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A three phase supplier selection method based on fuzzy preference degree

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Abstract As competition is growing high on this globalized world, the companies are imposing more and more importance on the process of supplier selection. After the foundation of fuzzy logic, the problem of supplier selection has been treated from the viewpoint of uncertainty. The present work reviews and classifies different approaches towards this problem. A new fuzzy preference degree between two triangular fuzzy numbers is introduced and a new approach is prescribed to solve the problem using this preference degree. The weights of the Decision Makers are considered and a methodology is proposed to determine the weights. Moreover, a unique process of classifying the suppliers in different groups is proposed. The methodologies are exemplified by a suitable case study.

1. Introduction

Supplier selection problem is one of the most significant one in Supply Chain Management (SCM). In today's extremely competitive corporate environment, it sounds airy to produce high quality and low cost products without a satisfactory supplier or a group of satisfactory suppliers. It is quite substantial that the better selection of suppliers reduces the purchasing cost and increases the competitive attitude of the companies. So the objective of the evaluation process of the supplier selection problems is to maximize the overall value to the purchaser and build proper relationship between buyers and suppliers. In literature several methods (Chen, 2000; Kheljani et al., 2010; Kumar et al., 2004; Li et al., 2007; Liu and Liu, 2010; Muralidharan et al., 2002; Shyur and Shih, 2006; Sreekumar, 2009; Vaezi et al., 2011; Wang, 2005) have been proposed to solve the problem through different kinds of methodology. Some of them are Weighting Method, Statistical Method, (Analytic Hierarchy Process) AHP, Data Envelopment Analysis (DEA), Technique for Ordered Preference by Similarity to Ideal Solution (TOPSIS) etc. In 2007, Li et al. proposed a grey based TOPSIS method to rank the suppliers by aggregating the DMs' opinion on the suppliers and attributes. The authors used grey possibility degree to compare each supplier with the ideal supplier. The methodology was followed in some other research articles too (Jadidi et al., 2008; Mukherjee and Kar, 2012). Muralidharan et al. (2002) employed a novel

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model based on aggregation technique for combining DMs' preferences into one consensus ranking. A brief of the existing literature will be discussed in Section 3.

The application and evaluation of the weights of the DMs is also a very crucial part of this type of decision making procedure. In literature, it is quite noticeable that there are only few techniques which involve the weights of the DMs in the methodology. But this assumption may affect the process of decision making, as a finite number of human beings' perception on a certain matter should not be considered as equally likely. In this paper a new technique is proposed for the said problem.

Our motive for the concerned problem certainly arises due to the following drawbacks of the existing literature.

1. Weights of the DMs have not been studied in most of the methods.
2. The suppliers have not been classified even after the completion of the ranking procedure, in any logical way.

Moreover the fuzzy preference degree (introduced in this paper) needs to be tested on a well known decision making platform and supplier selection is certainly a better choice for that.

The proposed methodology is a three phase algorithm. The first phase evaluates the weights of the DMs following a novel procedure. The second phase executes the ordering of the suppliers. And the final phase classifies the suppliers in different groups. While performing the second phase, each supplier is compared to the Positive Ideal Supplier (PIS) and Negative Ideal Supplier (NIS) by the fuzzy preference degree between two Triangular Fuzzy Numbers (TFN).

The paper is structured as follows. Section 2 discusses preliminary ideas and concepts, relevant to the topic. In Section 3, a brief review of the existing literature on supplier selection problem is described along with a moderate classification. Five significant supplier selection methods are picked out of them and we demonstrate their working algorithms. In Section 4, fuzzy preference degree is proposed along with its properties. Section 5 exhibits the projected methodology and in Section 6, a case study is provided to depict the effectiveness of the method. A comparative analysis is also provided, later in this section.

2. Preliminaries

The concept of fuzzy logic and fuzzy mathematics was introduced by Zadeh (1965) in 1965, when the two-valued logic completes its era. Initially it was given in prescribed form for engineering purposes and it got some time to accept this new methodology from different intellectuals. For a long time a lot of western scientists has been apathetic to use fuzzy logic because of its threatening to the integrity of older scientific thoughts. But once it got the stage, it performed fabulously. From mathematical aspects to engineering systems, it spreaded all over and the betterments of all types of systems were certainly there. After all, the society chose Fuzzy Logic as a better choice. In Japan, the first sub-way system was built by the use of fuzzy logic controllers in 1987. Since then almost every intelligent machine works with fuzzy logic based technology inside them.

In this section some preliminary concept on fuzzy and grey systems is overviewed.

Let X is a collection of objects called the universe of discourse. A fuzzy set denoted by $\widetilde{A}$ on X is the set of ordered pairs $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)) : x \in X\}$ where $\mu_{\widetilde{A}}(x)$ is the grade of membership of x in $\widetilde{A}$ and the function $\mu_{\widetilde{A}}(x) : \mathrm{X} \to [0, 1]$ is called the membership function.

2.1. Fuzzy numbers and TFNs

Definition 2.1.1. Let a be a given crisp number on the real line R. If there lies some uncertainty while defining a then we can represent a along with its uncertainty by an ordinary fuzzy number $\widetilde{A}$. To represent $\widetilde{A}$ mathematically and graphically a membership function $\mu_{\widetilde{A}}(x)$ is used which must satisfy the following conditions:

1. $\mu_{\widetilde{A}}(x)$ is upper semi continuous.
2. In a certain interval $[a, b]$ on R, $\mu_{\widetilde{A}}(x)$ is non zero, and otherwise it is zero.
3. There exists an interval $[c, d] \subset [a, b]$ such that
   (i) $\mu_{\widetilde{A}}(x)$ is increasing in $[a, c]$
   (ii) $\mu_{\widetilde{A}}(x)$ is decreasing in $[d, b]$, and
   (iii) $\mu_{\widetilde{A}}(x) = 1$ in $[c, d]$.

Now a TFN $\widetilde{A}$ satisfies all the above conditions and it is represented by $\widetilde{A} = (a, b, c)$.

Let us consider two TFNs $\widetilde{X} = (x_1, x_2, x_3)$, $\widetilde{Y} = (y_1, y_2, y_3)$ and a crisp number c. Then the basic arithmetic operations are as follows:

$\widetilde{X} \oplus \widetilde{Y} = (x_1 + y_1, x_2 + y_2, x_3 + y_3)$,
$\widetilde{X} \sim \widetilde{Y} = (x_1 - y_1, x_2 - y_2, x_3 - y_3)$,
$\widetilde{X} \otimes \widetilde{Y} \approx (x_1y_1, x_2y_2, x_3y_3)$ [Multiplication results an approximate fuzzy number]
and $\widetilde{X} \otimes c = (cx_1, cx_2, cx_3)$.

Definition 2.1.2. The distance between the TFNs $\widetilde{X} = (x_1, x_2, x_3)$ and $\widetilde{Y} = (y_1, y_2, y_3)$ is defined by Chen (2000) as:

$$d(\widetilde{X}, \widetilde{Y}) = \sqrt{\frac{1}{3}\left[(x_1 - y_1)^2 + (x_2 - y_2)^2 + (x_3 - y_3)^2\right]} \tag{2.1}$$

2.2. Grey system and interval grey numbers

Grey system theory (Deng, 1989) was proposed by Deng on the basis of grey sets. The systems that lack in information are pertained as Grey Systems. In the perspective of any type of numbers, Grey numbers represent the information between completely known and completely unknown situations, i.e., Grey System is the bridge connecting White System and Black System. We now take a look on some definitions of Grey theory.

Let X is the universal set of considerations. Then a Grey set G of X is defined by its two mappings $\bar{\mu}_G(x)$ and $\underline{\mu}_G(x)$:

$\bar{\mu}_G(x) : X \to [0, 1]$ and $\underline{\mu}_G(x) : X \to [0, 1]$ such that $\bar{\mu}_G(x) \geqslant \underline{\mu}_G(x), x \in X$. The Grey set G becomes a fuzzy set when the upper and lower membership functions in G are equal to each other, i.e., when $\bar{\mu}_G(x) = \underline{\mu}_G(x)$. When the lower and upper limits of any information can be estimated by real numbers, we certainly are able to express it by an IGN $\otimes G = [\underline{G}, \bar{G}] = \{\theta \in \otimes G : \underline{G} \leqslant \theta \leqslant \bar{G}\}$ where θ is an information and $\underline{G}, \overline{G}$ are respectively the lower and upper limits of the information's existence.

The degree of greyness, denoted by $\tilde{g}(\otimes G)$ is defined by a function of the two ends of the interval, i.e., $\tilde{g}(\otimes G) = f(\underline{G}, \overline{G})$.

An interval valued fuzzy set in X is given by A and is defined by $A = \{(x, \mu_A(x)) : x \in X\}$ where $\mu_A(x){:}X \to D[0, 1]$ defines the degree of membership of an element x to A and D[0, 1] denotes the family of sub closed intervals of [0, 1].

The degree of greyness of a grey set is the same as of the grey number with the same boundary of grey set.

According to Wang and Wu's approach (Wang and Wu) we now define some basic grey number operations:

$$\otimes G_1 + \otimes G_2 = [\underline{G}_1 + \underline{G}_2, \overline{G}_1 - \overline{G}_2]$$
$$\otimes G_1 - \otimes G_2 = [\underline{G}_1 - \overline{G}_2, \overline{G}_1 - \underline{G}_2]$$
$$\otimes G_1 - \otimes G_2 = [\min(\underline{G}_1\underline{G}_2, \underline{G}_1\overline{G}_2, \overline{G}_1\underline{G}_2, \overline{G}_1\overline{G}_2), \max(\underline{G}_1\underline{G}_2, \underline{G}_1\overline{G}_2, \overline{G}_1\underline{G}_2, \overline{G}_1\overline{G}_2)]$$
$$\otimes G_1 \div \otimes G_2 = [\underline{G}_1, \overline{G}_1] \times \left[\frac{1}{\underline{G}_2}, \frac{1}{\overline{G}}\right]$$

We cite (Li et al., 2007) to obtain the Grey Possibility Degree of $\otimes G_1 \leqslant \otimes G_2$ as

$$P\{\otimes G_1 \leqslant \otimes G_2\} = \frac{\max(0, L(\otimes G_1) + L(\otimes G_2) - \max(0, \overline{G}_1 - \underline{G}_2))}{L(\otimes G_1) + L(\otimes G_2)} \tag{2.2}$$

where $L(\otimes G) = \overline{G}_1 - \underline{G}_2$.

It is clear from the concept of possibility, that

i) when $\otimes G_1 = \otimes G_2$, then $P\{\otimes G_1 \leqslant \otimes G_2\} = 0.5$,
ii) when $\overline{G}_1 < \underline{G}_2$, then $P\{\otimes G_1 \leqslant \otimes G_2\} = 1$, and
iii) when $\overline{G}_1 > \underline{G}_2$, then $P\{\otimes G_1 \leqslant \otimes G_2\} = 0$.

Clearly these two stages (grey sets and interval valued fuzzy sets) represent two different kinds of approach towards representing uncertainty. They differ in both philosophical and practical concepts.

For the grey sets the degree of greyness is defined for the whole set while for the interval valued fuzzy sets, fuzziness is defined for individual elements. The relations $\leqslant$, $\prec$ and $=$ in grey sets occur for the components of two grey sets with members that may be different. But the same relations in interval valued fuzzy sets occur for two fuzzy sets with identical members. Philosophically greyness represents lack of knowledge about data. The interval of a grey set is the domain of definition corresponding to a white number. On the contrary, the membership degrees of the members of a fuzzy set represent measures of belief in some concepts. The interval of an interval valued fuzzy set is about the scope of its membership. Thus when additional information is supplied to a grey set, it becomes white. But when additional information is supplied to an interval valued fuzzy set, the belief measure gets stronger and a more precise membership value is obtained, the set remains fuzzy.

2.3. Linguistic term, linguistic scale and decision vector

Another important part in this aspect is linguistic terms and their expressions in fuzzy and grey systems. Sometimes, while dealing with scientific problems, we face both qualitative and quantitative aspects. The first one can be easily handled by precise numeric quantities. But for the qualitative aspects, we should not use precise or exact values, as uncertainty exists therein. For this problem of modeling uncertain information, sometimes linguistic terms are used. For a certain type of information, a fixed set of linguistic terms are employed. The mathematical representation of the linguistic terms is case wise different. Most popular approaches to this regard are based on fuzzy systems, grey systems, interval number systems, etc. In fuzzy system we can represent them as interval valued fuzzy numbers, triangular fuzzy numbers, trapezoidal fuzzy numbers, etc. The fuzzy and grey linguistic approaches are important tools for scientific problem solving, especially in the areas of information retrieval, human resource management, service revolution, service revolution, decision making and web equality.

Linguistic scale is a scale of representation that consists of linguistic terms. To rate some alternatives or attributes or any other set of classified things, it is sometimes necessary to restrict the decision inputs in a finite set, or rather, in a scale where the inputs are nothing but linguistic terms. The scale is independent of the choice of the type of the representation of the linguistic terms. For example, to represent the speed of a car, we can choose the linguistic scale {very high, high, medium, low, very low} or {extremely high, very high, medium high, medium, low, very low, extremely low} or any other scale.

Any non empty subset of the linguistic scale can be taken as a decision vector, provided the number of elements is equal to the number of attributes. In fact, decision vector is the decision of a DM on a certain alternative over all the attributes.

3. Literature review

In literature supplier selection problem has been viewed from different angles. We have classified them in mainly five categories: multi attribute decision making (MADM) approaches, multi objective decision making (MODM) approaches, statistical approaches (SA), intelligent approaches (IA) and other approaches.

Normally in an MADM approach, we have to choose the best alternative from a set of n alternatives $\{X_1, X_2, \ldots, X_n\}$ where the performance of the alternatives are judged on the basis of a set of m attributes $\{A_1, A_2, \ldots, A_m\}$ by a group of p DMs $\{E_1, E_2, \ldots, E_p\}$. The comments of each DM are recorded on a rectangular matrix,

$$\mathrm{G_k} = \begin{array}{c} \\ X_1 \\ \vdots \\ X_n \end{array} \begin{array}{c} \begin{array}{ccc} A_1 & \cdots & A_m \end{array} \\ \begin{pmatrix} G^k{}_{11} & \cdots & G^k{}_{1m} \\ \vdots & \ddots & \vdots \\ G^k{}_{n1} & \cdots & G^k{}_{nm} \end{pmatrix} \end{array},$$

where $k = 1, 2, \ldots, p$. Thus we have p such matrices.

Again sometimes the DMs are asked to weight the attributes. For this, we have another matrix

$$\text{matrix W=} \begin{array}{c} \\ A_1 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{ccc} E_1 & \cdots & E_p \end{array} \\ \begin{pmatrix} W_{11} & \cdots & W_{1p} \\ \vdots & \ddots & \vdots \\ W_{m1} & \cdots & W_{mp} \end{pmatrix} \end{array}.$$

In different methodologies proposed in the literature (Chen, 2000; Kheljani et al., 2010; Kumar et al., 2004; Li et al., 2007; Liu and Liu, 2010; Muralidharan et al., 2002; Shyur and Shih, 2006; Sreekumar, 2009; Vaezi et al., 2011; Wang, 2005), the researchers have tried to extract the decision or ranking on the basis of the above mentioned $p + 1$ matrix. Earlier the elements of the matrices G_k and W were considered to be real scalars. But in recent literature, it is counted that the entries are being taken as linguistic terms. Each method, using linguistic terms follow two different scales of decision inputs, one for G_k's and another for W. For example {VP, P, MP, F, MG, G, VG} is a seven point scale on alternative ranking and {VL, L, ML, M, MH, H, VH} is another seven point scale on attribute ranking. MADM is more general view of Linear Weighting model. Actually this model concentrates only on the supplier selection activities. TOPSIS, AHP, LW and ANP (Analytical Network Process) models can be classified in this category.

MODM is a more general view of Mathematical Programming (MP) or Linear Programming (LP) models. It involves the ideal model for the best supplier by interacting within the model constraints that best satisfy the decision maker. So there exists a constraint set and the suppliers are expressed in the feasible region of this set. Approaches like ε-constraint method, DEA, MP can be classified in this category. MP models actually formulate the given situation in terms of a mathematical objective function that afterwards needs to be either maximized or minimized, by varying the values of the variables in the objective function. In this category, fuzzy models have also been introduced. The best part in these models is that it impels the evaluator to express the objective function explicitly. Thus they are more objective than the other rating models.

Statistical Approaches are mainly based on the hypothesis that stochastic uncertainty is very much related to the supplier selection process. And thus it concentrates on the evaluation over a large number of surveys or deals. Cluster Analysis,

Table 3.1 Categories and sub-categories of supplier selection problem.

MADM	TOPSIS
	Li et al. (2007); Jadidi et al (2008); Sreekumar and Mahapatra (2009)
	Jadidi et al. (2010); Izadikhah (2011); Mukherjee and Kar (2012)
	AHP
	Narasimhan (1983); Nydick & Hill (1992); Barbarosoglu & Yazgac (1997)
	Masella & Rangone (2000); Hwang, Chuang & Jong (2003); Özkan, Başlıgil & Şahin (2011)
	Vaezi, Shahgholian, Shahraki (2011))
	LW
	(E. Timmermann (1986); K. N. Thompson (1990); Dobler & Burt (1996); Petroni & Braglia (2000))
	Outranking methods
	(de Boer et al. (1998))
	ANP
	(Sarkis & Talluri (2000); Shyur & Shih (2006))
MODM	ε Constraint method
	(Weber & Current (1993))
	DEA
	(Weber (1996); Papagapion et al. (1996); Weber et al. (1998); Liu et al. (2000)
	Forker and Mendez (2001); Talluri and Sarkis (2002); Sean (2006); Wu & Blackhurst (2009)
	Ramanathan (2007); Wu (2009))
	MP
	(Buffa & Jackson (1983); Chowdhury, Forst & Zydiac (1993); Kumar, Virat, and Shankar (2004)
	Kheljani, Ghodsypour & Ghomi (2010))
SA	Cluster analysis
	(Hinkle et al. (1969); Holt (1998))
	Uncertainty analysis
	(Soukoup (1987))
IA	CBR
	(Ng et al. (1995))
	Expert system
	(Cook (1997))
	GA
	(Ding et al. (2003))
	NN
	(Wei et al. (1997))
Others	TCO
	(Monezka & Trecha (1998); Smytka & Clemens (1993))
	ISM
	(Mandal & Desmukh (1994))
	Categorical methods
	(Zenz (1981); Timmermann (1986))

Uncertainty Analysis are two sub categories within it. Existing models only accommodate for uncertainty with regard to one criterion at a time.

Case Based Reasoning (CBR), Expert System analysis, Genetic Algorithm (GA) techniques, Neural network methods are classified in the category of Intelligent Approaches.

Some other techniques are also in the literatures, which belong to none of the above categories. Total Cost of Ownership (TCO), Interpretive Structural Model (ISM) and categorical methods are among them.

Now the supplier selection approaches considering each of these five categories with their sub categories are displayed in Table 3.1.

So, according to several authors, we conclude that TOPSIS is more reliable than LW, MP and AHP. Now two trends are generally observed in TOPSIS. The first type considers comparison of the suppliers with the positive ideal solution (PIS) only. But it is also noticeable that the best supplier (selected by first type of TOPSIS), which has smallest possibility degree from PIS, may has a lower possibility degree from the Negative Ideal Solution (NIS), as compared to other suppliers. So, second type of TOPSIS deals with both PIS and NIS and creates a closeness co-efficient which combines the possibility degree from both PIS and NIS. Here the concept is that the best supplier should be at a shortest distance from PIS and NIS.

More detailed review can be found in de Boer et al. (2001), Kontis and Vrysagotis (2011) and Shyur and Shih (2006).

In some of the existing literatures DMs' weights have been taken into considerations. Keeny and Kirkwood (1975) suggested the use of interpersonal comparison to obtain the weights on an additive scale. Bash (1975) used a Nash bargaining based approach to evaluate the weights intrinsically. Mirkin (1979) developed an eigenvalue method for deriving the same. In this method, a DM is asked to rank other DMs on a scale. The eigen values of the obtained square matrix are evaluated and the normalized values represent the weights. Mukherjee and Kar (2012) proposed another method by the distance from the mean of all DMs' decisions.

All of the above mentioned approaches have some sort of drawbacks. Those will be discussed in the comparative study in Section 7.

Now we summarize five important methods that obey the rules of TOPSIS to some extent.

3.1. Review of Grey based method by Li et al. (2007)

Considering m suppliers $\{S_1, S_2, \ldots, S_m\}$ and n attributes $\{A_1, A_2, \ldots, A_n\}$ a group of k decision makers $\{D_1, D_2, \ldots, D_k\}$ is appointed to order the preference of the suppliers and the attributes separately. The decisions on the supplier rating and linguistic terms those are restricted in the seven-point scale {very poor, poor, medium poor, fair, medium good, good, very good}. Also the decisions on the attribute rating are linguistic terms that are restricted in the seven point scale {very low, low, medium low, medium, medium high, high, very high}. All these linguistic terms are represented by interval valued grey numbers as shown in table. The algorithm is as follows:

1. Weights of the attributes are calculated by averaging the IGNs.
2. For each attribute the decision IGNs are averaged to constitute the Grey Decision Matrix.
3. Elements of the GDM are normalized by the maximum element to constitute the Normalized Grey Decision Matrix (NGDM).
4. Weighted NGDM is formed by multiplying elements with the corresponding attribute's weights.
5. The Ideal Supplier S^{Max} is constructed whose IGNs are the maximum elements of the corresponding branch.
6. The grey possibility degree $P\{S_i \leqslant S^{\mathrm{Max}}\}$ is calculated. If $P\{S_i \leqslant S^{\mathrm{Max}}\}$ is smaller, the rank of S_i is better.

3.2. Advancement of Li et al.'s method by Jadidi, Yusuff, Firouzi and Hong (Jadidi et al., 2008)

Jadidi, Yusuff and Hong proposed an advancement of Li et al.'s method. Firstly, they proposed the possibility degree of $G_1 \geqslant G_2$ as

$$P\{G_1 \geqslant G_2\} = \frac{\max(0, L^* - \max(0, \overline{G}_2 - \underline{G}_1))}{L^*}, \tag{3.1}$$

where $G_1 = [\underline{G}_1, \overline{G}_1], G_2 = [\underline{G}_2, \overline{G}_2]$ and $L^* = \mathrm{L(G_1)} + \mathrm{L(G_2)}$, L being the length of the IGN, which is quite similar to Eq. (2.2).

In their method, the previous six steps are similar to (Li et al., 2007). The next steps are as follows:

1. Construct the negative ideal solution S^{Min} whose IGNs are constituted by the minimum elements of the corresponding branches.
2. Calculate $\mathrm{P}(S_i \geqslant S^{\mathrm{Min}})$.
3. Calculate the relative closeness of each supplier to the IS by $\mathrm{C}_i = \frac{P(S_i \leqslant S^{\mathrm{Max}})}{P(S_i \geqslant S^{\mathrm{Min}})}$.
4. The supplier with minimum C_i is the best.

In both of the above methods, the concept of TOPSIS has been implemented.

3.3. Review of Sreekumar, Mahapatra's method (Sreekumar, 2009)

Considering m suppliers, n attributes and p Decision Makers the authors record the responses of the DMs in the form of linguistic terms in six point scale.

In this method, the authors proposed a fuzzy mathematics based TOPSIS type approach, where DMs are of variable importance. The weights of the DMs have been derived by AHP like procedure based on Eigen Value.

The algorithm of the proposed method is given below:

1. Weights of the DMs are computed.
2. From the set of TFNs in the decision matrix of attribute rating, the weights of the attributes are evaluated by Min-Average-Max principle.
3. The decision matrix of supplier rating is constituted. Each TFN of this decision matrix is multiplied by corresponding DM's weight.
4. The aggregate fuzzy rating of suppliers on the basis of the attributes is evaluated by the Min- Avg-Max principle.
5. The obtained matrix is normalized by the largest element.

6. The normalized matrix is weighted by the set of attribute weights.
7. Fuzzy Positive Ideal Solution (FPIS) and Fuzzy Negative Ideal Solution (FNIS) are constructed.
8. For each supplier, closeness co-efficient is calculated. The supplier with largest closeness co-efficient is the best choice.

3.4. Review of Vaezi, Shahgholian, Shahraki's method (Vaezi et al., 2011)

In this method, a defuzzification based supplier selection approach is proposed.

The algorithm is described as follows:

1. Weights of the DMs are obtained by assigning linguistic terms to them. The linguistic terms are defuzzified to get the weights.
2. The decision inputs for the attribute ratings are defuzzified and normalized (to lie between 0 and 1).
3. The decision inputs for supplier rating (on various attributes) are multiplied by corresponding DM's weights.
4. The values are weighted by the attribute weights.
5. The weighted fuzzy numbers are defuzzified. We denote them as X_i^* for the ith supplier.
6. If X_i be the score of the ith supplier, then X_i as $X_i = \frac{X_i^* - \min_{i=1,2,\ldots}\{X_i^*\}}{\max_{i=1,2,\ldots}\{X_i^*\} - \min_{i=1,2,\ldots}\{X_i^*\}}$. The supplier with best score is the first choice for the company.

3.5. Review of Mukherjee and Kar's method (Mukherjee and Kar, 2012)

This is a fuzzy mathematics based approach where the decision inputs have been considered as Triangular Fuzzy Numbers (TFN). Considering m attributes $C_1, C_2, \ldots, C_m$; p DMs $D_1, D_2, \ldots, D_p$ and n Suppliers $X_1, X_2, \ldots, X_n$ the authors have used six point linguistic scales for both type of decision inputs (supplier rating and attribute rating). Here DMs are not considered of equal importance. An algorithmic approach has been proposed to identify the weights of the DMs. The methodology is summarized as follows.

1. From the decision table of supplier rating, the TFNs of the inputs for a supplier X_i with respect to all attributes are averaged. The resulting mean is also a TFN. Each DM's decision inputs are now compared with the corresponding Mean by using the distance between two TFNs, proposed by Chen (2000).
2. The attribute weights are calculated using Min – Avg – Max principle.
3. The input TFNs are multiplied by the corresponding DM's weights.
4. The resulting elements are again aggregated by Min – Avg – Max principle to obtain the fuzzy decision matrix (FDM).
5. FDM is normalized by its largest element to obtain NFDM (Normalized Fuzzy Decision Matrix).
6. The TFNs of NFDM are multiplied by corresponding attribute weights to obtain the WNFDM (Weighted Normalized Fuzzy Decision Matrix).
7. A pseudo alternative S^{IA} is constructed by taking the minimum numbers with respect to all the attributes. Surely S^{IA} consists of m TFNs.
8. The distance of each TFN of the WNFDM from the TFN of the corresponding IA is calculated. The total distance is calculated.

Supplier with minimum distance gets the best rank.

4. Fuzzy preference degree between two TFNs

Comparing TFNs over real field is an important part in decision making problems. In this section, we introduce fuzzy preference degree between two TFNs. this preference degree can be applied not only to the supplier selection problems, but also to any other decision making problems where uncertain information is used to formulate the model.

Let us consider two TFNs $\widetilde{T}_1 = (a_1, b_1, c_1)$ and $\widetilde{T}_2 = (a_2, b_2, c_2)$. Also let x and y be two finite numbers satisfying $x \leqslant \mathrm{Min}\{a_1, a_2\}$ and $y \geqslant \mathrm{Max}\{c_1, c_2\}$. For a set of n TFNs $\widetilde{T}_i = (a_i, b_i, c_i), i = 1, 2, \ldots, n$, the choice of x and y should satisfy $x \leqslant \mathrm{Min}_i\{a_i\}$ and $y \geqslant \mathrm{Max}_i\{c_i\}$. Now in both of the above cases, the degree of preference of the ordered pair $(\widetilde{T}_1, \widetilde{T}_2)$ describes the degree of preferring $\widetilde{T}_2$ than $\widetilde{T}_1$ over the real finite scale $[x, y]$, denoted by $P(\widetilde{T}_1, \widetilde{T}_2)$, and is defined by

$$P(\widetilde{T}_1, \widetilde{T}_2) = \frac{\sqrt{(y-a_1)^2 + (y-b_1)^2 + (y-c_1)^2} - \sqrt{(y-a_2)^2 + (y-b_2)^2 + (y-c_2)^2}}{3(y-x)}. \tag{4.1}$$

Clearly, this preference degree is based on the distance between two TFNs, as shown in Eq. (2.1).

Some basic properties of $P(\widetilde{T}_1, \widetilde{T}_2)$

1. If $P(\widetilde{T}_1, \widetilde{T}_2) = 0$, then $\widetilde{T}_1 = \widetilde{T}_2$.
2. $-1 \leqslant P(\widetilde{T}_1, \widetilde{T}_2) \leqslant 1$.
3. If $P(\widetilde{T}_1, \widetilde{T}_2) \leqslant P(\widetilde{T}_1, \widetilde{T}_3)$, then we say that $\widetilde{T}_3 \geqslant \widetilde{T}_2$.
4. $P(\widetilde{T}_1, \widetilde{T}_2) = -P(\widetilde{T}_2, \widetilde{T}_1)$.

Example 4.1. Let us consider two TFNs $\widetilde{T}_1 = (1, 2, 3)$ and $\widetilde{T}_2 = (2, 4, 6)$.

If we consider $x = 1$ and $y = 6$, then $P(\widetilde{T}_1, \widetilde{T}_2) = 0.1733$.

Again, if we take $x = 0$ and $y = 10$, then $P(\widetilde{T}_1, \widetilde{T}_2) = 0.1053$.

Thus we observe that if we increase the length of the interval $[x, y]$, the value of $P(\widetilde{T}_1, \widetilde{T}_2)$ decreases and the vice versa.

5. Methodology

Let us deal with n suppliers $\{S_1, S_2, \ldots, S_n\}$ and m attributes $\{A_1, A_2, \ldots, A_m\}$. The objective is to choose the best supplier considering the status or position of the suppliers with respect to all the attributes. A group of p DMs $\{D_1, D_2, \ldots, D_p\}$ is appointed to submit their decisions on

i) the rating of the suppliers, and
ii) the rating of the attributes.

Table 5.1 Expression of the linguistic terms.

Linguistic term for attribute ratings	TFN	Linguistic term for attribute weights	TFN
Very poor	(1, 1, 2)	Very low	(0.1, 0.1, 0.2)
Poor	(2, 3, 4)	Low	(0.2, 0.3, 0.4)
Medium poor	(3, 4, 5)	Medium low	(0.3, 0.4, 0.5)
Fair	(4, 5, 6)	Medium	(0.4, 0.5, 0.6)
Medium good	(6, 7, 8)	Medium high	(0.6, 0.7, 0.8)
Good	(7, 8, 9)	High	(0.7, 0.8, 0.9)
Very good	(9, 10, 10)	Very high	(0.9, 1.0, 1.0)

The decisions are considered here as linguistic terms. For the rating of the suppliers, the scale of the linguistic terms is {VP, P, MP, F, MG, G, VG} and for the attributes, the scale is {VL, L, ML, M, MH, H, VH}. We represent these linguistic terms as TFNs as shown in Table 5.1.

We divide the methodology into three parts. The first part evaluates the weights of the DMs. In the second part, the ordering of the suppliers is performed. And in the last part, the suppliers are classified into groups.

The decisions on the supplier rating, given by the DMs, on all attributes, together constitute a decision matrix, say, Supplier Decision Matrix (SDM). Let us consider the general element of SDM by X_{ijk}, which is the decision of the kth DM on the ith supplier over jth attribute. Similarly, another decision matrix is constituted by the decisions on the attribute rating, given by the DMs. We call this Attribute Decision Matrix (ADM). Let us consider its general element by Y_{jk}.

5.1. Phase 1: finding DMs' weights

The proposed methodology is built on the hypothesis that the DMs should not be taken as equally important in the evaluation process. In Section 3, the existing approaches to evaluate DMs' weights have already been demonstrated along with their drawbacks. In this phase, a novel approach for the said problem is introduced.

At first, the company selects a particular supplier S_0 either from the given range or from any other list. The choice of S_0 demands that the company must have enough knowledge on S_0 from its previous performances over all the attributes. The decision vector for S_0, imposed by the company, will act as an ideal decision vector. Then the weights of the DMs will be extracted considering the mathematical distance between their own decision vectors and the ideal decision vector of S_0. So S_0 may or may not be a member of the set $\{S_1, S_2, \ldots, S_n\}$. The company pre-assumes the decisions on the rating of the supplier S_0 with respect to all the attributes. We consider the decision variable as X_{0j}, which is the decision on S_0 over jth attribute. The TFN of representation of X_{0j} is $T(X_0)$.

Now the decision makers are asked to submit their decisions on S_0 over all the attributes. Let the decision variable be X_{0jk} which is the decision of D_k on S_0 over A_j. The distance between $T(X_0)$ and $T(X_{0jk})$ reflects the gapping between two decisions. We denote the distance by $D\{T(X_{0j}), T(X_{0jk})\}$. Also we denote $d_k = \sum_j D\{T(X_{0j}), T(X_{0jk})\}$ which reflects the total distance of the kth DM from the standard decision made by the company, over all the attributes. Now the weight of the kth DM is denoted by $\omega(D_k)$ and is defined as

$$\omega(D_k) = \frac{1 - \frac{d_k}{\sum_k d_k}}{\sum_k \left(1 - \frac{d_k}{\sum_k d_k}\right)}. \tag{5.1}$$

Hence a decision maker D_j gets better weight than another decision maker D_k if the distance d_j is smaller than the distance d_k.

5.2. Phase 2

In this part, the most important task is performed the ordering of the suppliers. We describe this method stepwise. Let $T(X_{ijk}) = (a_{ijk}, b_{ijk}, c_{ijk})$, i.e., $T(X_{ijk})$ is the TFN of the linguistic variable X_{ijk}, defined earlier in this section.

Step 1

Let $T(Y_{jk}) = (a_{jk}, b_{jk}, c_{jk})$.

Now the weights of the jth attribute A_j is denoted by $\omega(A_j)$ and is defined by $\omega(A_j) = (a_j, b_j, c_j)$, where $a_j = \mathrm{Min}_k(a_{jk}\omega(D_k))$,

$b_j = \frac{1}{p}\sum_{k=1}^{p}(b_{jk}\omega(D_k))$, and
$c_j = \mathrm{Max}_k(c_{jk}\omega(D_k))$.

This aggregation technique is nothing but the Min – Avg – Max principle of combining a set of TFNs.

Step 2

The elements of the SDM are multiplied by the corresponding Decision Maker's weights. Let us denote this new variable by Z_{ijk}.

Then $T(Z_{ijk}) = (a_{ijk}\ \omega(D_k), b_{ijk}\ \omega(D_k), c_{ijk}\ \omega(D_k))$.

Now we construct the fuzzy decision matrix (FDM) whose general element $\widetilde{X}_{ij}$ is defined by its TFN $T(\widetilde{X}_{ij}) = (p_{ij}, q_{ij}, r_{ij})$ where

$p_{ij} = Min_k(a_{ijk}\omega(D_k))$,
$q_{ij} = \frac{1}{p}\sum_{k=1}^{p}(b_{ijk}\omega(D_k))$, and
$r_{ij} = Max_k(c_{ijk}\omega(D_k))$.Thus FDM is a $n \times m$ matrix where the DMs disappear as their decisions have already been aggregated by the Min-Avg-Max principle.

Step 3

The elements of the FDM are normalized by the greatest element $M = \mathrm{Max}_{i,j}\{p_{ij}, q_{ij}, r_{ij}\}$. Now the elements of the Normalized Fuzzy Decision Matrix (NFDM) certainly lie between 0 and 1.

Step 4

In this step we construct the Weighted Normalized Fuzzy Decision Matrix (WNFDM) by multiplying each element of the NFDM by its corresponding Attribute's weights. Let us denote this new variable by W_{ij} and consider $T(W_{ij}) = (l_{ij}, m_{ij}, n_{ij})$.

Thus $(l_{ij}, m_{ij}, n_{ij}) = \left(\frac{p_{ij}}{M}, \frac{q_{ij}}{M}, \frac{r_{ij}}{M}\right) \times (a_j, b_j, c_j)$,

i.e., $l_{ij} = \frac{p_{ij}}{M} \times a_j, m_{ij} = \frac{q_{ij}}{M} \times b_j$ and $n_{ij} = \frac{r_{ij}}{M} \times c_j$.

Step 5A pseudo alternative is constructed on the hypothesis that it is the best among all suppliers over all attributes. So we call it as Positive Ideal Supplier (PIS). So there must be a set of m TFNs to represent the PIS. For the jth attribute A_j, the TFN of the PIS is denoted by $(a_j^{PIS}, b_j^{PIS}, c_j^{PIS})$ and is defined by

$a_j^{PIS} = Max_i(l_{ij})$,
$b_j^{PIS} = Max_i(m_{ij})$, and
$c_j^{PIS} = Max_i(n_{ij})$.

Step 6

The fuzzy preference degree between two TFNs (discussed in section) is now applied to compare each supplier with the Positive Ideal Supplier. We compute the preference degree of each of the following pairs: $(T(W_{1j}), T(PIS_j))$, $(T(W_{2j}), T(PIS_j))$,...and so on.

Now we define $P_i^+ = \frac{1}{m}\sum_{j=1}^{m} P\{T(W_{ij}), T(PIS_j)\}$. Here P_i^+ represents the mean of the total preference degrees of the set of TFNs of the PIS and the set of TFNs of the ith supplier S_i.

Step 7

Again a pseudo alternative is constructed on the hypothesis that it is the negative best (worst) among all suppliers over all attributes. So we call it as Negative Ideal Supplier (NIS). So there must be a set of m TFNs to represent the NIS. For the jth attribute A_j, the TFN of the NIS is denoted by $(a_j^{\mathrm{NIS}}, b_j^{\mathrm{NIS}}, c_j^{\mathrm{NIS}})$ and is defined by

$a_j^{\mathrm{NIS}} = \mathrm{Min}_i(l_{ij})$,

$b_j^{\mathrm{NIS}} = \mathrm{Min}_i(m_{ij})$, and

$c_j^{\mathrm{NIS}} = Min_i(n_{ij})$.

Step 8

The preference degree of each of the pairs $(T(\mathrm{NIS}_j), T(W_{1j}))$, $(T(\mathrm{NIS}_j), T(W_{2j}))$,...and so on, are computed. Now we define $P_i^- = \frac{1}{m}\sum_{j=1}^{m} P\{T(\mathrm{NIS}_j), T(W_{ij})\}$. Here P_i^- represents the mean of the total preference degrees of the set of TFNs of the NIS and the set of TFNs of the ith supplier S_i.

Step 9

Finally the closeness co-efficient of each supplier S_i is defined by $\mathrm{CC}(S_i) = \frac{P_i^-}{P_i^+ + P_i^-}$. The supplier with maximum closeness co-efficient value will be the best choice for the company.

5.3. Phase 3

In this part the classification of the suppliers is prescribed. Normally, in the decision making process in supplier selection problems, the ordering of the suppliers is the final step. Now, if we consider the classification of the suppliers on a finite scale that will surely help the concerned company to shortlist the suppliers, in future. Here, in this paper, we propose three classifications: Below Average, Medium and Good. Following this methodology, one can classify the suppliers in more groups.

At first, to obtain the maximum value of the CC (Closeness Co-efficient) of the group Below Average, a fictitious supplier S_{BA} is considered whose rating vector from all DMs is considered as {MP, MP,..., MP}. This is based on the hypothesis that a poor supplier can mostly get these ratings from the DMs. The closeness co-efficient of S_{BA} is calculated and we denote $\mathrm{CC}(S_{\mathrm{BA}})$ by α_{BA}. Next to obtain the maximum CC of the group 'Medium', another fictitious supplier S_{M} is considered whose rating vector from all DMs is considered as {MG, MG,..., MG}. The closeness co-efficient of S_{M} is calculated and we denote $\mathrm{CC}(S_{\mathrm{M}})$ by α_M. Now it is obvious that the maximum CC of the group 'Good' is 1.

Thus we have three intervals of values of CC for three groups of the suppliers.

Then

1) $S_i \in$ BA Supplier if $\mathrm{CC}(S_i) \in [0, \alpha_{BA}]$,
2) $S_i \in$ Medium Supplier, if $\mathrm{CC}(S_i) \in (\alpha_{BA}, \alpha_M]$, and
3) $S_i \in$ Good Supplier, if $\mathrm{CC}(S_i) \in (\alpha_M, 1]$.

Decision making on the supplier selection is a dynamic process, where a decision on a supplier may be influenced by its previous performance. So this type of classification is also an important part of the problem.

6. Numerical example

Let us consider a situation of a company where it has to choose the best supplier from a panel of five suppliers $\{S_1, S_2, S_3, S_4, S_5\}$. The company appoints three DMs $\{D_1, D_2, D_3\}$ for the job. The DMs submit their valuable Decisions in linguistic terms for the suppliers over four attributes (Product Quality (A_1), Service Quality (A_2), Delivery Time (A_3) and Cost (A_4)). The decisions on supplier rating and attribute rating are respectively displayed in Tables 6.1 and 6.2.

6.1. Phase 1

Let the company pre-assume the Decision vector $\mathrm{C_V}$ = {G, MG, MP, F} for the supplier S_3. We calculate the distances between the DMs' decision vectors on S_3 and $\mathrm{C_V}$ and evaluate the weights of the DMs using (5.1).

We have $d_k = \sum_j D\{T(X_{\theta j}), T(X_{\theta jk})\}$Thus $d_1 = D\{(7, 8, 9), (7, 8, 9)\} + D\{(6, 7, 8), (9, 10, 10)\} + D\{(3, 4, 5), (3, 4, 5)\} + D\{(4, 5, 6), (3, 4, 5)\} = 3.708$.Similarly $d_2 = 6.440$, $d_3 = 3$. So $\sum_{k=1}^{3} d_k = 13.148$.Then by using (5.1) we get $w(D_1) = 0.3590$, $w(D_2) = 0.2551$, and $w(D_3) = 0.3859$.

6.2. Phase 2

Step 1

Using the aggregation technique (Min – Avg – Max), as described in the methodology, the weights of the attributes are evaluated as shown in Table 6.3.

Table 6.2 Decisions on attribute ratings.

	D_1	D_2	D_3
A_1	VH	H	VH
A_2	MH	MH	VH
A_3	H	VH	H
A_4	VH	VH	MH

Table 6.1 Decisions on supplier ratings.

S_i	S_1				S_2				S_3				S_4				S_5			
A_i	A_1	A_2	A_3	A_4	A_1	A_2	A_3	A_4	A_1	A_2	A_3	A_4	A_1	A_2	A_3	A_4	A_1	A_2	A_3	A_4
D_1	MG	G	F	VG	MP	G	G	F	G	VG	MP	MP	MP	MP	G	G	F	MG	MG	F
D_2	MG	G	F	MG	MP	MG	VG	MG	VG	VG	P	MP	P	MP	G	G	F	G	MG	F
D_3	MG	MG	F	G	P	MG	MG	F	G	G	P	MP	P	F	MG	MG	MP	MG	MG	MG

Table 6.3 Weights of the attributes.

	A_1	A_2	A_3	A_4
Weights	(0.7, 0.933, 1.0)	(0.6, 0.800, 1.0)	(0.7, 0.867, 1.0)	(0.6, 0.933, 1.0)

Table 6.4 FDM and the corresponding attribute weights.

	A_1	A_2	A_3	A_4
X_1	(1.5306, 2.2133, 3.0872)	(1.7857, 2.5380, 3.231)	(1.0204, 1.6667, 2.3154)	(1.5314, 2.8210, 3.590)
X_2	(0.7653, 1.2047, 1.7950)	(1.5306, 2.453, 3.231)	(2.2959, 2.7081, 3,271)	(1.436, 1.8367, 2.3154)
X_3	(2.2959, 2.8367, 3.4731)	(2.7013, 3.0761, 3.590)	(0.5102, 1.1197, 1.795)	(0.7653, 1.3333, 1.9295)
X_4	(0.5102, 1.1197, 1.795)	(0.7653, 1.462, 2.3154)	(1.7857, 2.538, 3.231)	(1.7857, 2.538, 3.231)
X_5	(1.0204, 1.538, 2.154)	(1.7857, 2.2984, 3.0872)	(1.5306, 2.2133, 3.0872)	(1.0204, 1.9239, 3.0872)
Weights	(0.7, 0.933, 1.0)	(0.6, 0.800, 1.0)	(0.7, 0.867, 1.0)	(0.6, 0.933, 1.0)

Table 6.5 NFDM with corresponding attribute weights.

	A_1	A_2	A_3	A_4
X_1	(0.4264, 0.6165, 0.8599)	(0.4974, 0.7070, 0.900)	(0.2842, 0.4643, 0.6450)	(0.4264, 0.7858, 1)
X_2	(0.2132, 0.3356, 0.5)	(0.4264, 0.6833, 0.900)	(0.6395, 0.7543, 0.9111)	(0.4, 0.5116, 0.645)
X_3	(0.6495, 0.7902, 0.9674)	(0.7524, 0.8568, 1)	(0.1421, 0.3119, 0.5)	(0.2132, 0.3714, 0.5375)
X_4	(0.1421, 0.3119, 0.5)	(0.2132, 0.4072, 0.645)	(0.4794, 0.7070, 0.900)	(0.4794, 0.7070, 0.900)
X_5	(0.2842, 0.4284, 0.6)	(0.4974, 0.6402, 0.8599)	(0.4264, 0.6165, 0.8599)	(0.2842, 0.5359, 0.8599)
Weights	(0.7, 0.933, 1.0)	(0.6, 0.800, 1.0)	(0.7, 0.867, 1.0)	(0.6, 0.933, 1.0)

Table 6.6 Weighted NFDM.

	A_1	A_2	A_3	A_4
S_1	(0.2985, 0.5752, 0.8599)	(0.2985, 0.5656, 0.9)	(0.1989, 0.4025, 0.645)	(0.2558, 0.7332, 1)
S_2	(0.1492, 0.3131, 0.500)	(0.2558, 0.5466, 0.9)	(0.4476, 0.645, 0.9111)	(0.24, 0.4773, 0.645)
S_3	(0.4476, 0.7372, 0.9674)	(0.4514, 0.6874, 1)	(0.0994, 0.2704, 0.5)	(0.1279, 0.3465, 0.5375)
S_4	(0.0995, 0.291, 0.5)	(0.1989, 0.3258, 0.645)	(0.3482, 0.613, 0.9)	(0.2984, 0.6596, 0.9)
S_5	(0.1989, 0.4, 0.6)	(0.2984, 0.5122, 0.8599)	(0.2985, 0.5345, 0.8599)	(0.1705, 0.5, 0.8599)

Table 6.7 TFNs for the PIS.

	A_1	A_2	A_3	A_4
PIS	(0.4476, 0.7372, 0.9674)	(0.4514, 0.6854, 1)	(0.4476, 0.6540, 0.9111)	(0.2984, 0.7332, 1)

Step 2

After multiplying all the TFNs of Table 6.1 by the corresponding weights of the DMs, we construct the FDM by the aggregation technique as described in the methodology.

Step3

We find the maximum element M = 3.590 in the FDM from Table 6.4. The elements of the FDM are normalized and the NFDM is shown in Table 6.5.

Step 4

The weighted NFDM is now constructed by multiplying the TFNs of NFDM with their corresponding attribute weights as shown in Table 6.6.

Step5

We construct the PIS as described in the methodology and we obtain the set of TFNs for PIS as shown in Table 6.7.

Step6

The Fuzzy Preference Degrees for the PIS are now calculated and shown in Table 6.8.

Step 7

We construct the NIS as described in the methodology and we obtain the set of TFNs for the NIS as shown in Table 6.9.

Step 8

The Fuzzy Preference Degrees for the NIS are now calculated and shown in Table 6.10.

Step 9

Finally the closeness co-efficients of the suppliers are obtained.

We have CC(S_1) = 0.5994, CC(S_2) = 0.5010, CC(S_3) 8 0.5171, CC(S_5) 8 0.4168 and CC(S_5) 8 0.4776.

Thus we conclude the ordering $S_4 < S_5 < S_2 < S_3 < S_1$.

Table 6.8 Fuzzy preference degrees for the PIS.

	A_1	A_2	A_3	A_4	Average
S_1	0.0731	0.0663	0.1342	0.0133	0.0717
S_2	0.1966	0.0816	0	0.0792	0.08935
S_3	0	0	0.2015	0.1444	0.0865
S_4	0.2126	0.1577	0.0356	0.1326	0.1044
S_5	0.1551	0.0779	0.0652	0.0760	0.09355

Table 6.9 TFNs for the NIS.

	A_1	A_2	A_3	A_4
NIS	(0.0995, 0.2910, 0.5)	(0.1989, 0.3258, 0.645)	(0.0994, 0.2704, 0.5)	(0.1279, 0.3465, 0.5375)

Table 6.10 Fuzzy preference degrees for the NIS.

	A_1	A_2	A_3	A_4	Average
S_1	0.1395	0.0914	0.0672	0.1311	0.1073
S_2	0.0160	0.0761	0.2015	0.0652	0.0897
S_3	0.2126	0.1577	0	0	0.092625
S_4	0	0	0.1659	0.1326	0.074625
S_5	0.0575	0.0799	0.1363	0.0684	0.085525

Table 6.11 Classification of the suppliers.

Group name	Intervals of CC	Suppliers in the group
Below average	[0, 0.072]	No supplier
Medium	(0.072, 0.5079]	S_4, S_5, S_2
Good	(0.5079, 0.6149]	S_1, S_3
Excellent	(0.6149, 1]	No supplier

6.3. Phase 3

In the final phase, we classify the suppliers in different groups. Here we choose four such groups: Below Average, Medium, Good and Excellent. Thus according to the methodology, three pseudo suppliers S_{BA}, S_{M}, S_{G} are considered. The decision vectors to obtain the maximum value of the CC of these groups are respectively {MP, MP, MP, MP}, {F, F, F, F} and {MG, MG, MG, MG}.

Putting these alternatives in the same evaluation process we obtain the values $\alpha_{BA} = 0.072$, $\alpha_M = 0.5079$ and $\alpha_G = 0.6149$. Certainly $\alpha_E = 1$, where α_E is the maximum value of the closeness co-efficient of the group 'Excellent'. Thus we have four intervals of separation. The intervals and the classified list of suppliers are displayed in Table 6.11.

Now we re-estimate the ranking of the supplies without considering the weights of the DMs. The PIS and NIS, obtained in this case are different and they are displayed in Table 6.12.

Table 6.12 PIS and NIS for the case where DMs' weights are not considered.

	A_1	A_2	A_3	A_4
PIS	(0.49, 0.809, 1)	(0.42, 0.746, 1)	(0.42, 0.722, 1)	(0.36, 0.75, 1)
NIS	(0.14, 0.311, 0.5)	(0.18, 0.346, 0.6)	(0.14, 0.289, 0.5)	(0.18, 0.36, 0.5)

Table 6.13 Fuzzy preference degrees where DMs' weights are not considered.

	A_1		A_2		A_3		A_4	
	PIS	NIS	PIS	NIS	PIS	NIS	PIS	NIS
S_1	0.0534	0.1684	0.0429	0.1228	0.1187	0.0744	0	O.1557
S_2	0.2160	0.0058	0.0452	0.1179	0	0.1932	0.0797	0.0760
S_3	0	0.2218	0	0.1631	0.1932	0	0.1557	0
S_4	0.2218	0	0.1631	0	0.0113	0.1818	0.0103	0.1453
S_5	0.1684	0.0534	0.0452	0.1179	0.0285	0.1647	0.0797	0.0760

The fuzzy preference degree for the PIS and NIS are shown in Table 6.13.

In the last stage the closeness co-efficients are calculated. And we obtain $CC(S_1) = 0.7078$, $CC(S_2) = 0.5354$, $CC(S_3) = 0.5245$, $CC(S_4) = 0.4460$ and $CC(S_5) = 0.5616$. Thus the ranking when DMS' weights are not considered is: $S_4 < S_3 < S_2 < S_5 < S_1$. It is distinctly observable that this ranking is different from the previous one. It establishes the necessity of considering the DMs' weights in the methodology.

7. Comparative study

At the end of our discussion we present a comparative analysis of several supplier selection methods and indicate some advantages of using our proposed methodology based on the preference degree of Triangular Fuzzy Numbers. Earlier in Section 3, we have briefly described four TOPSIS type approaches and one defuzzification based approach, in each of which linguistic terms have been used to represent uncertain information. The way of representing the linguistic terms is either TFNs or IGNs. Here we compare the proposed methodology with these five approaches.

At first, Table 7.1 shows the comparison based on different techniques used to obtain the solution.

Here we can clearly observe that Li et al.'s method and Jadidi et al.'s method suffer from the fact of inexistence of Decision Makers' weights. Again, apart from Sreekumar and Mahaptra's method, none has considered the classification of the ranked suppliers. Also, in Li et al.'s method and Mukherjee and Kar's method, Negative Ideal Solution (NIS) has not been taken into consideration. But the necessity of the NIS is already established in the literature.

Now we point out some additional drawbacks of some of these methods.

In Li et al.'s method, grey numbers have been compared by grey possibility degree which has been discussed in Section 2.2. There is a drawback of using this formula in decision making. Consider a situation where two interval grey numbers $\otimes G_1$ and $\otimes G_2$ are compared with another interval grey numbers $\otimes G_{\mathrm{Max}}$ and using Definition 3.1, we have obtained $P\{\otimes G_1 \leqslant \otimes G_{\mathrm{Max}}\} = 1$ as well as $P\{\otimes G_2 \leqslant \otimes G_{\mathrm{Max}}\} = 1$. In this case, no ranking or relation is derived between $\otimes G_1$ and $\otimes G_2$.

We illustrate this by an example.

Example 3.2. Consider a situation when two sets of grey numbers (G_1) and (G_2) are compared where $(G_1) = \{(1, 3), (2, 4), (5, 6)\}$ and $(G_2) = \{(2, 3), (3, 5), (7, 9)\}$.

Here

$$P\{(G_1) \leqslant (G_2)\} = \frac{1}{3}\left[\frac{2}{3} + \frac{3}{4} + 1\right] = 0.80556.$$

Again consider another situation where two sets of grey numbers (G_1) and (G_3) are compared where $(G_1) = \{(1, 3), (2, 4), (5, 6)\}$ and $(G_3) = \{(2, 3), (3, 5), (8, 10)\}$.

Here

$$P\{(G_1) \leqslant (G_3)\} = \frac{1}{3}\left[\frac{2}{3} + \frac{3}{4} + 1\right] = 0.80556.$$

Here we clearly observe that $P\{(G_1) \leqslant (G_2)\} = P\{(G_1) \leqslant (G_3)\}$, but $(G_2) \neq (G_3)$. Also it is understandable (considering the linear graphs of the grey numbers) that $P\{(G_1) \leqslant (G_3)\}$ should have got more values than $P\{(G_1) \leqslant (G_2)\}$.

Now we apply the Grey based method by Li et al. to the numerical data provided in Section 6. In the final step we obtain $S^{\mathrm{Max}} = \{[0.52, 0.93], [0.52, 0.73], [0.56, 0.93], [0.62, 0.87]\}$. And the grey possibility degrees are $P\{S_1 \leqslant S^{\mathrm{Max}}\} = 0.795$, $P\{S_2 \leqslant S^{\mathrm{Max}}\} = 0.88$, $P\{S_3 \leqslant S^{\mathrm{Max}}\} = 0.75$, $P\{S_4 \leqslant S^{\mathrm{Max}}\} = 0.797$, and $P\{S_5 \leqslant S^{\mathrm{Max}}\} = 0.96$. Thus the ranking is $S_5 < S_2 < S_4 < S_1 < S_3$ which is completely different from our obtained result.

In Sreekumar and Mahapatra's method, we have identified the following drawbacks.

1. In the procedure of identifying the weights of the DMs, the authors have used mutual ranking procedure. But the DMs may not know each other, or may know something about each other.
2. If a DM is biased to any particular supplier, then the biasness stays and effects in the evaluation process very strongly.
3. In step 9, a classification procedure is prescribed. But there is no strong logic behind the classification. In their method, the closeness co-efficient (CC_i for the ith supplier) certainly lies between 0 and 1. The authors have divided the suppliers into five classes as:

Table 7.1 Comparative analysis of six methods.

Methods	Way of representing linguistic terms	Whether DMs' weights are considered	Whether compared to the NIS	Whether classified
Li, Yamaguchi, Nagai (2007)	IGNs	No	No	No
Jadidi, Yusuff, Hong (2008)	IGNs	No	Yes	No
Sreekumar, Mahapatra (2009)	TFNs	Yes	Yes	Yes
Vaezi, Shahgholian, Shahraki (2011)	TFNs	Yes	Not applicable	Yes
Mukherjee, Kar (2012)	TFNs	Yes	No	No
Proposed approach	TFNs	Yes	Yes	Yes

Table 7.2 Distance of the suppliers from PIS and NIS (according to Sreekumar and Mahapatra's method).

	A_1		A_2		A_3		A_4	
	d^+	d^-	d^+	d^-	d^+	d^-	d^+	d^-
S_1	0.407	0.509	0.406	0.542	0.577	0.325	0.397	0.622
S_2	0.689	0.238	0.444	0.524	0.371	0.621	0.535	0.441
S_3	0.314	0.661	0.366	0.628	0.706	0.225	0.666	0.245
S_4	0.699	0.230	0.648	0.292	0.391	0.557	0.415	0.556
S_5	0.606	0.318	0.444	0.524	0.421	0.494	0.535	0.441

$CC_i \in [0, 0.2) \Rightarrow$ class I (Not Recommended)
$CC_i \in [0.2, 0.4) \Rightarrow$ class II (Recommended with high risk)
$CC_i \in [0.4, 0.6) \Rightarrow$ class III (Recommended with low risk)
$CC_i \in [0.6, 0.8) \Rightarrow$ class IV (Approved)
$CC_i \in [0.8, 1.0) \Rightarrow$ class V (Approved and highly recommended)

But a natural question arises here: why the length of the interval is taken as 0.2? Is there any connection of this classification with the computational algorithm? Neither of the facts of imposing decision inputs on attribute rating and supplier rating by the DMs has been considered here.

Now we again apply Sreekumar and Mahapatra's method to the same data provided in Section 6. In the final step, the PIS and NIS are obtained as$S^+ = \{(1, 1, 1), (1, 1, 1)\ (1, 1, 1), (1, 1, 1)\}$ and $S^- = \{(0.14, 0.14, 0.14), (0.14, 0.14, 0.14), (0.14, 0.14, 0.14), (0.14, 0.14, 0.14)\}$.Now the distance of each Supplier from the PIS and NIS are displayed in Table 7.2.

The closeness co-efficients are $CC(S_1) = 0.528$, $CC(S_2) = 0.472$, $CC(S_3) = 0.462$, $CC(S_4) = 0.432$ and $CC(S_5) = 0.469$. Thus the ranking is $S_4 < S_3 < S_5 < S_2 < S_1$ which is also different from the ranking evaluated in Section 6.

The following drawbacks are strongly present in Vaezi et al.'s approach:

1. In the first step of the algorithm, each decision maker's weight has been taken into consideration. But no methodology is defined for that.
2. Signed distance defuzzification method has been used for its simplicity, but it lacks in generality.

After applying the defuzzification based ranking method of Vaezi et al. to the same data provided in Section 6, we obtain the final rank as $S_4 < S_3 < S_2 < S_5 < S_1$, which is also different from our proposed ranking.

If we apply our proposed methodology to the numerical example given in Li et al. (2007), we notice that there is a change in the ranking. In (Li et al., 2007), the ranking is $S_1 > S_2 > S_4 > S_5 > S_3 > S_6$. But our method evaluates the ranking as $S_6 < S_5 < S_4 < S_3 < S_2 < S_1$ (without considering the weights of the DMs). The result is better as it is evaluated by using fuzzy preference degree.

Finally we apply the method proposed by Jadidi et al. to the same example provided in Section 6. Here $S^{Max} = \{[0.52, 0.93], [0.52, 0.73], [0.56, 0.93], [0.62, 0.87]\}$ and $S^{Min} = \{[0.12, 0.33], [0.21, 0.33], [0.14, 0.37], [0.28, 0.42]\}$. Thus the closeness co-efficients are obtained as $CC(S_1) = 0.93$, $CC(S_2) = 1.18$, $CC(S_3) = 1$, $CC(S_4) = 1.07$ and $CC(S_5) = 1.23$. The final rank is $S_5 < S_2 < S_4 < S_3 < S_1$ which is also a little bit different from our proposed ranking.

In view of the above discussion, we certainly claim that our proposed method stands better than the others. In our proposed approach, the above mentioned drawbacks are not present, as:

1. No mutual ranking is present here. The weights are evaluated by calculating total deviation from the individual decision to the ideal (pre-determined) decisions.
2. Biasness, if present, is reduced normally in this method.
3. A unique classification procedure of the suppliers is included in the methodology. As a result of this, the company becomes able to shortlist the suppliers for the future. It certainly helps the supply chain to be stronger.

Moreover this algorithmic approach can easily be implemented in the PC platform to avoid manual calculations. C, C^{++} and other programming languages can perform the trick easily. While running the program, there will be the following input va;ues asked by the PC:

1. Values of m, n, k, x and y,
2. mnk number of linguistic terms X_{ijk},
3. j number of linguistic terms X_{0j} for S_0, and
4. mk number of linguistic terms Y_{jk}.

And the output result will display:

1. the rank of the Suppliers, and
2. each Supplier's class to which it belongs.

So, the proposed approach is PC friendly and produces better judgment than the others.

8. Conclusion

In this paper we have developed a new three phase supplier section problem based on fuzzy preference degree. Each phase includes a new methodology which has structured the method stronger than the others. The comparative analysis describes the advantage of using this method as a better choice. The process of evaluation of fuzzy preference degree can be extended for the cases of Trapezoidal Numbers and Interval Valued Fuzzy Numbers also. The unique way of the evaluation of the weights of the Decision Makers can be used in other approaches.

The proposed fuzzy preference degree can be applied to other decision making problems, e.g., medical diagnosis, identification of poor households, etc.

More research on the properties of fuzzy preference degree between two fuzzy numbers can open a new area of research.

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