Journal of King Saud University – Computer and Information Sciences (2012) 24, 157–163

King Saud University

Journal of King Saud University – Computer and Information Sciences

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ORIGINAL ARTICLE

Application of fuzzy mathematics and grey systems in education

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Received 28 November 2011; revised 24 March 2012; accepted 4 April 2012
Available online 17 April 2012

KEYWORDS
Student evaluation;
Fuzzy logic;
Grey system;
Educational importance factor

Abstract In this paper two problems on the evaluation process of the education system are talked about. The methodologies to solve the problems are based on soft computing techniques. Fuzzy sets have been used to model and solve the problem of identifying the 'educational importance factor' of each academic year and grey numbers have been used to obtain the students' answer script evaluation process. The algorithmic approaches are supported by suitable examples.

1. Introduction

In today's hard battled life, competitive examinations are very important for almost all standards of students for their entry in professional life. Most of these involve a written examination, a personality test and students' academic qualifications. Our main interest is the latter one. A student gets certain marks for his previous obtained academic records. As for example, for the recruitment of Assistant Teachers for the post meant for Honours/Post Graduate vacancy in schools of West Bengal, a State of India, the mark distribution is summarized in Table 1.1.

Here we can clearly observe that a student with certain educational background gets a total award of marks which plays an important role in the selection procedure. Now we construe this problem from a different logical viewpoint. Students from different academic years get equal importance here, but the actual scenario is different. As days are passing somehow we see that academic results are going better for the students. Does it really mean that students' intelligence and excellence are improving constantly? Not always. Because the academic results certainly depend on the question pattern and standard, syllabus strength, ways of evaluating answer scripts, duration of the course, duration of the examination and the awarded marks distribution procedure. Since some of these attributes may vary for each academic year, it is injustice for the students from different academic years to avail equal importance; the evaluation procedure is not logically correct. To overcome this, a novel approach is prescribed on the basis of fuzzy logic in Section 3.1 and a counter example is demonstrated in Section 4.1 with a comparative study.

Another problem we are discussing here in this paper is the evaluation procedures of students' answer scripts in some examinations where the aim is to rank the students according to their merit. Normally in these examinations an answer script is evaluated on the basis of one time mark assignment. Here our main aim is to evaluate the answer scripts from different views of the decision makers. Grey theory has been implemented here to obtain the ordering. Lot of works has already been done on students' evaluation under different scenarios in

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Peer review under responsibility of King Saud University.

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http://dx.doi.org/10.1016/j.jksuci.2012.04.001

Table 1.1 Example of marks distribution in a competitive examination.

Examination	Full marks	Award of marks		
School Final (10th standard)	5	5 [1st division]	4 [2nd division]	3 [other division]
Higher Secondary (+2 stage)	5	5 [1st division]	4 [2nd division]	3 [other division]
Bachelor's Degree in Honours	6	6 [1st div/class]	5 [2nd div/class]	4 [other div]
Post Graduate Degree	6	6 [1st div/class]	5 [2nd div/class]	4 [other div]
Degree or Diploma in Teachers' Training	3	3 [1st div/class]	2 [2nd div/class]	1 [other div]

fuzzy environment. In Chen and Lee's method (1999), eleven satisfaction levels have been proposed and the final marks are obtained by the mean of the grades of these satisfaction levels weighted by the satisfaction grade given by the evaluator. The table of the performance here is called as extended fuzzy grade sheet. In Bai and Chen's method (2008), fuzzy rules and membership functions are used. The methodology is based on five matrices: Accuracy Matrix, Answer-time rate Matrix, Grade Matrix, Importance Matrix and Complexity Matrix. The basic problem regarding this method is that it consumes so much time and the procedure of calculation is too complex. Biswas (1995) introduced two methods Fuzzy Evaluation Method and Generalized Fuzzy Evaluation Method. These methods are based on some standard fuzzy sets with certain membership degrees. The ranking of the students is evaluated by the degrees of similarity between the fuzzy set of an individual student (obtained from the evaluators) and the standard fuzzy sets. The advantage of these methods is that they are easy to understand and easy to implement. The disadvantage is that they round off some grades during their calculation to their most similar grades which occurs errors. The defuzzification (quasi) applied just before the final step seems to lead to some errors.

The working methodology has been discussed in Section 3.2 and a suitable example has been placed in Section 4.2 to illustrate the proposed approach.

2. Preliminaries

The concept of fuzzy logic and fuzzy mathematics was introduced by Zadeh in 1965, when the two-valued logic completes its era. Initially it was given in prescribed form for engineering purposes and it got some time to accept this new methodology from different intellectuals. For a long time a lot of western scientists have been apathetic to use fuzzy logic because of its threatening to the integrity of older scientific thoughts. But once it got the stage, it performed fabulously. From mathematical aspects to engineering systems, it spread all over and the betterments of all types of systems were certainly there. After all, the society chose Fuzzy Logic as a better choice. In Japan, the first sub-way system was built by the use of fuzzy logic controllers in 1987. Since then almost every intelligent machine works with fuzzy logic based technology inside them.

In this section some preliminary concept on fuzzy and grey systems is overviewed. Linguistic terms are also defined as they have been used both for the proposed methodologies.

Let X is a collection of objects called the universe of discourse. A fuzzy set denoted by $\widetilde{A}$ on X is the set of ordered pairs $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)) : x \in X\}$ where $\mu_{\widetilde{A}}(x)$ is the grade of membership of x in $\widetilde{A}$ and the function $\mu_{\widetilde{A}}(x) : X \to [0,1]$ is called the membership function. Membership Function evaluation has been a challenging task in the literature. Generalized methodology is somehow missing. The actual reason is that the problem is very much context dependent.

Grey system theory (Deng, 1989) was proposed by Deng in 1982 on the basis of grey sets. The systems that lack in information are pertained as Grey Systems. In the perspective of any type of numbers, Grey numbers represent the information between completely known and completely unknown situations, i.e., Grey System is the bridge connecting White System and Black System. We now take a look on some definitions of Grey theory.

Let X is the universal set of considerations. Then a Grey set G of X is defined by its two mappings $\bar{\mu}_G(x)$ and $\underline{\mu}_G(x)$:

$\bar{\mu}_G(x) : X \to [0,1]$ and $\underline{\mu}_G(x) : X \to [0,1]$ such that $\bar{\mu}_G(x) \geqslant \underline{\mu}_G(x)$, $x \in X$. The Grey set G becomes a fuzzy set when the upper and lower membership functions in G are equal to each other, i.e., when $\bar{\mu}_G(x) = \underline{\mu}_G(x)$. When the lower and upper limits of any information can be estimated by real numbers, we certainly are able to express it by an interval Grey number $\otimes G = [\underline{G}, \overline{G}] = \{\theta \in \otimes G : \underline{G} \leqslant \theta \leqslant \overline{G}\}$ where θ is an information and $\underline{G}, \overline{G}$ are respectively the lower and upper limits of the information's existence.

The degree of greyness, denoted by $\tilde{g}(\otimes G)$ is defined by a function of the two ends of the interval, i.e., $\tilde{g}(\otimes G) = f(\underline{G}, \overline{G})$.

An interval valued fuzzy set in X is given by A and is defined by $A = \{(x, \mu_A(x)) : x \in X\}$ where $\mu_A(x) : X \to D[0,1]$ defines the degree of membership of an element x to A and $D[0, 1]$ denotes the family of sub closed intervals of [0, 1].

The degree of greyness of a grey set is the same as of the grey number with the same boundary of grey set.

According to Wang et al.'s approach (1988) we now define some basic grey number operations:

$$\otimes G_1 + \otimes G_2 = [\underline{G}_1 + \underline{G}_2, \overline{G}_1 - \overline{G}_2]$$

$$\otimes G_1 - \otimes G_2 = [\underline{G}_1 - \overline{G}_2, \overline{G}_1 - \underline{G}_2]$$

$$\otimes G_1 \times \otimes G_2 = [\min(\underline{G}_1\underline{G}_2, \underline{G}_1\overline{G}_2, \overline{G}_1\underline{G}_2, \overline{G}_1\overline{G}_2), \max(\underline{G}_1\underline{G}_2, \underline{G}_1\overline{G}_2, \overline{G}_1\underline{G}_2, \overline{G}_1\overline{G}_2)]$$

$$\otimes G_1 \div \otimes G_2 = [\underline{G}_1, \overline{G}_1] \times \left[\frac{1}{\underline{G}_2}, \frac{1}{\overline{G}_2}\right]$$

We cite (Guo-Dong et al., 2007) to obtain the Grey Possibility Degree of $\otimes G_1 \leqslant \otimes G_2$ as

$$P\{\otimes G_1 \leqslant \otimes G_2\} = \frac{\max(0, L(\otimes G_1) + L(\otimes G_2) - \max(0, \overline{G}_1 - \underline{G}_2))}{L(\otimes G_1) + L(\otimes G_2)},$$
$$\text{where } L(\otimes G) = \overline{G}_1 - \underline{G}_2.$$

It is clear from the concept of possibility, that

i) when $\otimes G_1 = \otimes G_2$, then $P\{\otimes G_1 \leqslant \otimes G_2\} = 0.5$,
ii) when $\overline{G}_1 < \underline{G}_2$, then $P\{\otimes G_1 \leqslant \otimes G_2\} = 1$, and
iii) when $\overline{G}_1 > \underline{G}_2$, then $P\{\otimes G_1 \leqslant \otimes G_2\} = 0$.

Clearly these two stages (grey sets and interval valued fuzzy sets) represent two different kinds of approach towards representing uncertainty. They differ in both philosophical and practical concepts.

For the grey sets the degree of greyness is defined for the whole set while for the interval valued fuzzy sets, fuzziness is defined for individual elements. The relations $\leqslant$, $\prec$ and $=$ in grey sets occur for the components of two grey sets with members that may be different. But the same relations in interval valued fuzzy sets occur for two fuzzy sets with identical members. Philosophically greyness represents lack of knowledge about data. The interval of a grey set is the domain of definition corresponding to a white number. On the contrary, the membership degrees of the members of a fuzzy set represent measures of belief in some concepts. The interval of an interval valued fuzzy set is about the scope of its membership. Thus when additional information is supplied to a grey set, it becomes white. But when additional information is supplied to an interval valued fuzzy set, the belief measure gets stronger and a more precise membership value is obtained, the set remains fuzzy.

Other important parts in this aspect are linguistic terms and their expressions in fuzzy and grey systems. Sometimes, while dealing with scientific problems, we face both qualitative and quantitative aspects. The first one can be easily handled by precise numeric quantities. But for the qualitative aspects, we should not use precise or exact values, as uncertainty exists therein. For this problem of modeling uncertain information, sometimes linguistic terms are used. For a certain type of information, a fixed set of linguistic terms are employed. The mathematical representation of the linguistic terms is case wise different. Most popular approaches to this regard are based on fuzzy systems, grey systems, interval number systems, etc. In fuzzy system we can represent them as Interval Valued Fuzzy Numbers, Triangular Fuzzy Numbers, Trapezoidal Fuzzy Numbers, etc. The fuzzy and grey linguistic approaches are important tools for scientific problem solving, especially in the areas of information retrieval, human resource management, service revolution, service revolution, decision making and web equality.

3. Proposed methodology

In this section we apply fuzzy logic and grey theoretic techniques to two valuable problems of our educational system.

3.1. Educational importance factor evaluation approach for each academic year

For a particular academic year of considerations, some attributes are considered here in this approach, e.g., the standard of the question (A_1), ratio of the students qualified and total students (A_2), ratio of the students qualified in the first division and total students (A_3) and the ratio of the highest marks obtained by the student attaining the first position and total marks (A_4). Let there be m decision makers $D_1, D_2, \ldots, D_m$ for this problem and w_i^k be the weight of importance given by the decision makers D_k for the ith attribute, $k = 1, 2, \ldots, m$; $i = 1, 2, \ldots, n$, n being the total number of attributes. The aim is to determine the educational importance factor (EIF) for each academic year so that the evaluation of the students passed out in different academic years becomes logically justified.

Let us commence our method with p academic years $Y_1, Y_2, \ldots, Y_p$. The attribute A_1 is clearly a fuzzy linguistic term and the other attributes are non fuzzy as we can get specific values for them. Now all the membership values of the attributes for each Y_j, $j = 1, 2, \ldots, p$ are shaped in the region [0, 1] to avoid computational complexity. To extract one membership value of the attribute A_1 for each Y_j, we introduce here a new technique illustrated as follows.

Let there be q number of subjects $S_1, S_2, \ldots, S_q$. The decision makers have been asked to submit their opinions about the standard of the questions of each subject in terms of linguistic terms from the five members of the set {Very Easy (VE), Easy (E), Medium (M), Hard (H) and Very Hard (VH)} and Table 3.1.1 is thus constructed.

Here the entries are among the above mentioned five linguistic terms.

The individual total counts (number of appearing in the table) of each fuzzy set is calculated for each year Y_j. Let $n_j(\mathrm{VE}) = n_1^j, n_j(\mathrm{E}) = n_2^j, n_j(\mathrm{M}) = n_3^j, n_j(\mathrm{H}) = n_4^j$ and $n_j(\mathrm{VH}) = n_5^j$ where $n_j(\mathrm{X})$ is the total count of the fuzzy set X in the academic year Y_j. Now let us obtrude weights for each of these five fuzzy sets in such a manner that harder questions' linguistic fuzzy sets get larger weights. We impose here simple weights 0.10, 0.15, 0.20, 0.25 and 0.30 for the fuzzy sets VE, E, M, H and VH, respectively. These weights may also be determined by the decision makers. We define then the membership degree of a particular academic year Y_j for the attribute: Standard of question as

$$\mu_{A_1}(Y_j) = \frac{n_1^j}{mq} \times 0.10 + \frac{n_2^j}{mq} \times 0.15 + \frac{n_3^j}{mq} \times 0.20 + \frac{n_4^j}{mq} \times 0.25 + \frac{n_5^j}{mq} \times 0.30 = \frac{1}{mq}\sum_{r=1}^{5} w_r n_r^j$$

where w_r is the weight of the rth fuzzy set (if determined by the decision makers).

Now for the other attributes, membership degrees are constructed from the logical viewpoint that the way of evaluating answer scripts has a certain impact on the attributes A_2, A_3 and A_4. The instruction given in different academic years to the evaluators may vary so as the way of evaluation. The standard of the syllabus of all subjects of that corresponding academic year has also an influence on these attributes. Keeping this in mind the membership functions are formulated in such a manner that academic year with better result gets smaller membership values. Again the academic result of the students also depends on the attribute A_1, which has already been considered. Thus we impose a restriction on the attribute weights as $w_1^k \geqslant w_2^k + w_3^k + w_4^k$. In other words, $w_1^k \geqslant 0.5$.

Let $\mu_{A_2}(Y_j)$, $\mu_{A_3}(Y_j)$ and $\mu_{A_4}(Y_j)$ be the membership values of the attributes A_2, A_3 and A_4, respectively, for the academic

Table 3.1.1 Decision Makers' opinion on standard of questions of all subjects in p academic years.

	S_1				S_2				…	S_q			
	D_1	D_2	…	D_m	D_1	D_2	…	D_m		D_1	D_2	…	D_m
Y_1													
Y_2													
⋮													
Y_p													

year Y_j. Thus we construct $\mu_{A_2}(Y_j) = 1 - \frac{q_j}{p_j}$, $\mu_{A_3}(Y_j) = 1 - \frac{r_j}{p_j}$ and $\mu_{A_4}(Y_j) = 1 - \frac{h_j}{t_j}$ where p_j = total number of students, q_j = total number of qualified students, r_j = total number of students qualified in the first division, h_j = highest marks obtained by the first ranked student and t_j = total marks of the concerned examination for the jth academic year Y_j. Finally the educational importance factor $E(Y_j)$of the jth year is defined as $E(Y_j) = \sum_{i=1}^{4} w_i \mu_{A_i}(Y_j)$ where $w_i = \frac{1}{m}\sum_{k=1}^{m} w_i^k$.

3.2. Student evaluation by grey theory

In this section a new approach based on grey theory is proposed for ranking students by evaluating their answer scripts. The procedure is same as multi criteria decision making in uncertain environment. Let us consider a discrete set of m students $S = \{S_1, S_2, \ldots, S_m\}$ and a set of n attributes, $A = \{A_1, A_2, \ldots, A_n\}$. These attributes are additively dependent. Also consider $\otimes w = \{\otimes w_1, \otimes w_2, \ldots, \otimes w_n\}$ as a vector of attribute weights which are realized to be linguistic variables. Now these linguistic weights and attribute ratings can be expressed in grey numbers shown in Table 3.2.1.

At first a group of k Decision Makers $D = \{D_1, D_2, \ldots, D_k\}$ is formed. Then the weight $\otimes w_j$ of the jth attribute A_j is calculated as $\otimes w_j = \frac{1}{k}\left[\otimes w_j^1 + \otimes w_j^2 \ldots + \otimes w_j^k\right]$ where $\otimes w_j^k$ is the attribute weight given by the kth decision maker, described by the grey number $\left[\underline{w}_j^k, \bar{w}_j^k\right]$. Next the linguistic variables for the ratings are used to construct attribute rating value, calculated as $\otimes G_{ij} = \frac{1}{k}\left[\otimes G_{ij}^1 + \otimes G_{ij}^2 + \cdots + \otimes G_{ij}^k\right]$ where $\otimes G_{ij}^k$ is the attribute rating value given by the kth decision maker, described by the grey number $\left[\underline{G}_{ij}^k, \overline{G}_{ij}^k\right]$. Thus the grey decision matrix GDM is established as shown in Table 3.2.2.

The elements in GDM are realized to be the average grey sets. In the next step these elements are normalized by the maximum of the upper limits of the grey numbers of the matrix. Hence we find the Normalized Grey Decision Matrix NGDM by the elements $\otimes G'_{ij} = \left[\frac{\underline{G}_{ij}}{G_j^{\max}}, \frac{\overline{G}_{ij}}{G_j^{\max}}\right]$ where $G_j^{\max} = \max_{1 \leqslant i \leqslant m}\{\overline{G}_{ij}\}$ shown in Table 3.2.3.

Now each normalized element is multiplied by their corresponding weights and we get the weighted normalized grey decision matrix WNGDM by the elements $\otimes T_{ij} = \otimes G'_{ij} \times \otimes w_j$ as shown in Table 3.2.4.

At the final stage of this method we construct a pseudo alternative, named as Best Student BS, defined by

$$BS = \left\{\otimes G_1^{\max} = [\max_i \underline{T}_{i1}, \max_i \overline{T}_{i1}], \otimes G_2^{\max} = \left[\max_i \underline{T}_{i2}, \max_i \overline{T}_{i2}\right], \ldots, \otimes G_n^{\max} = [\max_i \underline{T}_{in}, \max_i \overline{T}_{in}]\right\}.$$

Table 3.2.2 GDM (Grey Decision Matrix).

$\otimes G_{11}$	$\otimes G_{12}$	…	$\otimes G_{1n}$
$\otimes G_{21}$	$\otimes G_{22}$	…	$\otimes G_{2n}$
⋮			
$\otimes G_{m1}$	$\otimes G_{m2}$	…	$\otimes G_{mn}$

Table 3.2.3 NGDM (Normalized Grey Decision Matrix).

$\otimes G'_{11}$	$\otimes G'_{12}$	…	$\otimes G'_{1n}$
$\otimes G'_{21}$	$\otimes G'_{22}$	…	$\otimes G'_{2n}$
⋮			
$\otimes G'_{m1}$	$\otimes G'_{m2}$	…	$\otimes G'_{mn}$

Table 3.2.4 WNGDM (Weighted Normalized Grey Decision Matrix).

$\otimes T_{11}$	$\otimes T_{12}$	…	$\otimes T_{1n}$
$\otimes T_{21}$	$\otimes T_{22}$	…	$\otimes T_{2n}$
⋮			
$\otimes T_{m1}$	$\otimes T_{m2}$	…	$\otimes T_{mn}$

Table 3.2.1 expression of linguistic terms in grey numbers.

Linguistic term for attribute weights	Grey numbers	Linguistic term for attribute ratings	Grey numbers
Very low	[0.0,0.1]	Very poor	[0,1]
Low	[0.1,0.3]	Poor	[1,3]
Medium low	[0.3,0.4]	Medium poor	[3,4]
Medium	[0.4,0.5]	Fair	[4,5]
Medium high	[0.5,0.6]	Medium good	[5,6]
High	[0.6,0.9]	Good	[6,9]
Very high	[0.9,1.0]	Very good	[9,10]

Table 4.1.1 Decision Makers' linguistic comments on the standard of questions of all subjects – part I.

	Beng-I				Beng-II				History				Geography			
	D_1	D_2	D_3	D_4	D_1	D_2	D_3	D_4	D_1	D_2	D_3	D_4	D_1	D_2	D_3	D_4
2005 (Y_1)	VE	E	M	M	M	M	E	E	H	H	M	H	M	M	M	M
2007 (Y_2)	VE	E	E	E	VE	E	E	VE	M	M	M	E	M	M	E	E
2009 (Y_3)	E	VE	E	VE	VE	VE	VE	VE	M	E	E	VE	M	M	E	E

Table 4.1.2 Decision Makers' linguistic comments on the standard of questions of all subjects – part II.

	English				Life sciences				Physical sc.				Mathematics			
	D_1	D_2	D_3	D_4	D_1	D_2	D_3	D_4	D_1	D_2	D_3	D_4	D_1	D_2	D_3	D_4
2005 (Y_1)	M	M	M	M	E	E	M	E	M	M	E	M	VH	H	H	M
2007 (Y_2)	E	M	M	E	E	VE	E	E	M	M	E	E	H	M	M	M
2009 (Y_3)	E	VE	E	E	VE	VE	E	VE	E	M	E	E	M	M	E	E

Table 4.1.3 Data for attributes A_2, A_3 and A_4.

	A_2: ratio of the students qualified and total students	A_3: ratio of the students qualified above 60% marks and total students	A_4: ratio of highest getting marks and total marks
2005 (Y_1)	0.705	0.270	0.981
2007 (Y_2)	0.746	0.321	0.994
2009 (Y_3)	0.762	0.353	0.945

Table 4.1.4 Attribute weights in crisp numbers.

	A_1	A_2	A_3	A_4
D_1	0.50	0.25	0.20	0.05
D_2	0.60	0.20	0.15	0.05
D_3	0.50	0.20	0.20	0.10
D_4	0.65	0.20	0.10	0.05

Table 4.2.1 Attribute weights in linguistic terms.

$\otimes w_j$	D_1	D_2	D_3	D_4
A_1	VH	VH	H	VH
A_2	H	MH	MH	H
A_3	M	ML	M	M

This pseudo alternative is used to determine the orderings. Each student is compared with BS by the grey possibility degree $P\{S_i \leqslant BS\} = \frac{1}{n}\sum_{j=1}^{n} P\left\{\otimes T_{ij} \leqslant \otimes G_j^{\max}\right\}$. The ranking is done according to this possibility values and higher possibilistic alternative gets better rank.

4. Example

The methodologies provided in Section 3 can be applied to competitive examinations globally. This is not our intention to decrease the EIF with time. It is totally context dependent. In Section 4.1, we illustrate the construction of EIF by a real case study. In Section 4.2, the student evaluation procedure based on grey numbers is also exemplified by a suitable case study.

4.1. A case study taken from WBBSE of India

In this section we start with an example taken from a state educational board of India, viz, West Bengal Board of Secondary Education (WBBSE). The secondary examination of the students of 10th standard is conducted under this board. Before 2007, the examination was executed on the basis of two years' syllabus pattern. But then onwards it is based on only one year evaluation process. The students in this board sit for eight subjects: Bengali-I, Bengali-II (Regional Language Paper), English, History, Geography, Life Sciences, Physical Sciences and Mathematics. Each subject paper is evaluated with equal importance with a maximum mark of 100. The example is constituted with data from three academic years 2005, 2007 and 2009. A group of three decision makers (D_1, D_2, D_3) is organized and for the attribute A_1 we gather the comments of the decision makers in fuzzy linguistic term in Tables 4.1.1 and 4.1.2.

Now for the other attributes the authors have collected the required information which is displayed in Table 4.1.3.

Again for the attribute weights, Table 4.1.4 is formed after getting those weights from the Decision Makers.

Thus we have $\frac{n_1^1}{mq} = \frac{1}{32} = 0.031, \frac{n_2^1}{mq} = \frac{7}{32} = 0.219, \frac{n_3^1}{mq} = \frac{18}{32} = 0.562, \frac{n_4^1}{mq} = \frac{5}{32} = 0.156, \frac{n_5^1}{mq} = \frac{1}{32} = 0.031$. Thus

Table 4.2.2 Decision Makers' comments on the students' answer-scripts for different attributes.

$S_{it\ i}$	D_1	D_2	D_3	D_4
A_1				
S_1	P	MP	P	MP
S_2	F	MG	MG	MG
S_3	MP	F	F	F
S_4	G	MG	G	MG
S_5	MG	MG	G	G
A_2				
S_1	MP	F	F	F
S_2	G	MG	G	G
S_3	P	MP	MP	P
S_4	VG	G	G	G
S_5	MG	F	F	MG
A_3				
S_1	MP	P	P	MP
S_2	VG	VG	G	VG
S_3	F	F	MP	MP
S_4	VG	G	G	G
S_5	F	F	MG	G

Table 4.2.3 GDM for 5 students and 3 attributes.

	A_1	A_2	A_3
S_1	[2, 3.5]	[3.75, 4.75]	[2, 3.5]
S_2	[4.75, 5.75]	[5.75, 8.25]	[8.25, 9.75]
S_3	[3.75, 4.75]	[2, 3.5]	[3.5, 4.5]
S_4	[5.5, 7.5]	[6.75, 9.25]	[6.75, 9.25]
S_5	[5.5, 7.5]	[4.5, 5.5]	[4.75, 6.25]

Table 4.2.4 NGDM for 5 students and 3 attributes.

	A_1	A_2	A_3
S_1	[0.205, 0.359]	[0.385, 0.487]	[0.205, 0.359]
S_2	[0.487, 0.59]	[0.59, 0.846]	[0.846, 1]
S_3	[0.385, 0.487]	[0.205, 0.359]	[0.359, 0.462]
S_4	[0.564, 0.769]	[0.692, 0.949]	[0.692, 0.949]
S_5	[0.564, 0.769]	[0.462, 0.564]	[0.487, 0.641]

Table 4.2.5 WNGDM for 5 students and 3 attributes.

	A_1	A_2	A_3
S_1	[0.169, 0.35]	[0.212, 0.365]	[0.077, 0.152]
S_2	[0.402, 0.575]	[0.324, 0.634]	[0.317, 0.425]
S_3	[0.318, 0.475]	[0.113, 0.269]	[0.135, 0.196]
S_4	[0.465, 0.75]	[0.381, 0.712]	[0.26, 0.403]
S_5	[0.465, 0.75]	[0.254, 0.423]	[0.183, 0.272]

$$\mu_{A_1}(Y_1) = \frac{n_1^1}{mq} \times 0.10 + \frac{n_2^1}{mq} \times 0.15 + \frac{n_3^1}{mq} \times 0.20 + \frac{n_4^1}{mq} \times 0.25 + \frac{n_5^1}{mq} \times 0.30 = 0.197.$$

By similar procedure we obtain $\mu_{A_1}(Y_2)$ =0.166 and $\mu_{A_1}(Y_3) =$ 0.142. Also $\mu_{A_2}(Y_1) = 1 - 0.705 = 0.295$, $\mu_{A_3}(Y_1) = 1 - 0.270 =$ 0.73, $\mu_{A_4}(Y_1) = 1 - 0.981 = 0.019$; $\mu_{A_2}(Y_2) = 1 - 0.746 =$ 0.254, $\mu_{A_3}(Y_2) = 1 - 0.321 = 0.679$, $\mu_{A_4}(Y_2) = 1 - 0.994 =$ 0.006; $\mu_{A_2}(Y_3) = 1 - 0.762 = 0.238$, $\mu_{A_3}(Y_3) = 1 - 0.353 =$ 0.647, $\mu_{A_4}(Y_3) = 1 - 0.945 = 0.055$.

Table 4.2.6 Grey possibilistic degrees and ranking.

$P\{S_i \leqslant BS\}$	Rank
$P\{S_1 \leqslant BS\} = 1$	5
$P\{S_2 \leqslant BS\} = 0.622$	2
$P\{S_3 \leqslant BS\} = 0.992$	4
$P\{S_4 \leqslant BS\} = 0.552$	1
$P\{S_5 \leqslant BS\} = 0.805$	3

Now we evaluate the average weights of the attributes imposed by the Decision Makers as $w_i = \frac{1}{m}\sum_{k=1}^{m} w_i^k, i = 1, 2, 3, 4$. Thus $w_1 = 0.5625$, $w_2 = 0.2125$, $w_3 = 0.162$ and $w_4 = 0.0625$. Finally the educational importance factor (EIF) of the jth year is calculated as $E(Y_j) = \sum_{i=1}^{4} w_i \mu_{A_i}(Y_j)$.

Thus $E(Y_1) = 0.293$, $E(Y_2) = 0.258$ and $E(Y_3) = 0.239$.

Let us compare our proposed approach with the existing evaluation procedure. So in this case study we clearly observe that the EIFs are decreasing as years depart. Since this is a specific case study we cannot generalize this fact and different rankings are also possible. However this technique should be applied for competitive examinations where students' academic records matter. Now for more illustration let us consider three students X_1, X_2 and X_3 who have passed their Secondary Examinations under WBBSE in respective years 2005, 2007 and 2009. Also let the obtained marks are 58.5%, 59.25% and 70%, respectively. From Table 1.1, it is clear that X_1 and X_2 get 4 marks while X_3 gets 5 marks as reward for the secondary exam. Now to execute our proposed approach the awarding marks system should be changed a little. If the maximum allotted marks for secondary examination be fixed as 5, a student with x% marks in the academic year Y_j should be awarded $\frac{x}{100} \times 5 \times \{1 - (\max_j E(Y_j) - E(Y_j))\}$. Following this, X_1 will get 2.925, X_2 will get 2.859 and X_3 will get 3.311. It is significant that X_2 gets lesser marks than X_1 while his obtained mark in the secondary examination is greater than that of X_1. It is just because of the EIF differences in two academic years here.

4.2. Example of a student evaluation approach by grey theory

This is the illustration of the methodology described in Section 3.2. Here we will deal with five students and four decision makers. The answer-scripts of the students are evaluated on the basis of three attributes: Average time accuracy of the student (A_1), Answering to the point (A_2) and Presentation (A_3). Now these attributes may change in number and nature for different subjects' answer-scripts. As for example, for a subject of regional language or English, the attributes Standard of language, Handwriting and Spelling accuracy should be considered, while for science subjects these additional attributes may not come into consideration. However this example is constituted with the attributes A_1, A_2 and A_3.

First of all the response of the Decision Makers on attribute weights is recorded in Table 4.2.1.

Then the Decision Makers are asked to submit their opinions on the attribute ratings of the answer-scripts in linguistic terms as shown in Table 4.2.1. The ratings are described in Table 4.2.2.

Now the GDM is constructed by averaging the corresponding grey numbers and is displayed in Table 4.2.3.

The elements of the GDM are normalized by the maximum number 9.75 and we get the NGDM as shown in Table 4.2.4.

The grey numbers in the NGDM are now multiplied by their corresponding attribute weights which are also grey numbers. Thus the WNGDM is constructed and exhibited in Table 4.2.5.

So Table 4.2.5 reflects the position of the students' answer-scripts on the basis of the three attributes A_1, A_2 and A_3. As stated clearly in the methodology in Section 3.2 the pseudo alternative Best Student (BS) is now formed and BS = {[0.465, 0.75], [0.381, 0.712], [0.317, 0.425]}. Hence our task is to evaluate the grey possibilistic degrees $P\{S_i \leqslant BS\}$. These degrees as well as the rank of the students are demonstrated in Table 4.2.6.

Thus we have the ordering of students $S_4 > S_2 > S_5 > S_3 > S_1$.

5. Conclusion

In this paper we have implemented fuzzy mathematics and grey theory to solve two genuine problems of education system. The methodologies have been supported by two examples and the obtained results show the effectiveness of the approaches. More scope of research is there in this field of education, the evaluation procedures under various types of uncertainty can be made less complex using this type of techniques.

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