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A new greedy search method for the design of digital IIR filter

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Abstract A new greedy search method is applied in this paper to design the optimal digital infinite impulse response (IIR) filter. The greedy search method is based on binary successive approximation (BSA) and evolutionary search (ES). The suggested greedy search method optimizes the magnitude response and the phase response simultaneously and also finds the lowest order of the filter. The order of the filter is controlled by a control gene whose value is also optimized along with the filter coefficients to obtain optimum order of designed IIR filter. The stability constraints of IIR filter are taken care of during the design procedure. To determine the trade-off relationship between conflicting objectives in the non-inferior domain, the weighting method is exploited. The proposed approach is effectively applied to solve the multiobjective optimization problems of designing the digital low-pass (LP), high-pass (HP), bandpass (BP), and bandstop (BS) filters. It has been demonstrated that this technique not only fulfills all types of filter performance requirements, but also the lowest order of the filter can be found. The computational experiments show that the proposed approach gives better digital IIR filters than the existing evolutionary algorithm (EA) based methods.
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1. Introduction

Digital filters, both Finite Impulse Response (FIR) and infinite impulse response (IIR), have received immense consideration from researchers in recent years. This is due to the fact that digital filters have numerous applications in scientific disciplines and engineering areas such as geophysics, multidimensional signal processing, radar systems and high speed communication systems. IIR digital filters in comparison with FIR digital filters, offer improved selectivity, computational efficiency, and reduced system delay with comparable approximation accuracy (Lutovac et al., 2001). Design of IIR digital

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filters involves multiple, often conflicting, design criteria and finding an optimum design is, therefore, not a simple task. Digital IIR filter design principally follows two techniques: traditional design technique and optimization technique. Traditional design techniques involved in the design of digital IIR filters require analog-to-digital transformation based on a given set of predefined specifications and is commonly known as bilinear transformation approach (Antoniou, 2005). The design of digital IIR filters by using the transformation approach needs too much preknowledge and shows impaired performance in most cases (Tsai and Chou, 2006). The design of digital IIR filter involves optimization of various parameters involving different criteria with multiple local optima of error surface. The gradient-based optimization algorithms such as the steepest-descent and quasi-Newton (QN) algorithms are effectively used for the design of IIR filters (Antoniou, 2005; Lu and Antoniou, 2000), but these methods easily get stuck in the local minima of multimodal error surface. So there is a need for efficient optimization algorithms for the design of digital IIR filters. A variety of heuristic optimization algorithms (Chen et al., 2001; Dai et al., 2010; Harris and Ifeachor, 1998; Karaboga et al., 2004; Kalinli and Karaboga, 2005; Liu et al., 2004; Mandal et al., 2012; Nilsson et al., 2004; Sun et al., 2004; Tsai et al., 2006; Tsai and Chou, 2006; Vanuytsel et al., 2002; Zhang et al., 2003) have been proposed for digital IIR filter design such as particle swarm optimization (Sun et al., 2004), seeker-optimization-algorithm-based evolutionary method (Dai et al., 2010), simulated annealing (Sun et al., 2004), tabu search (Kalinli and Karaboga, 2005), ant colony optimization (Karaboga et al., 2004), artificial bee colony algorithm (Karaboga and Çetinkaya, 2011; Karaboga and Latifoglu, 2013), hybrid taguchi genetic algorithm (HTGA) (Tsai et al., 2006), immune algorithm (TIA) (Tsai and Chou, 2006), gravitation search algorithm (GSA) (Saha et al., 2014) and many more.

Mostly digital IIR filter design is treated as a single objective problem with some supplementary conditions, but secondary objectives like phase response and filter order are not considered. Some researchers have used the traditional iterative algorithms to design an IIR filter with minimum phase response error (Lu, 1999; Lang, 2000; Sullivan and Adams, 1998; Shyu et al., 2009), but to secure the stability of the filter, more computation needs to be incorporated in these methods. The order of IIR digital filter needs to be considered in the design process to minimize the computational complexity. In hierarchical genetic algorithm (HGA) (Tang et al., 1998) filter structure is optimized, but linear phase response error is neglected.

The formulation of IIR filter design as a multi-objective optimization problem involves numerous objectives; like linear phase response, minimum magnitude response error and lowest filter order; and requires vast experience in the areas of digital filters design and optimization algorithms. Recently, some researchers (Tang et al., 1998; Wang et al., 2011; Yu and Xinjie, 2007) have designed the IIR digital filter by considering phase response, magnitude response and filter order simultaneously. In the cooperative co-evolutionary genetic algorithm (CCGA) (Yu and Xinjie, 2007) structure and the coefficients of the digital IIR filter are coded separately. The nondominated sorting genetic algorithm-II (Deb et al.,2002) is applied to control the species and guide the algorithms toward achieving three objectives simultaneously. A new local search operator enhanced multi-objective evolutionary algorithm (LS-MOEA) (Wang et al., 2011) is applied to design IIR filter with equivalent consideration of linear phase response, magnitude response and the order of the filter.

The paper proposes a binary successive approximation based evolutionary search (BSA-ES) method for the design of stable digital IIR filter by considering phase response, magnitude response and filter order simultaneously. The direct search is performed in two moves. The first move is exploratory move which acquires knowledge about the behavior of the function. In exploratory move, search is assumed in the neighborhood of the current point and the best point around the current point of a hypercube is recorded. BSA-ES strategy has been proposed to reduce the computational burden involved in exploring the corner points of a hypercube. The two best points recorded after the outcome of exploratory move are used further for pattern move. The values of the filter coefficients are optimized to achieve minimum magnitude response error and phase response error along with optimal order of the filter simultaneously with BSA-ES approach. Multiobjective constrained optimization problem is converted into scalar objective constrained optimization problem using the weighting method. The weighting technique is used to generate non-inferior solutions, which allow explicit trade-off between conflicting objective levels. The weighting patterns are either presumed on the basis of decision maker's intuition or simulated with suitable step size variation. In the paper, the weighting pattern is searched using the evolutionary search (ES) technique along with the decision vector. On violation of inequality constraints, decision variables are updated randomly till inequality constraints are satisfied. The credibility of the proposed method has been demonstrated in Dhillon et al. (2009) to solve the economic-emission load dispatch problem by searching the generation pattern of committed units.

The paper is structured as follows. Section 2 describes the IIR filter design problem statement. Binary successive approximation based evolutionary search method for designing the optimal digital IIR filters is described in Section 3. In Section 4, the performance of the proposed method has been evaluated and achieved results are compared to the design obtained in Tang et al., 1998; Wang et al., 2011; Yu and Xinjie, 2007 for the LP, HP, BP, and BS filters. Finally, the conclusions and discussions are outlined in Section 5.

2. IIR filter design problem

The cascading is the most commonly used form to implement IIR filter that avoids the coefficient quantization problem which causes instability. The fundamental cascaded structure regardless of the filter type is (Tang et al., 1998):

$$H(z) = x_1 \prod_{k=1}^{m} \frac{1 + x_{2k} z^{-1}}{1 + x_{2k+1} z^{-1}} \prod_{i=1}^{n} \frac{1 + x_{4i+2m-2} z^{-1} + x_{4i+2m-1} z^{-2}}{1 + x_{4i+2m} z^{-1} + x_{4i+2m+1} z^{-2}} \tag{1}$$

x is a vector decision variable of dimension $V \times 1$ with $V = 2m + 4n + 1$. x_1 represents the gain, $[x_2, x_3, \ldots x_{2m+4n+1}]$ denotes the filter coefficients of first and second order sections.

2.1. Magnitude response error

To design IIR filter, the main aim is to minimize the magnitude response error at defined frequency bands in which either frequency is allowed to pass or restricted. Magnitude response error for passband frequency is stated as (Tang et al., 1998), (Yu and Xinjie, 2007) and (Wang et al., 2011):

$$\Delta H_P(\omega) = \begin{cases} 1 - \delta_P - |H(e^{j\omega})|, & |H(e^{j\omega})| < 1 - \delta_P \\ 0, & |H(e^{j\omega})| \geqslant 1 - \delta_P \end{cases} \quad (2)$$

where ω represents passband frequency and $\Delta H_p(\omega)$ is the magnitude response error in passband.

Similarly magnitude response error is stated below for stopband frequency.

$$\Delta H_s(\omega) = \begin{cases} |H(e^{j\omega})| - \delta_s, & |H(e^{j\omega})| > \delta_s \\ 0, & |H(e^{j\omega})| \leqslant \delta_s \end{cases} \quad (3)$$

where ω represents stop band frequency and $\Delta H_s(\omega)$ is the magnitude response error in stop band.

Our main aim is to minimize the magnitude response error in passband and stopband. Mathematically this objective is written as:

$$\text{Minimize } O_1 = \frac{1}{A_n}\sum_{j=1}^{A_n} \Delta H_P(\omega_j) + \frac{1}{B_n}\sum_{k=1}^{B_n} \Delta H_s(\omega_k) \quad (4)$$

where A_n and B_n are the number of sampling points in passband and stopband, respectively.

In the IIR filter design problem, fixed grid approach is used (Deczky, 1972). Frequency range from 0 to π is divided into 200 equally spaced sample points. The best fitness function value is achieved when magnitude response of the designed IIR filter lies within the prescribed range in passband and stopband.

2.2. Phase response error

The linear phase response needs to be optimized for both passband and transition band (Yu and Xinjie, 2007; Wang et al., 2011), because sometimes non-linearity in phase response of transition band may cause distortion. The phase response β is calculated at different frequency sampling points $\{\beta_1, \beta_2, \ldots, \beta_s\}$. The first order difference in phase response can be calculated as:

$$O_2 = \Delta phase = \{\Delta\beta_1, \Delta\beta_2, \ldots, \Delta\beta_{s-1}\} \quad (5)$$

where $\Delta\beta_j = \beta_{j+1} - \beta_j$, $j = 1, 2, 3, \ldots s - 1$; s is the total number of sampling points in passband and transition band. The phase response is considered as linear if all the elements of $\Delta phase$ have the same value. The second objective function in terms of linear phase response error is represented as variance of phase differences.

$$\text{Minimize } O_2 = \text{var}\{\Delta\beta_s | \beta_s \in \text{passband} \cup \text{transitionband}\} \quad (6)$$

2.3. Multiobjective IIR filter problem formulation

The design of IIR filter involves obtaining the optimal structure of filter having optimal order, minimum magnitude and minimum phase response error. Mathematically, multiobjective optimization problem for the design of IIR filter is stated below:

$$\text{Minimize } [O_1(x), O_2(x)]^T \quad (7)$$

Subject to: Stability constraints (Jury, 1964)

$$1 + x_{2k+1} \geqslant 0 \quad (k = 1, 2, \ldots, m) \quad (7a)$$

$$1 - x_{2k+1} \geqslant 0 \quad (k = 1, 2, \ldots, m) \quad (7b)$$

$$1 - x_{4i+2m+1} \geqslant 0 \quad (i = 1, 2, \ldots, n) \quad (7c)$$

$$1 + x_{4i+2m} + x_{4i+2m+1} \geqslant 0 \quad (i = 1, 2, \ldots, n) \quad (7d)$$

$$1 - x_{4i+2m} + x_{4i+2m+1} \geqslant 0 \quad (i = 1, 2, \ldots, n) \quad (7e)$$

where $O_1(x)$ given by (4) is magnitude response error, and $O_2(x)$ given by (6) is the variance of phase differences. x is a vector decision variable of dimension $V \times 1$ with $V = 2m + 4n + 1$. The aim is to find the value of filter coefficients being decision variables, x which optimizes all the objective functions, simultaneously.

The multiobjective constrained optimization problem for the design of digital IIR filter is converted into a scalar objective constrained optimization problem by using a weighted sum of the objectives of $O_1(x)$ and $O_2(x)$ to generate non-inferior solutions.

$$\text{Minimize } f = \sum_{j=1}^{M} w_j O_j(x) \quad (8a)$$

Subject to :

$$\text{(i) } w_j = \frac{\alpha_j}{\sum_{j=1}^{M} \alpha_j} \text{ and } \alpha_j \geqslant 0 \quad (j = 1, 2, \ldots, M) \quad (8b)$$

(ii) Satisfaction of stability constraints given by (7a)–(7e).

where

$O_j(x)$ is the jth objective function.

α is non-negative real weights varying between 0 and 100, assigned to j[th] objective.

M is number of participating objectives.

A binary string based decimal value representing weight is exploited to search the value of weights, assigned to different objectives corresponding to each individual particle. The optimal weight pattern is searched with the help of successive approximation method thus overcoming the limitation that decision maker may not be provided with the weight set that corresponds to actual best solution. The decision variable becomes $x = [x_1, x_2, \ldots, x_{2m+4n+1}, x_{2m+4n+2}, x_{2m+4n+3}]$. $x_{2m+4n+2}$ corresponds to w_1 and $x_{2m+4n+3}$ corresponds to w_2.

2.4. Order

The order of the digital IIR filter is determined as follows:

$$Order = \sum_{j=1}^{m} p_j + 2\sum_{k=1}^{n} q_k \quad (9)$$

where p_j and q_k are jth and kth control genes respectively for corresponding first order and second order blocks, m and n are the number of first and second order blocks respectively. The maximum order of the filter is $m + 2n$.

The structure of digital IIR filter is represented by control gene (Fig. 1). The coding method followed has been inherited from the papers (Tang et al., 1998; Wang et al., 2011; Yu and Xinjie, 2007). The control genes determine activation/deactivation of corresponding blocks of filter coefficients by setting 1/0, respectively. The value of binary bits used to generate control genes is evaluated based on the integer value of variable, $x_{2m+4n+4}$ of decision vector x. The integer value of variable $x_{2m+4n+4}$ is optimized along with the filter coefficients to obtain optimum *Order* of designed IIR filter. $x = [x_1, x_2, \ldots, x_{2m+4n+1}, x_{2m+4n+2}, x_{2m+4n+3}, x_{2m+4n+4}]^T_{V\times 1}$ is the final decision variable where $V = 2M + 4N + 4$.

2.5. Constraint handling

The design of causal recursive filters requires the inclusion of stability constraints. The stability constraints on the coefficients of the digital IIR filter are obtained by using the Jury method (Jury, 1964) in (1).

The stability constraints given by 7a, 7b, 7c, 7d, 7e have been forced to be satisfied by updating the coefficients with random variation as given below. The variation is kept small so that the characteristic of population should not be changed.

$$x_{2k+1} = \begin{cases} x_{2k+1}(1-r)^2; & (1+x_{2k+1}) < 0,\ or\ (1-x_{2k+1}) < 0 \\ x_{2k+1}; & Otherwise \end{cases} \tag{10a}$$

$$x_{4i+2m+1} = \begin{cases} x_{4i+2m+1}(1-r)^2; & (1-x_{4i+2m+1}) < 0 \\ & or\ (1-x_{4i+2m+1}) \geqslant 0 \\ x_{4i+2m+1}; & Otherwise \end{cases} \tag{10b}$$

$$x_{4i+2m} = \begin{cases} x_{4i+2m}(1-r)^2; & (1+x_{4i+2m}+x_{4i+2m+1}) < 0 \\ & or\ (1-x_{4i+2m}+x_{4i+2m+1}) < 0 \\ x_{4i+2m}; & Otherwise \end{cases} \tag{10c}$$

where r is any uniform random number which is varied between [0, 1]. Square term gives small increment.

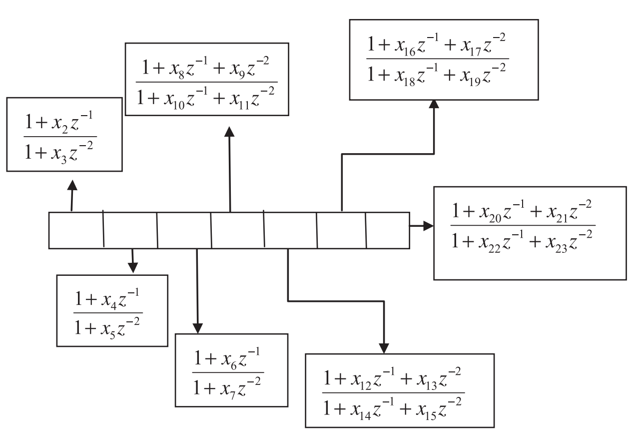

Fig. 1 Activation/deactivation of filter coefficients with control gene.

3. Binary successive approximation based evolutionary search (BSA-ES) method

In the proposed BSA-ES method the trial solutions are sequentially examined using a direct search methodology. The process of departing from a given point to the next improved point is called a *move*. A move is termed a *success* if objective improves; otherwise, it is a *failure*. The direct search technique makes two types of move. The first move is exploratory move which acquires the knowledge concerning the behavior of the function. In exploratory move search is performed in the neighborhood of the current point and the best point around the current point of a hypercube is recorded. Second move is pattern move (Kothari and Dhillon, 2011).

3.1. Exploratory move

In exploratory move, the current point is oscillated in positive and negative directions along each variable one at a time and the best point is recorded. The current point is changed to the best point at the end of each variable oscillation. If the point found at the end of all variable oscillations is different than the original point, the exploratory move is a success; otherwise the exploratory move is a failure. In any case, the best point is considered to be the outcome of the exploratory move.

3.2. Evolutionary search method

Evolutionary method is applied to search the optimal value of filter coefficients. In this method, 2^V feasible solutions are generated for V number of filter coefficients. A (V) dimensional hypercube of side λ is formed around the point. x_i^C represents filter coefficients from the current point in the hyperspace. The solution is considered as better if it gives improved objective function value of the problem. At the end of iteration better point is recorded and next hypercube is formed around the better point. All the corners of the hypercube represented in binary (V) bits equivalent code, generated around the current set of filter coefficients, are explored iteratively for the desired solution simultaneously. Table 1 shows the pattern of filter coefficients for 3-filter coefficients where 3 bits code is considered to represent the corners of the 3-dimensional hypercube (Fig. 2).

Serial numbers of hypercube corners in decimal are converted into their binary equivalent code. The shift/divergence from the current center point is calculated by replacing 0's with $-\lambda$ and 1's with $+\lambda$ in code associated with hypercube corners. As the number of filter coefficients increases, it becomes time consuming and cumbersome exercise to hunt for the better solution among all the hypercube corners. To accomplish the task of exploring all the corners of the hypercube with minimum number of function evaluations and comparisons an efficient search methodology is required.

3.3. Binary successive approximation (BSA) strategy

The search is performed on filter coefficients pattern exploiting evolutionary optimization and BSA strategy to search the optimal solution with minimum number of computations. BSA strategy to search the filter coefficients is depicted in Fig. 3.

Table 1 Filter coefficient vector at hypercube corners.

Hyper cubecorners	Possible combinations of 3-bits $C_2\ C_1\ C_0$	Distance of hypercube corners from center point x_1^c, x_2^c, x_3^c	Pattern of filter coefficients at the hypercube corners		
0	000	$-\lambda_1 - \lambda_2 - \lambda_3$	$x_1^c - \lambda_1$	$x_2^c - \lambda_2$	$x_3^c - \lambda_3$
1	001	$-\lambda_1 - \lambda_2 + \lambda_3$	$x_1^c - \lambda_1$	$x_2^c - \lambda_2$	$x_3^c + \lambda_3$
2	010	$-\lambda_1 + \lambda_2 - \lambda_3$	$x_1^c - \lambda_1$	$x_2^c + \lambda_2$	$x_3^c - \lambda_3$
3	011	$-\lambda_1 + \lambda_2 + \lambda_3$	$x_1^c - \lambda_1$	$x_2^c + \lambda_2$	$x_3^c + \lambda_3$
4	100	$+\lambda_1 - \lambda_2 - \lambda_3$	$x_1^c + \lambda_1$	$x_2^c - \lambda_2$	$x_3^c - \lambda_3$
5	101	$+\lambda_1 - \lambda_2 + \lambda_3$	$x_1^c + \lambda_1$	$x_2^c - \lambda_2$	$x_3^c + \lambda_3$
6	110	$+\lambda_1 + \lambda_2 - \lambda_3$	$x_1^c + \lambda_1$	$x_2^c + \lambda_2$	$x_3^c - \lambda_3$
7	111	$+\lambda_1 + \lambda_2 + \lambda_3$	$x_1^c + \lambda_1$	$x_2^c + \lambda_2$	$x_3^c + \lambda_3$

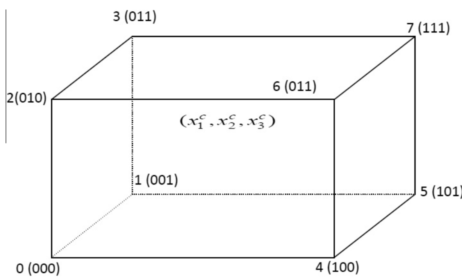

Fig. 2 Three dimensional hypercube representing corners in decimal.

As shown the solution procedure moves toward the optimal solution by comparing two solutions at a time represented by two corners of hypercube.

The process of searching for the best solution is started by initializing decision vector variable x_i^{ck}, giving objective F^c. To implement the BSA strategy by the iterative process, C_i^k is initialized as below:

$$C_i^k = \begin{cases} 1; & \text{for } (i = 1) \\ 0; & \text{for } (i = 2, 3, \ldots, V) \end{cases} \tag{11}$$

The two corners of hypercube, in reference to selected corner, are produced for the comparison purpose as below:

$$C_{1i}^k = \begin{cases} 1; & \text{for } \ i+1 \\ C_j^k; & \text{for } (j = 1, 2, \ldots, \text{i}, (\text{i}+2), \ldots, \text{V}) \end{cases} \tag{12}$$

$$C_{2i}^k = \begin{cases} 0; & \text{for } i \\ C_{1j}^k; & \text{for } (j = 1, 2, \ldots, (i-1), (\text{i}+1), \ldots, \text{V}) \end{cases} \tag{13}$$

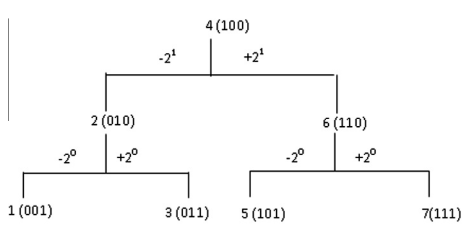

Fig. 3 Binary successive approximation strategy for 3-bits code.

The vectors of filter coefficients are generated in reference to these two corners, as depicted in Table 1. Mathematically, it is represented as follows:

$$x_{mi}^k = x_i^{ck} + \Delta_{mi}^k; \quad (m = 1, 2)\ (i = 1, 2, \ldots, V) \tag{14}$$

where

$$\Delta_{mi}^k = \begin{cases} +\lambda_i & \text{if } C_{mi}^k = 1 \\ -\lambda_i & \text{if } C_{mi}^k = 0 \end{cases} \quad (m = 1, 2)\ (i = 1, 2, \cdots, V) \tag{15}$$

The value by which the filter coefficients are incremented is decided by

$$\lambda_i = (x_i^{\max} - x_i^{\min})/\delta_3 \tag{16}$$

Compute x_{mi}^k using (14) and then objective functions at x_{1i}^k and x_{2i}^k are recorded as follows:

$$F_m^k = f(x_{mi}^k) \quad (m = 1, 2) \tag{17}$$

Minimum of the two points is selected based upon the function values computed by (17) and is compared with the rest of the corners, generated subsequently by (12) and (13).

$$F^k = \min\{F_1^k, F_2^k\}; \tag{18}$$

The corner shortlisted for the generating next two corners is

$$C_i^k = \begin{cases} C_{1i}^k & \text{if } F_1^k < F_2^k \\ C_{2i}^k & \text{if } F_2^k < F_1^k \end{cases} \quad (i = 1, 2, \cdots, V) \tag{19}$$

The search is repeatedly provoked till all hypercube corners are explored by BSA method and overall minimum is selected which becomes the new center point for the next iteration. The procedure ends when last element of vector C_i^k contains 1 or the last branch of the binary successive approximation tree is reached thus exploring all the corners of hypercube. This method drastically reduces the number of comparisons. This is depicted in Table 2 for different number of filter coefficients.

3.4. Pattern move

The pattern move speeds up the search by utilizing the information acquired in the exploratory move, and ascertain that the minimization of the function is achieved by moving in the direction of the established "pattern". In pattern move a new point is recorded by moving from the current best point x_i^k along a direction connecting the previous best point x_i^{k-1} and is implemented as given below:

Table 2 Comparison of number of function evaluations.

Number of filter coefficients (V)	Number of corners of hypercube (2^V)	Number of comparisons by BSA method ($2 \times V$)
7	128	14
9	512	18
11	2048	22
13	8192	26
15	32,768	30
17	13,1072	34

$$x_i^{k+1} = x_i^k + \rho_i(x_i^k - x_i^{k-1}) \quad (i = 1, 2, \ldots, V)$$
where (20)
$$\rho_i = \rho_i \pm \Delta\rho_i$$

Algorithm. Binary successive approximation based evolutionary search method.

To implement the direct search explained in proceeding section, the step wise procedure is elaborated here:

1. Initialize vector decision variable as x_i^0, $F^0 = f(x_i^0)$ and $k = 0$.
2. DO
   - 2.1. $x_i^c = x_i^k$ $(i = 1,2,\ldots, V)$
   - 2.2. Initialize $i = 0$, C_i^k $(i = 1,2,\ldots,V)$ following (11), $k = k + 1$.
   - 2.3. Compute C_{1i}^k $(i = 1,2,\ldots,V)$ using (12).
   - 2.4. Compute C_{2i}^k $(i = 1,2,\ldots,V)$ using (13).
   - 2.5. Obtain x_{mi}^k and, Δ_{mi}^k $(m = 1,2)$ $(i = 1,2,\ldots,V)$ from (14) and (15). respectively.
   - 2.6. Evaluate F_1^k and F_2^k from (8a).
   - 2.7. Evaluate F^k from (18).
   - 2.8. DO
     - 2.8.1. $j = j + 1$.
     - 2.8.2. Obtain C_{mi}^k, x_{mi}^k and, Δ_{mi}^k $(m = 1,2)$ $(i = 1,2,\ldots,V)$ from (12)–(15), respectively.
   - 2.9. Evaluate F_1^k and F_2^k from (8a).
   - 2.10. Evaluate F^k from (18).
     - 2.10.1. Obtain C_i^k $(i = 1,2,\ldots,V)$ from (19).
   - 2.11. WHILE $(j < (V - 1))$.
   - 2.12. IF $(F^k < F^{k-1})$ THEN
     - 2.12.1. DO
     - 2.12.2. Obtain x_i^{k+1} $(i = 1,2,\ldots,V)$ from (20).
     - 2.12.3. $x_i^{k-1} = x_i^k (i = 1, 2, ..., V)$ and $F^{k-1} = F^k$
     - 2.12.4. $x_i^k = x_i^{k+1} (i = 1, 2, ..., V)$, and $F^k = F^{k+1}$
     - 2.12.5. WHILE $(F_i^k < F^{k-1})$
     - 2.12.6. $x_i^k = x_i^{k-1} (i = 1, 2, ..., V)$, and $F^k = F^{k-1}$
   - 2.13. ELSE
     - 2.13.1. $\lambda_i = \lambda_i/\sigma$ $(i = 1,2,..,V)$
   - 2.14. END
3. WHILE $(\|\lambda\| \leqslant err)$
   STOP

4. Design examples and comparisons

The validity of the proposed method for designing of digital IIR filters is illustrated by attaining the optimum design results for all four types namely LP, HP, BP and BS digital IIR filters. The values of different parameters used in BSA-ES algorithm are shown in Table 3.

The prescribed design conditions followed in BSA-ES method for the design of digital IIR filter are given in Table 4.

The filter coefficients obtained by the BSA-ES approach for LP, HP, BP and BS filters are presented in Table 5.

The magnitude and phase response diagrams of LP and HP filters are presented in Fig. 4. Fig. 5 presents magnitude and phase response diagrams of BP and BS filters. The trend of obtained magnitude and phase of digital IIR filters obtained with BSA-ES approach clearly indicates that the designed filters meet the requisite design specifications as ripples in pass-band are less than δ_p and ripples in stop-band are less than δ_s. The phase response of designed digital IIR filters is linear in both pass-band and stop-band.

The obtained results in terms of lowest filter order, pass-band magnitude performance, stop-band magnitude performance and phase response error with BSA-ES method are shown in Table 6. To establish the predominance of obtained results with BSA-ES method, comparison is done with the results obtained by HGA (Tang et al., 1998), CCGA (Yu and Xinjie, 2007), NSGA-II (Deb et al., 2002) and LS-MOEA (Wang et al., 2011).

By analyzing the data in Table 6, it is concluded:

- In comparison with EAs (HGA and CCGA), BSA-ES offers better performance in terms of phase response for LP, HP, BP and BS filters. In terms of lowest order: BSA-ES and CCGA are equivalent and are better than HGA. The magnitude response obtained with BSA-ES is better in almost all types of digital IIR filters.
- When compared with optimization algorithms (LS-MOEA and NSGA-II), the magnitude response obtained with BSA-ES is better in almost all types of digital IIR filters. In terms of lowest order BSA-ES is comparable with LS-MOEA and NSGA-II. The phase response quality obtained with BSA-ES is comparable in almost all types of digital IIR filters.

The pole-zero diagrams for LP, HP, BP and BS digital IIR filters are presented in Fig. 6. It is observed from Fig. 6 that the designed filters follow the stability constraints imposed in the design procedure as all the poles lie inside the unit circle.

The trend of fitness function obtained with BSA-ES method with respect to iterations for LP, HP, BP and BS digital IIR filters is shown in Fig. 7. Hence it is proved from Fig. 7 that that the proposed BSA-ES method attains the solution in small number of iterations.

Table 3 Parameter values for BSA-ES algorithm.

Parameter	Value
δ_3	4.0
ρ_i	.01
σ	1.618
err	1.0E−12

Table 4 Design conditions for LP, HP, BP and BS filters.

Filter type	Pass-band ($\delta_P = 0.1088$)	Stop-band ($\delta_s = 0.17783$)	Order
Low-pass (LP)	$0 \leqslant \omega \leqslant 0.2\pi$	$0.3\pi \leqslant \omega \leqslant \pi$	11
High-pass (HP)	$0.8\pi \leqslant \omega \leqslant \pi$	$0 \leqslant \omega \leqslant 0.7\pi$	11
Band-pass (BP)	$0.4\pi \leqslant \omega \leqslant 0.6\pi$	$0 \leqslant \omega \leqslant 0.25\pi$ $0.75 \leqslant \omega \leqslant \pi$	11
Band-stop (BS)	$0 \leqslant \omega \leqslant 0.25\pi$ $0.75 \leqslant \omega \leqslant \pi$	$0.4\pi \leqslant \omega \leqslant 0.6\pi$	11

Table 5 The values of coefficients for LP, HP, BP and BS IIR digital filter designed by BSA-ES.

Coefficients	LP filter	HP filter	BP filter	BS filter
Gain x_1	0.190025	0.196578	0.232502	0.466631
x_2	0.251280	−0.533238	−1.687329	0.284547
x_3	−0.430135	0.321605	1.061928	0.904613
x_4	−0.927912	0.934002	−0.635597	0.739555
x_5	0.947152	0.930848	0.508129	0.465761
x_6	−1.223384	1.170541	1.646724	−0.310116
x_7	0.648660	0.614222	1.018342	0.851182
x_8	–	–	−0.635005	−0.767862
x_9	–	–	0.506126	0.485811

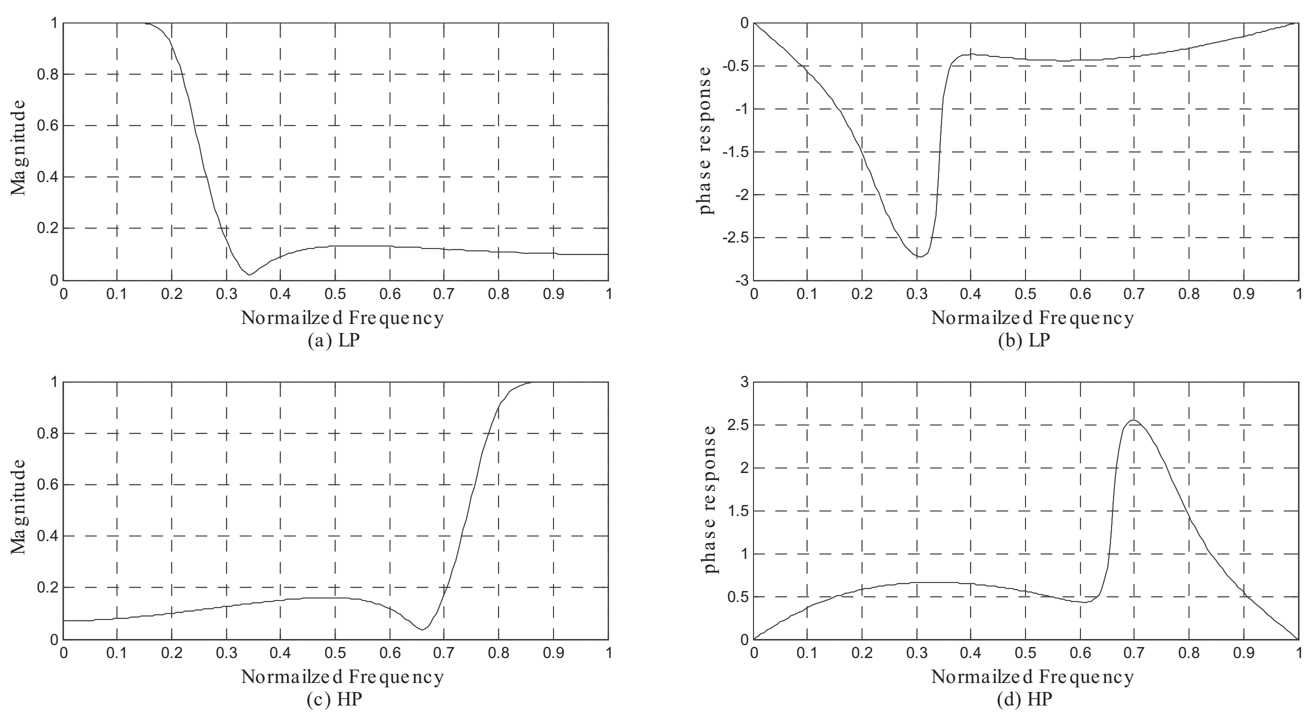

Fig. 4 Magnitude and phase response of LP and HP filter.

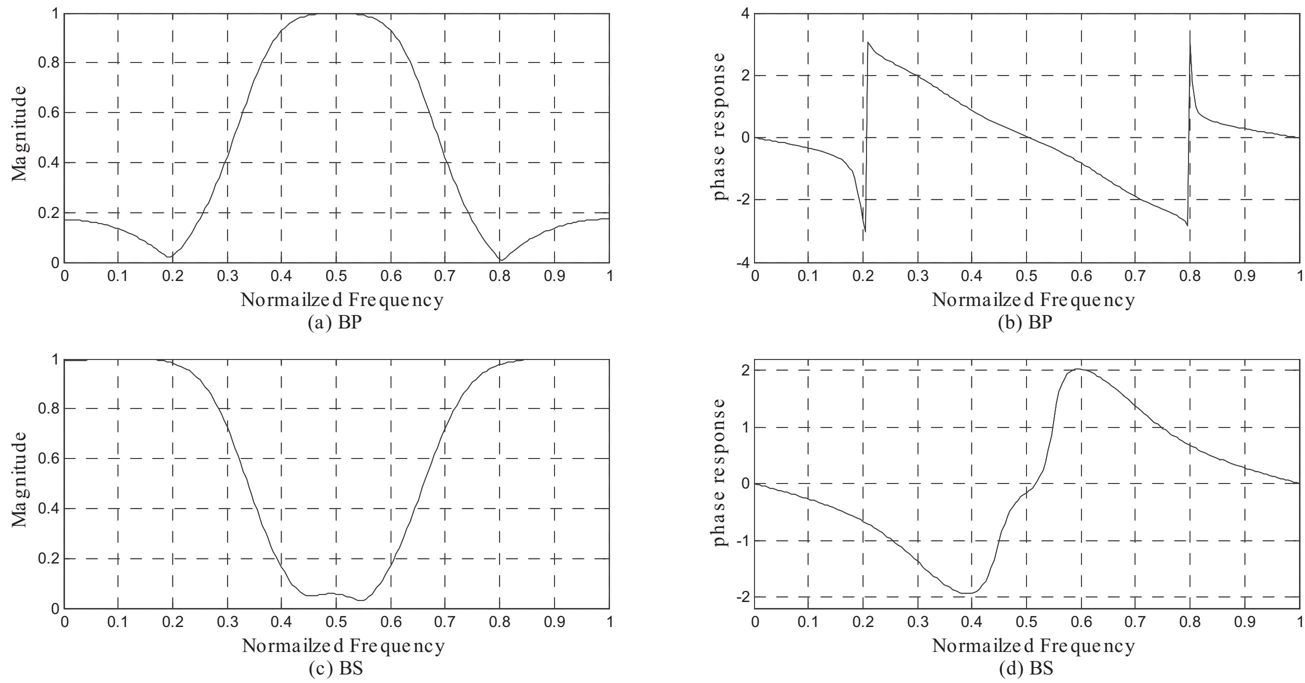

Fig. 5 Magnitude and phase response of BP and BS filter.

Table 6 Comparison of design results for LP, HP, BP and BS filters.

	Lowest filter order	Pass-band magnitude performance	Stop-band magnitude performance	Phase response error
LP filter				
HGA	3	$0.8862 \leqslant \lvert H(e^{j\omega})\rvert \leqslant 1.0$	$\lvert H(e^{j\omega})\rvert \leqslant 0.1800$	1.6485×10^{-4}
CCGA	3	$0.9034 \leqslant \lvert H(e^{j\omega})\rvert \leqslant 1.0$	$\lvert H(e^{j\omega})\rvert \leqslant 0.1669$	1.4749×10^{-4}
NSGA-II	3	$0.9117 \leqslant \lvert H(e^{j\omega})\rvert \leqslant 1.0$	$\lvert H(e^{j\omega})\rvert \leqslant 0.1719$	1.2662×10^{-4}
LS-MOEA	3	$0.9083 \leqslant \lvert H(e^{j\omega})\rvert \leqslant 1.0$	$\lvert H(e^{j\omega})\rvert \leqslant 0.1586$	1.0959×10^{-4}
BSA-EV	3	$0.9117 \leqslant \lvert H(e^{j\omega})\rvert \leqslant 1.0$	$\lvert H(e^{j\omega})\rvert \leqslant 0.1555$	1.1918×10^{-4}
HP filter				
HGA	3	$0.9221 \leqslant \lvert H(e^{j\omega})\rvert \leqslant 1.0$	$\lvert H(e^{j\omega})\rvert \leqslant 0.1819$	1.1212×10^{-4}
CCGA	3	$0.9044 \leqslant \lvert H(e^{j\omega})\rvert \leqslant 1.0$	$\lvert H(e^{j\omega})\rvert \leqslant 0.1749$	9.7746×10^{-4}
NSGA-II	3	$0.8960 \leqslant \lvert H(e^{j\omega})\rvert \leqslant 1.0$	$\lvert H(e^{j\omega})\rvert \leqslant 0.1769$	9.1419×10^{-5}
LS-MOEA	3	$0.9004 \leqslant \lvert H(e^{j\omega})\rvert \leqslant 1.0$	$\lvert H(e^{j\omega})\rvert \leqslant 0.1746$	9.6150×10^{-5}
BSA-EV	3	$0.8990 \leqslant \lvert H(e^{j\omega})\rvert \leqslant 1.0$	$\lvert H(e^{j\omega})\rvert \leqslant 0.1738$	9.4648×10^{-5}
BP filter				
HGA	6	$0.8956 \leqslant \lvert H(e^{j\omega})\rvert \leqslant 1.0$	$\lvert H(e^{j\omega})\rvert \leqslant 0.1772$	1.1222×10^{-4}
CCGA	4	$0.8920 \leqslant \lvert H(e^{j\omega})\rvert \leqslant 1.0$	$\lvert H(e^{j\omega})\rvert \leqslant 0.1654$	8.1751×10^{-5}
NSGA-II	4	$0.9100 \leqslant \lvert H(e^{j\omega})\rvert \leqslant 1.0$	$\lvert H(e^{j\omega})\rvert \leqslant 0.1771$	3.6503×10^{-4}
LS-MOEA	4	$0.9285 \leqslant \lvert H(e^{j\omega})\rvert \leqslant 1.0$	$\lvert H(e^{j\omega})\rvert \leqslant 0.1734$	6.0371×10^{-5}
BSA-EV	4	$0.9290 \leqslant \lvert H(e^{j\omega})\rvert \leqslant 1.0$	$\lvert H(e^{j\omega})\rvert \leqslant 0.1733$	4.6918×10^{-5}
BS filter				
HGA	4	$0.8920 \leqslant \lvert H(e^{j\omega})\rvert \leqslant 1.0$	$\lvert H(e^{j\omega})\rvert \leqslant 0.1726$	2.7074×10^{-4}
CCGA	4	$0.8966 \leqslant \lvert H(e^{j\omega})\rvert \leqslant 1.0$	$\lvert H(e^{j\omega})\rvert \leqslant 0.1733$	1.6119×10^{-4}
NSGA-II	4	$0.8917 \leqslant \lvert H(e^{j\omega})\rvert \leqslant 1.0$	$\lvert H(e^{j\omega})\rvert \leqslant 0.1770$	1.5190×10^{-4}
LS-MOEA	4	$0.8967 \leqslant \lvert H(e^{j\omega})\rvert \leqslant 1.0$	$\lvert H(e^{j\omega})\rvert \leqslant 0.1725$	1.5084×10^{-4}
BSA-EV	4	$0.9026 \leqslant \lvert H(e^{j\omega})\rvert \leqslant 1.0$	$\lvert H(e^{j\omega})\rvert \leqslant 0.1716$	1.5818×10^{-4}

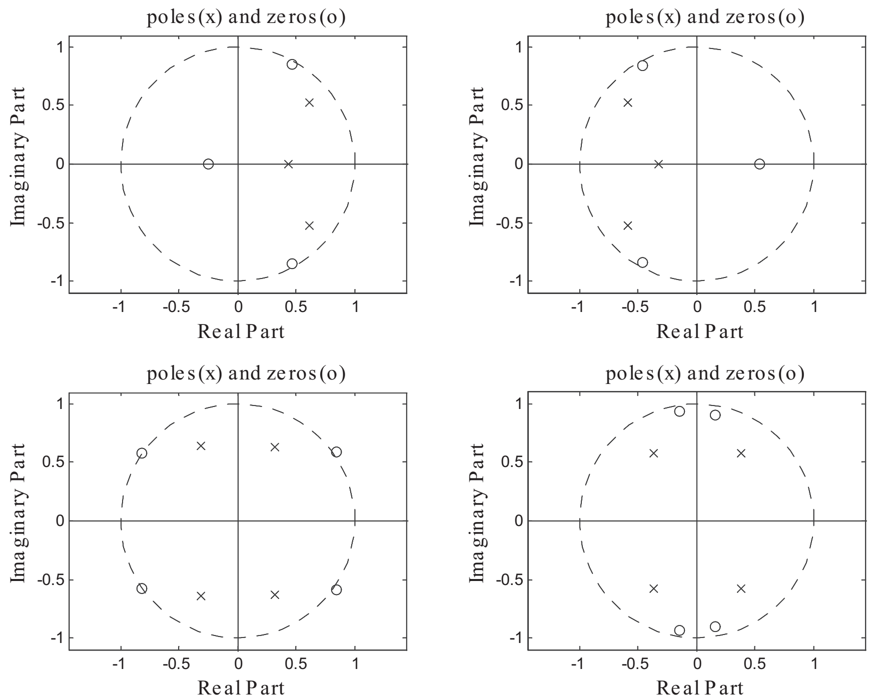

Fig. 6 Pole-zero plots for LP, HP, BP and BS filter respectively.

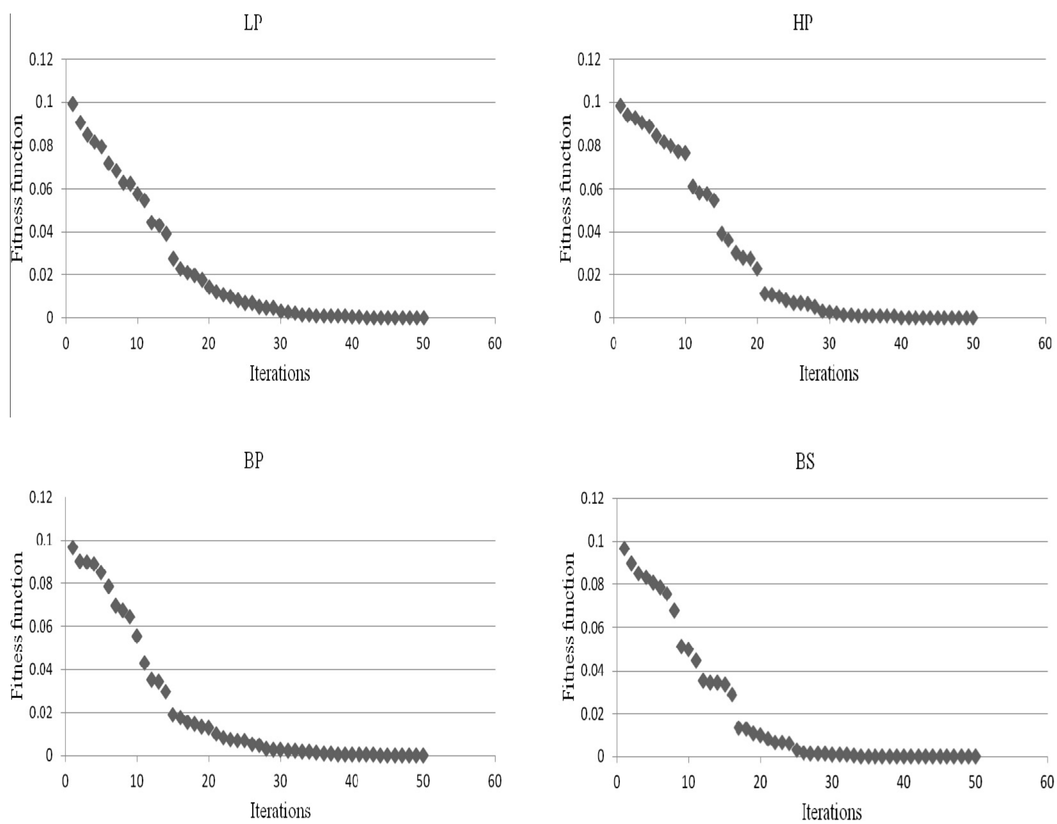

Fig. 7 Variation of fitness function with iterations for digital IIR filters designed with BSA-ES.

5. Conclusions

In this paper, a novel binary successive approximation (BSA) based evolutionary search (ES) methodology has been applied to achieve minimum magnitude response error and phase response error in conjunction with optimal order of the filter simultaneously. In BSA-EV derivative of the function need not be calculated thus reducing number of comparisons and function evaluations required to attain the solution. The results obtained depict that the BSA-EV method is an efficient optimizer as compared to existing evolutionary methods for digital IIR filter design. The proposed method can also be applied to solve multi-objective optimization problems.

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