces. We will start with a graphical test to have an a priori sense of the existence of self-similarity in these traces. From the essence of self-similarity, we present two such methods:

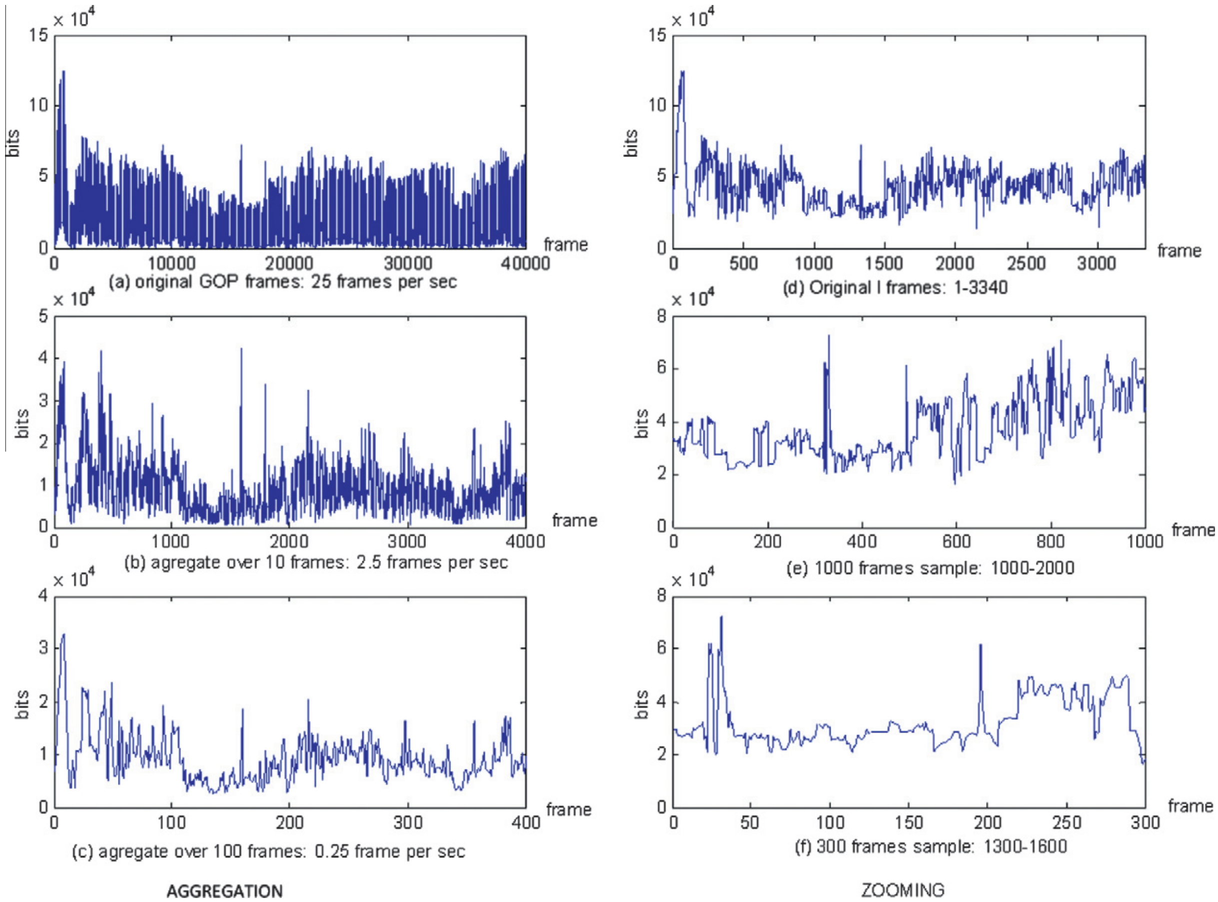


**Figure 1** Starwars traces under various time scales and zooming views.

(1) Either the signal may be aggregated by replacing contiguous and non-overlapping blocks of frames with their averages (in a manner similar to Eq. (A.1) which will be used as a stochastic definition of *self-similarity*). The results are shown in Fig. 1a–c.
(2) Or, the signal may be zoomed-in and zoomed-out by considering only subsets of the whole trace, a method that has been used in Leland et al. (1994). The corresponding result is shown in Fig. 1d–f.

It is clear from all these figures, that the burstiness is kept in all time scales. Had the traffic been Markovian, then the curves would have been smoothed out.

Then, in Table 2 are shown estimates of the Hurst parameter $H$ using the R/S plot and the Periodogram. Knowing that the Hurst parameter has to be $1/2 < H < 1$ for a traffic sequence to be self-similar, it is clear that all traces may be considered self-similar.[1]

### 3.2. Aggregation using wavelets

The method to be used to test for the self-similarity property of video traces (or series, as denoted in mathematical terms) will be based on the work presented in Abry et al. (1998), which will be customized to our work.

[1] Although not with the same extent.

**Table 2** H parameter estimation using R/S plot and Periodogram.

| Video trace | R/S plot | Peridogram |
|---|---|---|
| Lambs | 0.79 | 0.74 |
| Race | 0.80 | 0.92 |
| News | 0.82 | 0.73 |
| Talk | 0.77 | 0.78 |
| Dino | 0.76 | 0.80 |
| Starwars | 0.80 | 0.78 |
| Soccer | 0.80 | 0.81 |
| Terminator | 0.78 | 0.83 |
| Starwars (long) | 0.79 | 0.71 |
| LAN | 0.80 | 0.83 |

#### 3.2.1. Wavelet scaling

Many recent papers have proven the equivalence between *Multiscale-based* and *Wavelet-based* tests on one hand (Abry et al., 1998, 2002; Soltani et al., 2000; von Sachs and Neumann,2000), and the feasibility of Wavelet techniques in the characterization of network traffic on the other (Riedi et al., 1999). Also, these papers have shown the computational efficiency and accuracy of the Wavelet-based techniques. Which is due to the fact that eventhough LRD signals are highly correlated in the time domain, they become nearly decorrelated in the Wavelet domain.

The wavelet method was one of three methods used in Hong et al. (2001) to estimate the H parameter for various MPEG VBR traces. The Hurst parameter was estimated using the ratio of energies between two adjacent resolutions. Then, the effects of H have been studied on the efficiency of bandwidth allocation in an ATM network.

*3.2.2. Test tool description*

The video trace will be represented by a sequence $X_N = \{x_n : n = 1,2,\ldots,N\}$ where each element $x_n$ represents the number of bits in the $n^{th}$ element in the sequence (GOP, or I, P, or B frames), and $N$ represents the total number of frames in the trace.

Let $\{\psi_{j,k}(t)\}$ represent the Wavelet templates related to the mother Wavelet $\psi_0(t)$ by:

$$\psi_{j,k}(t) = 2^{-j/2}\psi_0(2^{-j}t - k) \tag{1}$$

Define the coefficients of the discrete wavelet transform (DWT) $y^{(j)}(k)$ by the inner product:

$$y^{(j)}(k) = < X_N, \psi_{j,k} > \tag{2}$$

In what follows these coefficients will be denoted by $d_X(j,k)$.

It is worthwhile noting that this transformation represents a band-pass filtering of the sequence $X_N$ with width $2^j$. Put in other words, these coefficients represent the details of the data rather than the approximations (Abry et al., 1998), as shown in Fig. 2. If $X_N$ exhibits long-range dependence (LRD) characteristics, then the variance of the aggregated process should have a power-law behavior when represented as a function of the aggregated level, i.e.:

$$\mathrm{Var}\{d_X(j,k)\} \sim 2^{j(1-\beta)};\ j \to \infty, \forall k \tag{3}$$

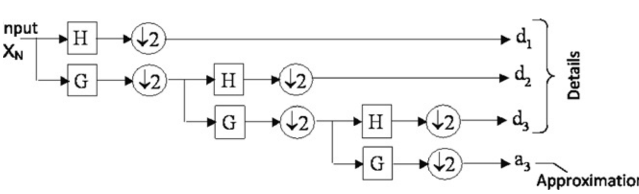


**Figure 2** Wavelet signal decomposition.

Furthermore, it has been shown in Abry and Aldroubi (1995) that the mean of the squares of the Wavelet coefficients $\frac{1}{N_j}\sum_{k=1}^{N_j} d_X(j,k)^2$ is an unbiased, asymptotically efficient estimate of $\mathrm{var}\{d_X(j,k)\}$ . This leads us to the final result to be used here, which is:

If the sequence $X_N$ possesses LRD characteristics, then the plot of $\log_2(2^j)$ versus $\log_2\left\{\frac{2^j}{N_0}\sum_{k=1}^{N_j} d_X(j,k)^2\right\}$ is approximately linear. Consequently, the corresponding slope is an estimator of the LRD parameter $\beta$.

In this part, our work is similar to Ma and Ji (1998), except in using an estimate of the variance of the Wavelet coefficients instead of the variance itself. Additionally, the test will be applied to both the GOP and the frame sequences. As an example, the results obtained when applying the tool to the I frames of the Starwars trace are shown in Fig. 3.

## 4. The MVH-CV testing technique

A major requirement for using the tool in Abry et al. (2002) was that when the wavelet transform is applied to self-similar random processes with stationary increments, for each fixed scale $j$ the details $\{d_X(j,k), k \in \mathscr{Z}\}$ are stationary processes. Meaning that the considered video traces should have stationary increments.

Put into simple mathematical terms, this means that the probability distribution of the increments $\{X_{n+m} - X_n\}$ should not depend on $n$. To test such property from the available sample data is not an easy matter and may not be even possible. Nevertheless, some necessary but not sufficient tests for the stationarity of the increments will be proposed. They are based on the computation of some statistical measures, namely: the mean, the variance, and the histogram for various values of the increment $m$, and for both the $I$ and $IPB$ frame sequences.

The technique will be denoted by $MVH\text{-}CV$ since it bases its tests on the computation of the coefficient of variation ($CV$) for the mean, variance, and histogram ($MVH$) of the pertinent sequences.

Because of the size limitation of the data being used, the video sequence $\{X_n : n = 1,2,\ldots,N\}$ was subdivided into subsequences $\{x_{kn} : k = 0,1,\ldots,N-S, n = 1,2,\ldots,S\}$, where $N$ is the length of the subsequence, $S$ is the length of each

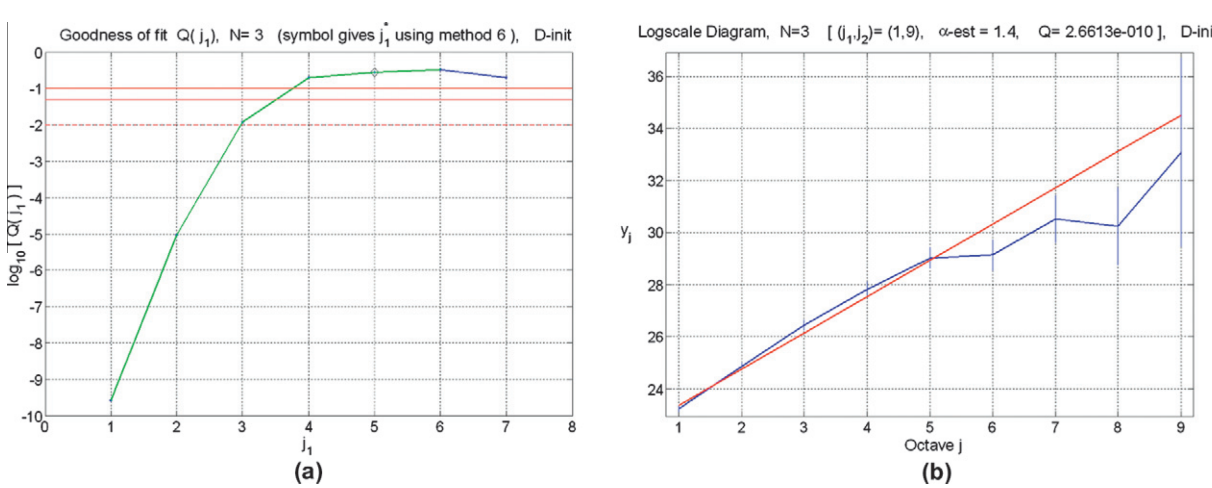


**Figure 3** Results for the Starwars I frames sequence.

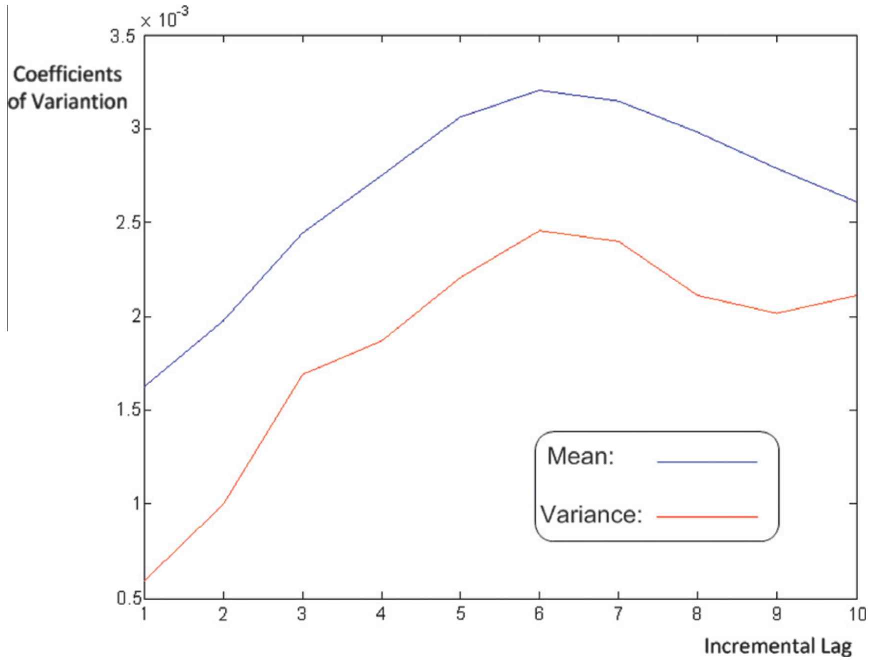


**Figure 4** Coefficients of variation for the mean and variance of the frame size increments.

subsequence, and $k$ is the subsequence number.[2] The subsequences $x_{kn}$ are constructed from the sequence $X_n$ according to the following equation:

$$x_{kn} = X_{k+n}; \quad n = 1, 2, \ldots S, \; k = 0, 1, \ldots, N - S \tag{4}$$

The increments $d_{mkn}$ are defined as the absolute value of the difference between the sizes of the frames with an increment $m$ in the sequence numbers:

$$d_{mkn} = |x_{(k+m)n} - x_{kn}|; \quad n = 1, 2, \ldots S, k = 0, 1, \ldots, N - S \tag{5}$$

So, for each increment $m$, there will be $N - S + 1$ vectors with $S$ elements in each. The average $d^a_{mk}$ in each vector is defined as the sample mean, and the variance $d^v_{mk}$ by the sample variance. Then, for each increment $m$, is defined a coefficient of variation $r_m$ for the mean by:

$$r^2_m = \frac{\mathrm{Var}(d^a_{mk})}{E(d^a_{mk})^2} \tag{6}$$

and a coefficient of variation $c_m$ for the variance by:

$$c^2_m = \frac{\mathrm{Var}(d^v_{mk})}{E(d^v_{mk})^2} \tag{7}$$

## 5. Results

The results will be categorized into two main sets. In the first set, some tests will be run to establish the stationarity of the increments in the trace sequences, followed by a second type of tests to reveal any effects of the number of vanishing moments. In the second set, the estimated $H$ parameter will be related in a first stage to the variability parameters, and then in a second to the time scale of the used traces.

### *5.1. Stationarity of the increments*

#### *5.1.1. Coefficient of variation for the mean and variance*

In Fig. 4 are plotted the coefficients of variation for both the mean and variance in the frame size increments for various time incremental lags. The statistical mean values range from $1.6\times\ 10^{-3}$ to $3.2\times10^{-3}$, while the statistical variance values range from $0.6\times10^{-3}$ to $2.4\times10^{-3}$. Knowing that the coefficient of variation provides a measure of data dispersion about the mean, these very small numbers signify that both computed sample mean and sample variance are practically time invariant to the incremental lags.

#### *5.1.2. Coefficient of variation for the histograms*

To further confirm these results, a plot is shown in Figs. 5 and 6 for the coefficients of variation for the histograms of the various subsequences and for increments from 1 to 10. It is to be noted that the tool, used to get the histograms, divides the range of values taken by the differences between the frame sizes into ten equal intervals and then counts the number of differences falling in each.

The histogram intervals are used as abscissa in Fig. 5, with, in each interval, the values corresponding to each increment. For instance, in the first interval, only the first three increments have a non zero correlation coefficient. While at the tenth interval, only the first four and the sixth increment are non-zero.

The same data is again shown in Fig. 6 but with the increments as abscissa. Now, it can be seen that the correlation coefficients for the first three increments are non-zero in all the ten intervals. For an increment of four, only the last seven are non-zero. In the last four increments, the coefficients are all zeros.

[2] Not all subsequences need to be used.

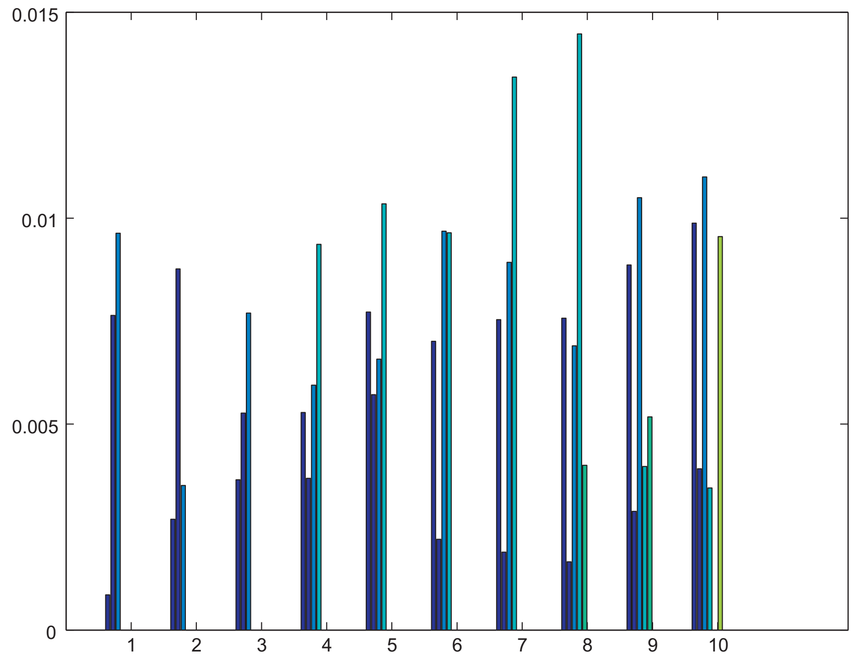


**Figure 5** The coefficients of variation for the histograms as a function of the intervals.

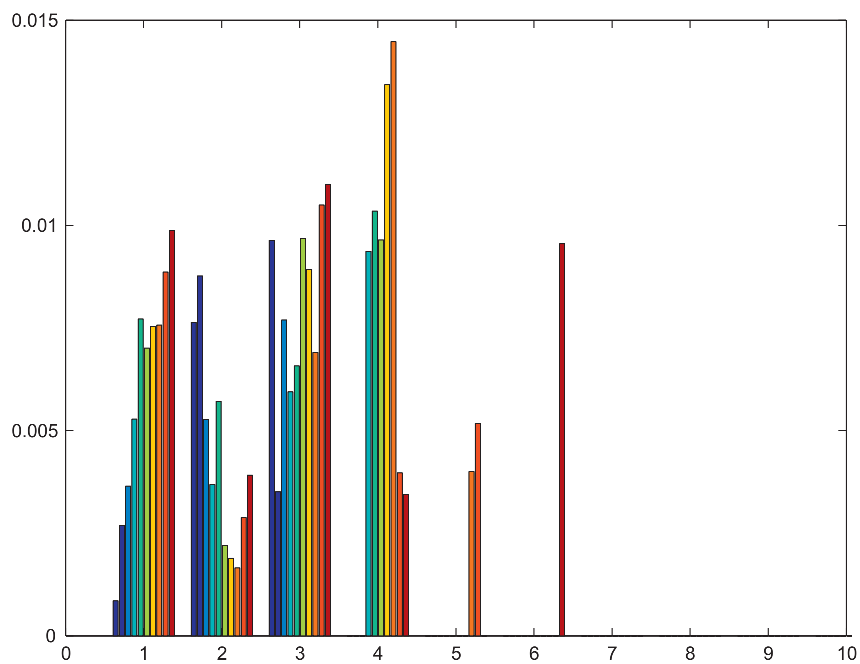


**Figure 6** The coefficients of variation for the histograms as a function of the increments.

The coefficient of variation for the histograms varies between zero and a maximum of 0.014. From these very small values, it can be deduced that the used video trace has effectively stationary increments.

### 5.2. Effects of the number of vanishing moments

As a last test before using the Wavelet tool, it is important to check how the number of vanishing moments used in the wavelet transform affects the new estimator results. In Fig. 7 are plotted the Hurst parameter estimates for various traces as a function of the number of vanishing moments (Abry et al. (1998) requires that the number of vanishing moments $N \geqslant 2$).

It may be noticed that all estimates are in the range of [0.7, 0.95], and that most estimates are in the range of [0.8, 0.9].

### 5.3. Relation to degree of spread

To check for the effects of the degree of statistical variability and time scale on the self-similarity aspects of VBR video,

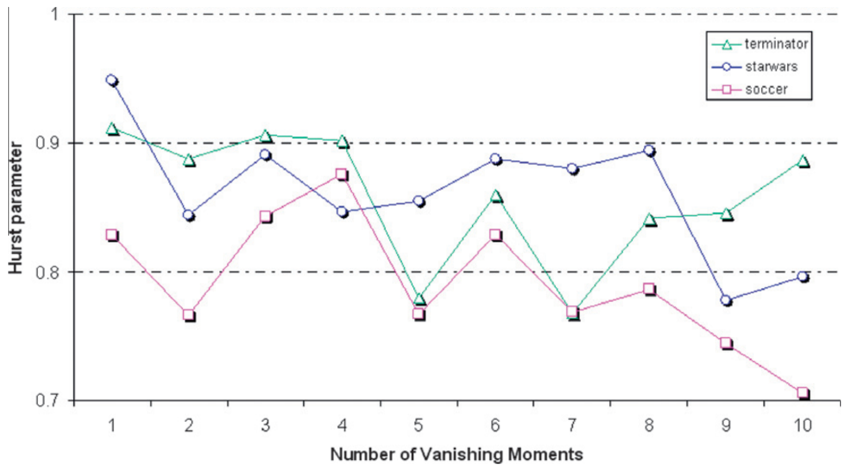


**Figure 7** Effects of the number of vanishing moments on the estimated H parameter (Daubechies wavelet).

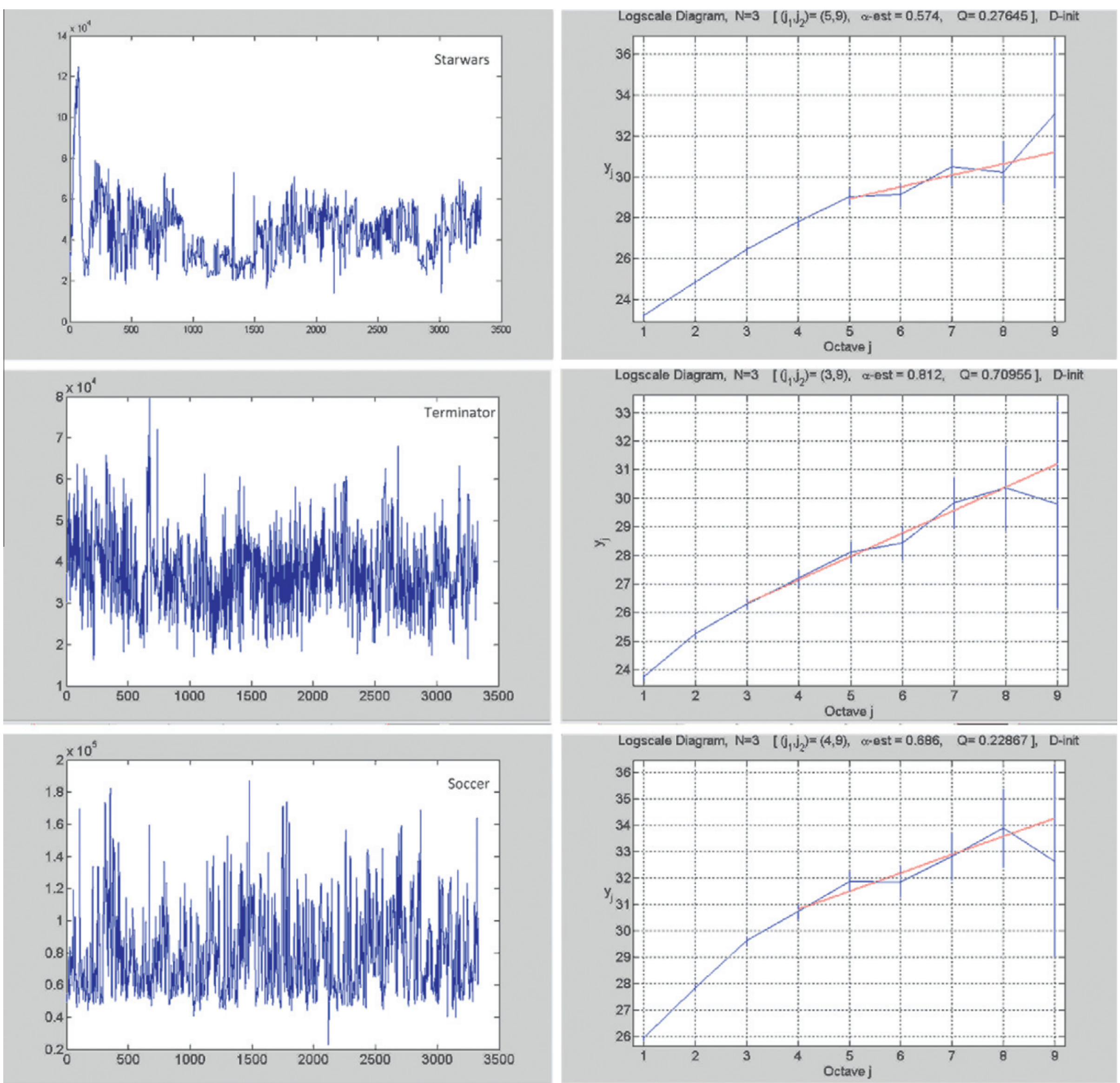


**Figure 8** Temporal representation and log-log plot for various traffic traces.

three traces were selected, namely: Starwars, Terminator, and Soccer. For each video trace, three sets of graphs are plotted:

(1) The frame sizes as a function of time;
(2) A log–log plot of the variance of the Wavelet coefficients as a function of the number of octaves (similar to the one in Fig. 3);
(3) Estimate of the Hurst parameter for various time scales.

The first two sets for each one of the three traces are shown in Fig. 8. By comparing these results with the statistical measures shown in Table 1, it may be noticed that eventhough the soccer trace has the highest mean it has the lowest coefficient of variation. As expected, the signal magnitude has no effect on the inter-frames dependance. Also, it is clear from the figure that the starwars and soccer traces have a higher variability than the terminator trace, which has been translated

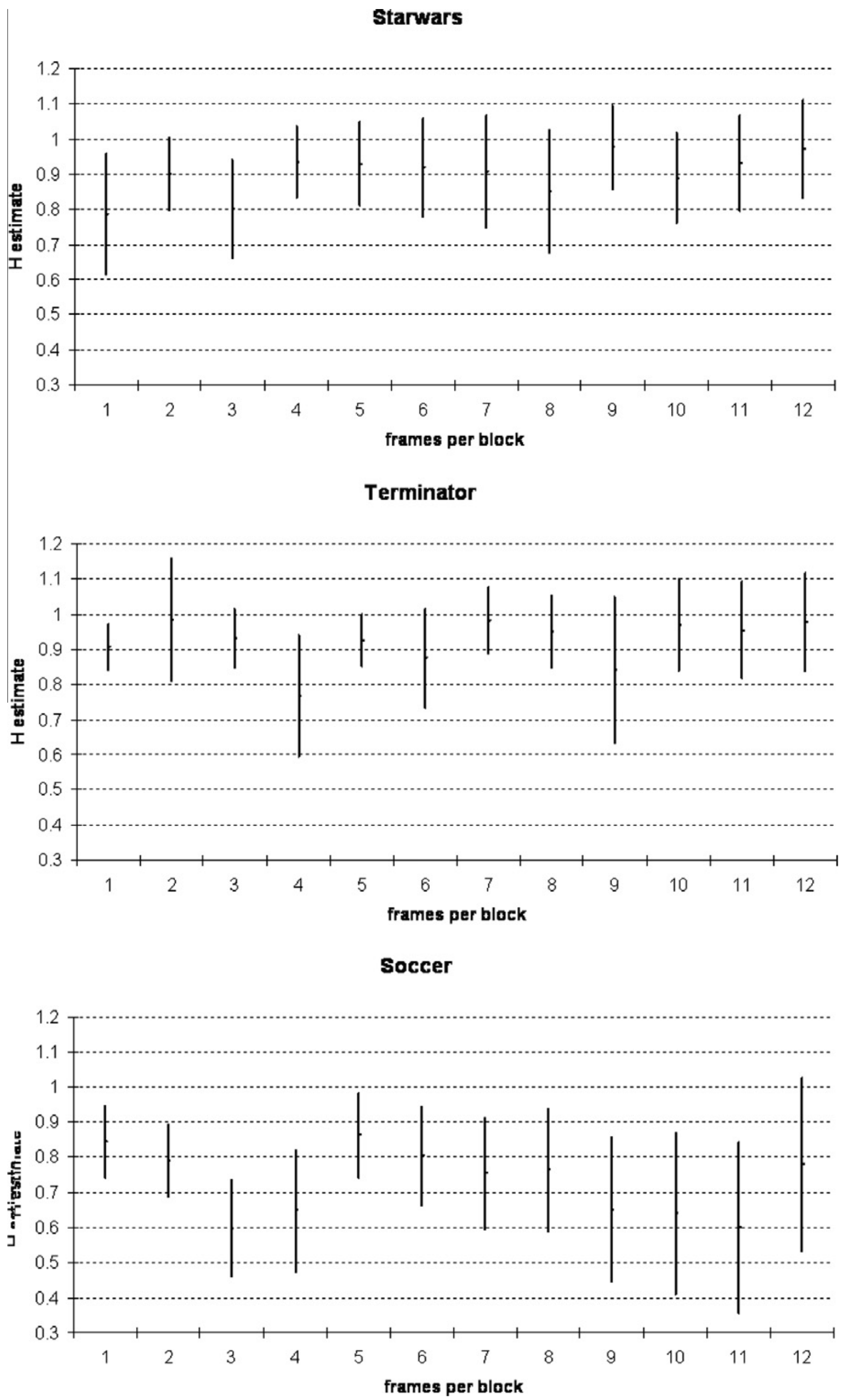


**Figure 9** Time scale effects on the estimation of the H parameter for various traffic traces.

in the Table 1 by a higher coefficient of variation and peak-to-mean ratio. Lastly, the expected Hurst parameter was in reverse relation with the variability of the traces. Thus, the Terminator trace had the lowest coefficient of variation but the highest $H$, while the starwars and soccer traces had the same coefficient of variation, but since the former had a higher peak-to-mean ratio it got a smaller $H$.

From these observations (and similar ones using other traces, and which are not displayed for compactness), the following rules may be deduced:

- The variability of a sequence may be measured through its coefficient of variation and peak-to-mean ratio, i.e. a higher coefficient of variation or peak-to-mean ratio means a higher degree of variability.
- The coefficient of variation and peak-to-mean ratio of a sequence have an opposite effect on the estimated H parameter, i.e. as the coefficient of variation and peak-to-mean ratio increase, the estimated $H$ parameter decreases.
- The coefficient of variation has precedence over peak-to-mean ratio in the effect on the estimated H parameter, i.e. if two traces have different coefficients of variation, then the peak-to-mean ratios have no effect on the relation between the estimated H parameters. It is only when the coefficients of variation are equal that the peak-to-mean ratios are brought in.

### 5.4. Effects of time-scale

The last set of graphs is shown in Fig. 9, and represents an estimate of the Hurst parameter for various time scales. The estimation of the Hurst parameter was based on the method used in the second graph of Fig. 8, but with the video trace replaced with an equivalent trace seen at various time scales. For instance, a trace seen at time scale $i$ was obtained by replacing non-overlapping blocks of $i$ frames by their corresponding averages. Knowing that the frames were generated at a rate of 24 frames per second, a time scale $i$ will correspond to $i/24$ seconds. The estimated H is shown along with its corresponding 95% confidence interval (under Gaussian assumption).

From Fig. 9, the following remarks may be recorded:

- At most time scales, the estimated $H$ parameter for the Terminator trace stayed in the range of 0.9 to 1.0.
- Almost the same thing happened for the Starwars trace, with the main difference being at the lowest scale where it started at around 0.8.
- In the case of the soccer trace things were much different:
  - the variability in the estimated $H$ was much larger (from 0.6 to 0.86).
  - all values are below 0.9.
  - the confidence intervals were the largest.
  - above a time scale of about 5, an almost steady decrease in $H$ was observed.
- These results do not follow the same trends as in the previous subsection:
  - Even though the Soccer trace had a higher coefficient of variation than the Terminator trace, its estimated $H$ parameter over various time scales was smaller.
  - Also, the Starwars and Soccer traces had the same coefficient of variation, but since the former had a higher peak-to-mean ratio, it kept the same trend for $H$ when the time scale was increased above 1.

Using these observations along with the results shown in Table 1, the following remarks may be deduced:

(1) The coefficient of variation and the time-scale have the same effects on the estimated H parameter, i.e. as the coefficient of variation gets smaller, the effects of the time-scale gets smaller.
(2) For the same coefficients of variation, a sequence with lower peak-to-mean ratio is affected more by the time-scale than the peak-to-mean ratio.

## 6. Conclusions

In this paper, some representative video traces were used to test the self-similarity aspects of VBR video. First, some validation tests were carried out to make sure that the obtained results were credible. Thus, some novel testing criteria were proposed to check for the stationarity of increments in the used video traces. After applying our proposed test to the traces, all tested positive.

Then, the effects of the number of vanishing moments used in the wavelet-based test tool on the estimated H parameter were studied. All traces showed a tolerable variation within the expected range of H.

In the second part of the results, first possible relations between the statistical variability measures, namely the coefficient of variation and the peak-to-mean ratio, and the $H$ parameter were studied. It was found that the effects of the variability parameters have an inverse effect on H, and that the coefficient of variation has a precedence effect over the peak-to-mean ratio.

Then, the effects of the time-scale on the estimated Hurst parameter have been studied, and some relations to the statistical spreading parameters have been drawn. So, in addition to the confirmation of previous results that suggested the existence of the self-similarity trail in VBR video, we succeeded in relating its level and trend to the sequence time scale and to the traffic spreading parameters.

Lastly, and as was pointed out previously, using these results the generation of video traffic may become more realistic, leading to a more thorough study of its effects on the performance of computer networks systems, in regard to the buffer management policies and call admission control.

## Appendix A. Self-similarity and long-range dependence

### A.1. Self-similarity

Given a wide-sense stationary time series $X = (X_n; n = 1,2,3,\ldots)$, with mean $\mu$ and variance $\sigma^2$. Define the $m$-aggregated series $X^{(m)} = \left(X_k^{(m)};\ k = 1,2,3,\ldots\right)$ by summing the original series $X$ over non-overlapping blocks of size $m$. $X$ is said to be *H-self-similar* if for all positive $m$, $X^{(m)}$ has the same distribution as $X$ rescaled by $m^H$. That is:

$$X_n^{(m)} \equiv \frac{1}{m^H} \sum_{i=(n-1)m+1}^{nm} X_i; \text{ for all } m \in N \tag{A.1}$$

Furthermore, if $X$ is *H-self-similar*, it has the same autocorrelation function $r(k)$, defined by:

$$r(k) = \frac{1}{\sigma^2} E[(X_n - \mu)(X_{n+k} - \mu)] \tag{A.2}$$

as the series $X^{(m)}$ for all $m$.

### A.2. Long-range dependence

A wide-sense stationary time series $X = (X_n;\ n = 1,2,3,\ldots)$ with autocorrelation function $r(k)$ is said to be *long-range dependent* if it has a non-summable autocorrelation function:

$$\sum_{k=-\infty}^{\infty} r(k) = \infty \tag{A.3}$$

The special processes that have the characteristic of: $\mathrm{Var}(X) = m^{2-2H}\mathrm{Var}(X^m)$ are *long-range dependent*. They are also known as *second-order self-similar* (Abry et al.,1998; Riedi et al., 1999).

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