



ORIGINAL ARTICLE

Entropy based fuzzy classification of images on quality assessment

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Abstract Referenced image quality assessment methods require huge memory and time involvement, therefore not suitable to use in real time environment. On the other hand development of an automated system to assessing quality of images without reference to the original image is difficult due to uncertainty in relations between features and quality of images. The paper aims at developing a fuzzy based no-reference image quality assessment system by utilizing human perception and entropy of images. The proposed approach selects important features to reduce complexity of the system and based on entropy of feature vector the images are partitioned into different clusters. To assign soft class labels to different images, continuous weights are estimated using entropy of mean opinion score (MOS) unlike the previous works where crisp weights were used. Finally, fuzzy relational classifier (FRC) has been built using MOS based weight matrix and fuzzy partition matrix to establish correlation between features and class labels. Quality of the distorted/decompressed test images are predicted using the proposed fuzzy system, showing satisfactory results with the existing no-reference techniques.

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1. Introduction

Digital images are subjected to loss of information, various ways of distortions during compression (Sayood, 2000) and transmission, which deteriorate visual quality of the images at the receiving end. Quality of an image plays fundamental role to take vital decision and therefore, its assessment is essential prior to application. Modeling physiological and psycho

visual features of the human visual system (Pappas and Safranek, 2000; Watson, 1993; Watson et al., 1997) and signal fidelity criteria (Sonka et al., 1999) based quality assessment are reported (Pappas and Safranek, 2000; Watson, 1993) though each of these approaches has several shortcomings. The most reliable means of measuring image quality is subjective evaluation based on the opinion of the human observers (Wang and Bovik, 2002, 2006). However, subjective testing is not automatic and expensive too. On the other hand, most objective image quality assessment methods (Sheikh et al., 2006; VQEG, 2000) either require access to the original image as reference (Sheikh and Bovik, 2004) or only can evaluate images, degraded with predefined distortions and therefore, lacking generalization approach. Two prominent works have been reported relating to no-reference image quality evaluation: (i)

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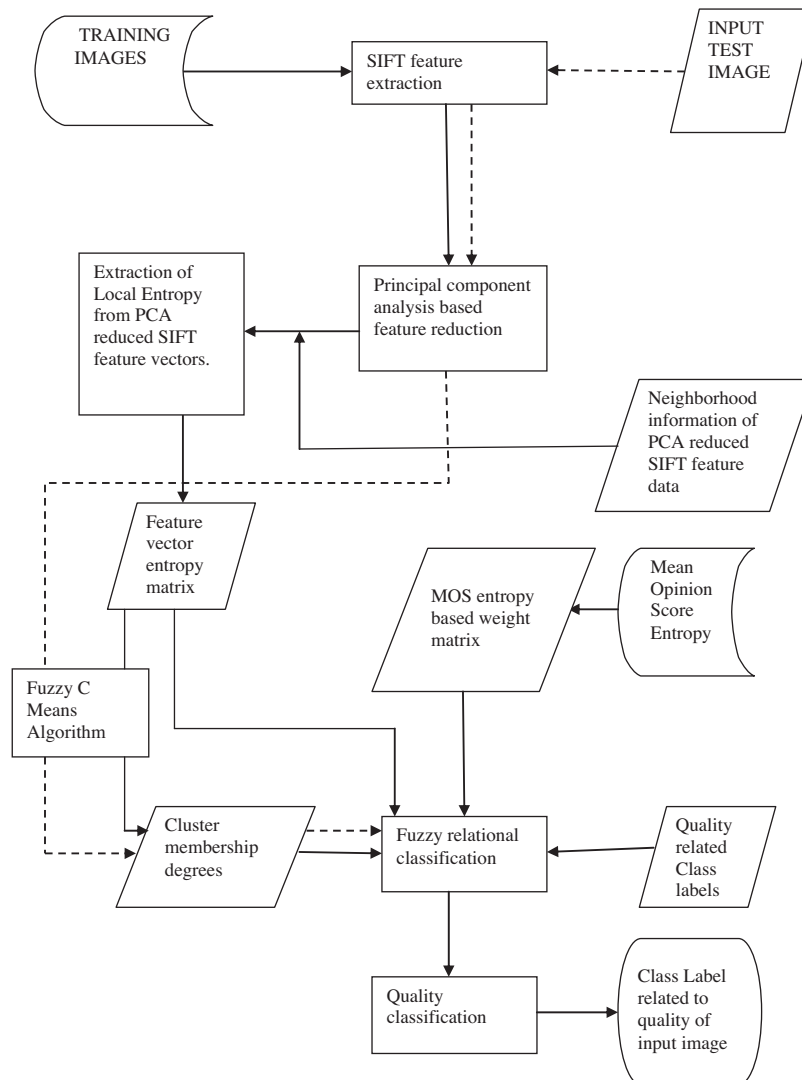


Figure 1 Feature entropy based image quality classification.

Wang, Bovik and Sheikh's no-reference JPEG image quality index (Wang et al., 2002, 2006) and (ii) Sheikh's quality metric based on natural scene statistics (NSS) model, applied on JPEG2000 compressed images (Sheikh et al., 2005). Very recently, three different types of metrics are reported to assess quality of an image namely, extreme learning machine classifier based mean opinion score (MOS) estimator Suresh et al., 2009, discrete cosine transform (DCT) domain statistics based metric (Brandão et al., 2008) and blind image quality index (Moorthy and Bovik, 2009a,b). Since human being is the ultimate evaluator of the images, the best suited method based on human perception has not been exploited yet to assess quality of images. None of the existing metric incorporates human centric computational intelligence approaches, to assessing image quality based on human visual system. On the other hand fuzzy classification techniques (Castiello, 2003) are used for image classification for quite long time back by allowing pixels to have membership in more than one class. However, handling information at pixel level is time consuming and there is high chance of biased assessment of images if class labels are assigned by a single human observer. Even considering multiple observers' opinions do not able to reflect individual's

perception in assessing quality of images, if it is crisp. It is worth to mention here that in the paper the FRC is used to assess quality of images distorted by information loss or noise, unlike the earlier methods (Lu and Weng, 2007) where images are preprocessed to remove the noise before classification.

In the paper, a no-reference image quality assessment technique is proposed using entropy of significant features by capturing local information variation in training images. The proposed quality metric is estimated by a fuzzy relational classifier (FRC) where variations of human perceptions to assess a particular image are incorporated by generating continuous weights based on which MOS based class labels are assigned. First different scale invariant local features are extracted and after removing redundancy significant features are selected using principal component analysis (PCA) algorithm. To remove uncertainty in assigning images into different class labels, fuzzy *c*-means (FCM) clustering algorithm is applied using entropy of features. As a next step, logical relation has been established (by designing the FRC) between information contained in the images (fuzzy partition matrix) and the human perception about the visual quality of the images (continuous weighted MOS matrix) using ϕ -composition (a fuzzy

implication) (Lin and George Lee, 1993) and conjunctive aggregation methods. Quality of test images are assessed or predicted in terms of degree of membership of the pattern in the given classes by applying fuzzy relational operator. The flowchart of the entire procedure is described in Fig. 1.

The paper is organized into five sections. Section 2 describes the feature selection process along with feature entropy calculation while Section 3 states designing of FRC applied to predict image quality. Section 4 present results of experiments and conclusions with future scope of work are summarized in Section 5.

2. Feature selection

Scale Invariant Feature Transform (SIFT) is an approach for detecting and extracting local feature descriptors, reasonably invariant to changes in illumination, noise, rotation, scaling and small changes in viewpoint. Here, different scale invariant local image features are extracted from gray level (PGM for-

mat) training images of TAMPERE database (Ponomarenko et al., 2009) by applying David Lowe’s (Lowe, 2004) algorithm. However, all extracted features are not equally important and redundant, so might not play significant role to assessing quality of images. Significant features are selected by applying Principal Component Analysis (PCA) algorithm that effectively reduces dimension of SIFT feature vector corresponding to each training images. Approximately 20,000 features have been reduced to 128 only considering TAMPERE database.

2.1. Entropy of features

Entropy of features are calculated by forming a feature matrix (F) with number of rows corresponds to number of training images and number of columns representing the dimension of each feature vector. Information contained in each feature is obtained using local Shannon entropy (Gonzalez et al., 2003) as defined in the below equation:

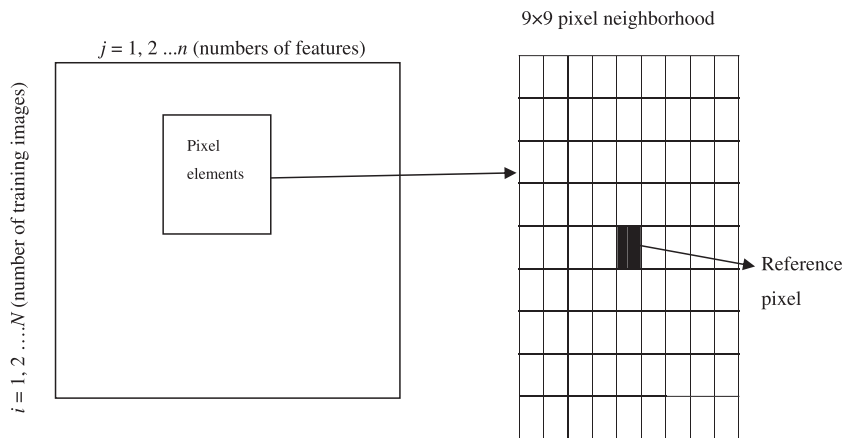


Figure 2 Feature entropy calculation method.

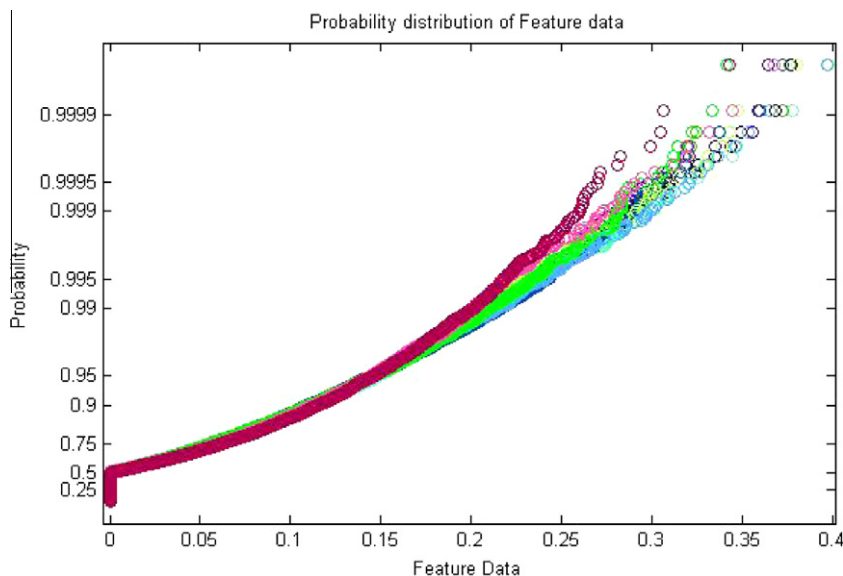


Figure 3 Probability distribution of features of training images represented by different colors.

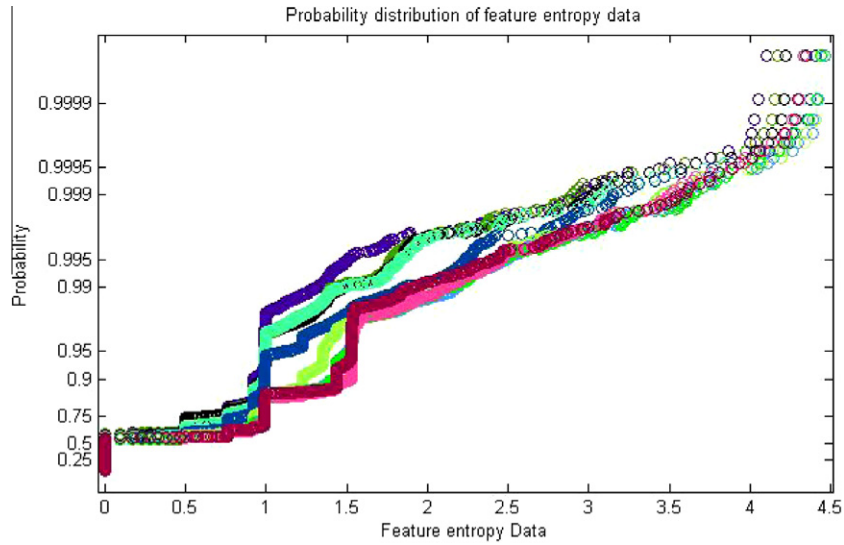


Figure 4 Probability distribution of entropy of feature vectors of training images represented by different colors.

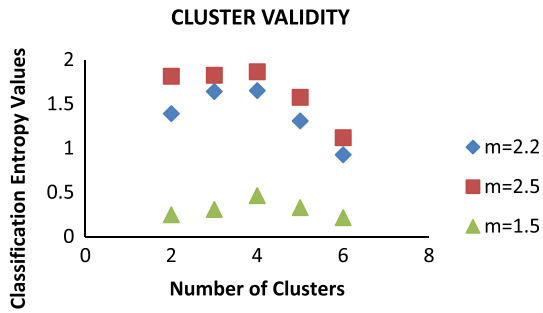


Figure 5 Bezdek's cluster validity measure for different 'm'.

$$E_j = -(p_j \log p_j) \quad (1)$$

where p_j represents probability of occurrence of j -th feature in a particular training image.

To compute the local entropy of each feature vector, the feature matrix F is considered as representation of an image and each element of F denotes pixel value of the image. The F matrix has been scanned from left hand top corner pixel to right hand bottom corner pixel using a 9×9 neighborhood pixel window, shown in Fig. 2. Each pixel is considered as a reference pixel that corresponds to a particular feature in an

image. The frequency of occurrence of each reference pixel in the neighboring pixel region (9×9) is calculated and the process is repeated for n number of reference pixels ($j = 1, \dots, n$). Finally, entropy of a feature vector for a particular image (i -th image) is calculated using the following equation:

$$E_i = -(\sum_{k=1}^{81} p_k \log p_k) \quad (2)$$

Algorithm 1 (entropy of feature)

Input: Feature matrix F of dimension 10 (No. of images) \times 16384 (No. of feature vectors)

Begin

Step 1 Scan F from left to right and top to bottom.

Step1.1 Select the first matrix element and keep it at the center of the 9×9 window.

Step1.2 Compute local entropy of the selected matrix element using Eq. (2) and store in the output matrix.

Step1.3 Repeat step 1.2 for the entire matrix F .

End.

The probability distributions of the selected features extracted from 10 different images are shown in Fig. 3, exhibiting exponential nature of variation of feature values in different train-

Table 1 Training image features extracted from TAMPERE database with varied level of distortion.

| Image name | Distortion type | Distortion level | Mean opinion score (MOS) |
|------------|----------------------------|------------------|--------------------------|
| Image 1 | Spatially correlated noise | 3 | 3.3529 |
| Image 2 | Additive Gaussian noise | 3 | 4.6176 |
| Image 3 | JPEG transmission errors | 4 | 2.3333 |
| Image 4 | JPEG transmission errors | 4 | 1.8710 |
| Image 5 | Gaussian blur | 4 | 2.1765 |
| Image 6 | JPEG2000 compression | 4 | 1.0000 |
| Image 7 | Additive Gaussian noise | 1 | 5.9706 |
| Image 8 | Additive Gaussian noise | 2 | 5.4167 |
| Image 9 | Additive Gaussian noise | 3 | 4.5556 |
| Image 10 | Spatially correlated noise | 4 | 3.1176 |

Table 2 MOS weight matrix W (highlighted value indicates estimated MOS entropy value for a particular class).

| Images | Class labels | Excellent | Good | Average | Bad | Poor |
|----------|--------------|---------------|---------------|---------------|---------------|---------------|
| Image 7 | | 0.132 | 0.217 | 0.217 | 0.217 | 0.217 |
| Image 8 | | 0.1264 | 0.2184 | 0.2184 | 0.2184 | 0.2184 |
| Image 9 | | 0.2207 | 0.117 | 0.2207 | 0.2207 | 0.2207 |
| Image 2 | | 0.2209 | 0.1163 | 0.2209 | 0.2209 | 0.2209 |
| Image 1 | | 0.2254 | 0.2254 | 0.0985 | 0.2254 | 0.2254 |
| Image 10 | | 0.2263 | 0.2263 | 0.0945 | 0.2263 | 0.2263 |
| Image 5 | | 0.2302 | 0.2302 | 0.2302 | 0.0792 | 0.2302 |
| Image 3 | | 0.2310 | 0.2310 | 0.2310 | 0.0758 | 0.2310 |
| Image 4 | | 0.2328 | 0.2328 | 0.2328 | 0.0688 | 0.2328 |
| Image 6 | | 0.2388 | 0.2388 | 0.2388 | 0.2388 | 0.0447 |

ing images. The result implies that information contained in different features is wide apart and so effective to partition the images using FCM clustering algorithm.

Probability distribution of entropy of feature vectors of different training images is shown in Fig. 4. The distribution result shows step functional nature at lower entropy values while becoming smooth and exponential in nature at higher local entropy values. Moreover, distinct characteristics among training images are evident when local entropy values of selected SIFT features (Fig. 4) are considered compare to feature values (Fig. 3). Therefore, in the proposed method local entropy of selected SIFT features are used to partition the images.

3. Fuzzy relational classification

Fuzzy relational classification (Setnes and Babuska, 1999) establishes correspondence between structures in feature space of the training instances and the class labels. By using fuzzy logic in classification, one avoids the uncertainty of hard labeling the prototypes and easily captures the partial sharing of structures among several classes. In the training phase, two steps are performed to build the proposed classifier: (a) exploratory data analysis using unsupervised fuzzy clustering and (b) establishing a logical relation between the structures of the feature space and the class labels using fuzzy MOS based weight matrix. Simultaneously, a MOS weight matrix $W(N \times C)$ is formed by incorporating human perception on each training images where each element w_{ij} represents degree of belongingness of i -th image in j -th classes.

3.1. MOS entropy based weight matrix

Utilizing human perception about the visual quality of the images, MOS entropies are computed and classified using Algorithm 2 and the following equation:

$w_{iq} = E_{x_i}$ for class q and

$$w_{il} = \frac{1 - E_{x_i}}{C - 1} \quad \text{for other classes}(l \neq q) \quad (3)$$

where x_i is the MOS of i -th image, w_{il} represents the class membership of i -th image to class l , C is the total number of classes and E_{x_i} stands for Shannon's entropy of image i , defined in the below equation:

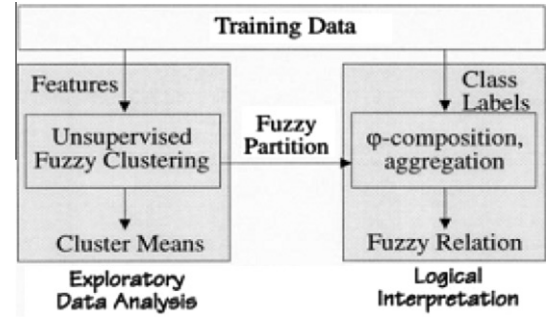


Figure 6 Fuzzy relational classifier training phase.

$$E_{x_i} = -p_{x_i} \log [(p)_{x_i}] \quad \text{where } p_{x_i} = \frac{x_i}{\sum_{i=1}^N x_i} \quad (4)$$

and N is the total number of images.

Algorithm 2 (classifying MOS entropies)

Input: Five class labels: “Excellent”, “Good”, “Average”, “Bad” and “Poor” with rank from high to low.

Begin

Step 1. Sort MOS entropy values of images in descending order

Step 2. Compute mean (M) of the Entropy data sets

Step 3. Denote maximum value of the data as E_{\max} and minimum value as E_{\min} .

Step 4. If entropy value of an image $\geq M$ and $\leq E_{\max}$ then Assign Class label to the image $>$ “Average” (i.e. “Excellent”, “Good”)

Else

Assign Class label to the image \leq “Average” (i.e. “Average”, “Bad”, “Poor”)

Step 5. Set $E_{\min} = M$ and compute new mean ($m1$) of the data having range E_{\max} to E_{\min}

If entropy value of an image $\geq m1$ and $\leq E_{\max}$ then

Assign Class label to the image $>$ “Good” (i.e. “Excellent”)

Else

Assign Class label to the image \leq “Good” (i.e. “Good” as classification under “Average” category is already done)

Step 6. Set $E_{\max} = M$ and repeat step 5 with assignment of the class label of the image being changed to “Bad”.

Step 7. Repeat step 5 and step 6 until all Entropy values are covered.

End.

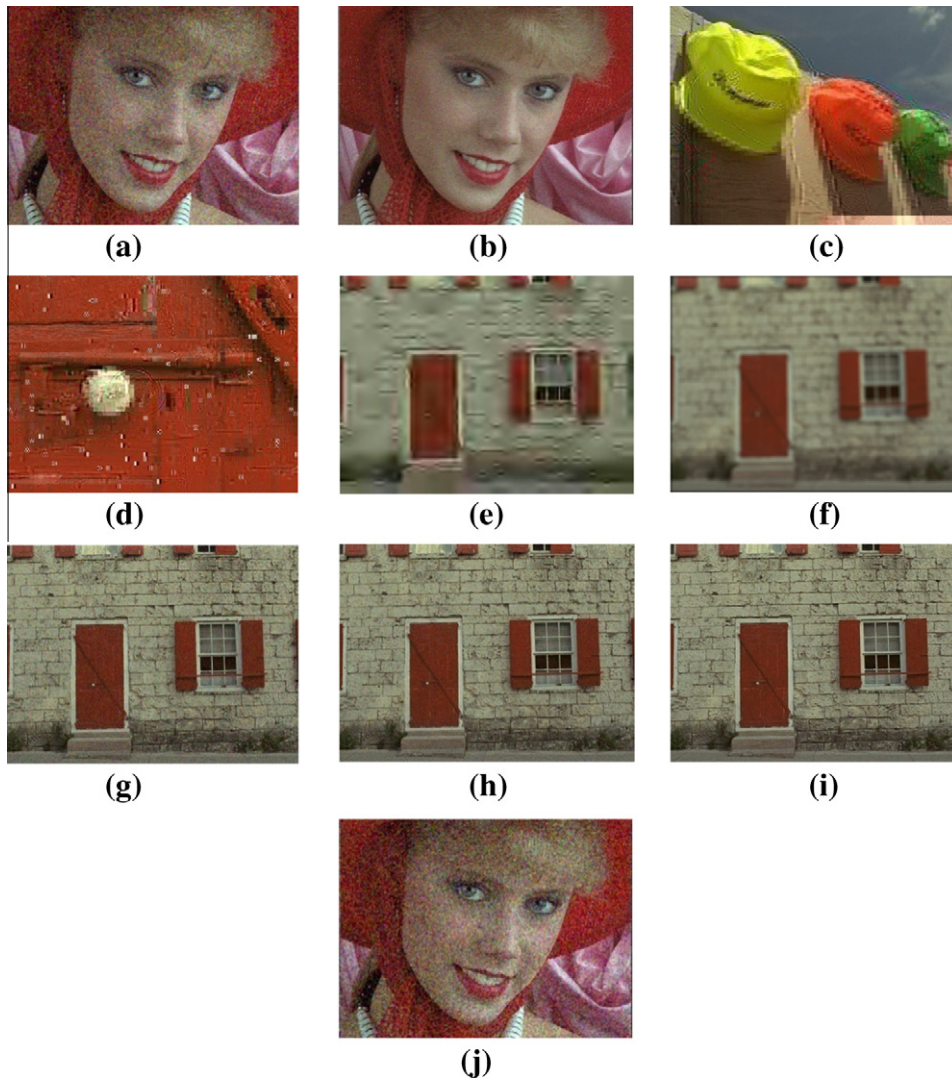


Figure 7 Training images: (a) Image 1, (b) Image 2, (c) Image 3, (d) Image 4, (e) Image 5, (f) Image 6, (g) Image 7, (h) Image 8, (i) Image 9 and (j) Image 10.

Table 3 Fuzzy relational matrix (R).

| Clusters ↓ | Class labels → | Excellent | Good | Average | Bad | Poor |
|---------------|-------------------|-----------|--------|---------|--------|--------|
| Cluster 1 | | 0.0223 | 0.01 | 0.0208 | 0.0126 | 0.0223 |
| Cluster 2 | | 0.01 | 0.0208 | 0.0126 | 0.0223 | 0.0281 |
| Cluster 3 | | 0.0208 | 0.0126 | 0.0223 | 0.01 | 0.0208 |
| Cluster 4 | | 0.0126 | 0.0388 | 0.01 | 0.0393 | 0.0306 |

The output matrix provides the MOS entropy based classification weight, which is used to obtain fuzzy relational matrix. Table 2 shows MOS weight matrix generated using 10 training images of Fig. 5.

3.2. Image partitioning

Ten training images (Fig. 7) from TAMPERE databases (Ponomarenko et al., 2009) are collected and different kinds of distortion is applied, as listed in Table 1 with corresponding MOS

value. Images are partitioned based on the entropy of feature sets using FCM (Dunn, 1973) algorithm. The element μ_{ij} of the partition matrix P specifies degree of membership of i -th image ($i = 1, \dots, N$) in j -th clusters ($j = 1, \dots, c$). Number of clusters is set to four ($c = 4$) with fuzziness exponent value 2.5 ($m = 2.5$) as determined experimentally. After clustering, the dataset has been validated (Fig. 5) using Bezdek's classification entropy index (Bezdek, 1980). Xie-Beni index is also tried but it fails to predict optimal number of clusters for very large number of clusters (Xie and Beni, 1991). Number of clusters vs. classification entropy graph (Fig. 6) for different fuzz-



Figure 8 Test images, from left to right and top to bottom – Img132, Img162, I04, I01, Chinacongress distorted, Chinacongress original, Bernstein distorted, Annan original, Afghan distorted and compressed Lena image.

Table 4 The comparison of the proposed quality metric with other quality metrics.

| Image, taken from different databases, indicated in parentheses | Fuzzyness exponent, value (m) | Fuzzy relation based image, quality in linguistic variable term | Blind image, quality index (linguistic variable) (Moorthy and Bovik) | Jpeg quality, score (linguistic variable) (Wang et al.) | JP2KNR (linguistic variable) (Sheikh et al.) |
|--|-----------------------------------|---|--|---|--|
| Img162 (LIVE) | 2.5 | Excellent | Good | Excellent | Good |
| Img132 (LIVE) | 2.5 | Excellent | Average | Average | Good |
| Chinacongress distorted (PROFILE) http://www.vasc.ri.cmu.edu/idb/html/face/profile_images/index.html | 0.75 | Poor | Average | Average | Good |
| Chinacongress original (PROFILE) | 2.5 | Excellent | Good | Excellent | Good |
| Annan original (PROFILE) | 0.75 | Excellent | Good | Excellent | Good |
| Lena image decompressed with codebook size 1024 | 2.5 | Excellent | Good | Average | Good |
| I01 (TAMPERE) | 2.5 | Excellent | Good | Excellent | Good |
| I04 (TAMPERE) | 2.5 | Excellent | Good | Excellent | Good |
| Afghan Gaussian distorted (PROFILE) | 2.5 | Average | Average | Average | Good |
| Bernstein distorted (PROFILE) | 0.75 | Poor | Poor | Average | Good |

iness exponents (m) (Yu et al., 2004) reveals the fact that classification entropy (negative value) is minimized at number of clusters being 4. Therefore, while preparing the fuzzy relational matrix the number of clusters is taken as 4.

3.3. Class membership generation

In the second step of designing the FRC, ϕ -composition (a fuzzy implication) and conjunctive aggregation operators are applied, specifying the logical relationship between the cluster membership (partition matrix \mathbf{P}) and the class membership values (\mathbf{W} matrix). To classify new patterns, the membership of each pattern in the clusters (fuzzy prototypes) is computed by measuring its distance from the respective cluster centers, giving a fuzzy set of prototype membership. Then, applying relational composition operator an output fuzzy set is obtained that classifies the new pattern in terms of membership degrees in the respective classes. The process is described using Fig. 6.

The fuzzy relational matrix (\mathbf{R}) and the class membership of a test image in a particular class are obtained by executing the following steps:

Step 1: The partition matrix (\mathbf{P}) is combined with the MOS weight matrix (\mathbf{W}) using product implication method, given in the equation below (Setnes and Babuska, 1999):

$$(r_{ij}) = \min_{k=1,2,\dots,N} [\mu_{ik} \times w_{jk}] \quad (5)$$

where $(r_{ij})_k = \mu_{ik} + w_{jk} - \mu_{ik} \times w_{jk}$.

Each element of \mathbf{R} , r_{ij} represents degree of relation between i -th cluster and j -th class ($i = 1 \dots c$, $j = 1 \dots C$), μ_{ik} is the element of \mathbf{P} representing degree of membership of k -th image in i -th cluster ($k = 1 \dots 10$ and $i = 1 \dots 4$) and w_{jk} is the element of \mathbf{W} (MOS weight matrix) provides weight of image k in class j .

Step 2: For a particular image, say s , the class membership Ω_j in a particular class j is computed from the relational composition, as given in the following equation:

$$\Omega_j = \max_{(i=1,\dots,c)} [\mu_{is} \times r_{ij}] \quad (6)$$

Step 3: The class membership value is converted to linguistic information to obtain the quality metric of the test images.

Table 3 shows the fuzzy relational matrix \mathbf{R} obtained using Eq. (5).

The class memberships of the test images (Fig. 8) are computed using Eq. (6) and accordingly its quality label is determined. Table 4 shows comparison with the proposed quality metric and other no-reference quality metrics.

4. Results

To build the MOS weight matrix (\mathbf{W}), Mean Opinion scores of ten training images are used as given in Table 2.

5. Conclusions and future works

The concept of fuzzy relational classifier has been utilized in the paper to develop a no-reference image quality assessment technique of distorted and decompressed images. Important scale invariant local features are selected and then partitioned based on the entropy of features, thereby reducing dimensionality and complexity of the system and at the same time avoid-

ing information loss. The variation of selected features are wide enough for capturing important information from the images. To avoid human biasing in assigning class label to the test images, variation of human observations are incorporated by estimating continuous weight based on entropy of MOS in different classes. The effect of fuzziness exponent on classification process is also studied in assessing quality of image. The proposed no-reference image quality metric has been compared with the existing quality metric producing satisfactory result.

The future work can be performed with the objectives: (i) improve classification accuracy by optimizing the MOS weight matrix using evolutionary algorithms and (ii) enhance the quality of the images by parametric learning methods.

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