Chapter 7
Distinctions Between Displacement and Separation

The previous chapter documents that the separation index (S) can reveal the presence of important aspects of residential segregation that cannot be reliably established by examining the more widely used dissimilarity index (D). Specifically, S reliably indicates whether groups are separated and live apart from each other in different areas of the city and experience substantially different residential outcomes – at minimum with respect to area racial composition and potentially also on other neighborhood outcomes that co-vary with area racial composition. High values on S thus signal that groups are residentially separated and reside apart from each other in areas that are polarized on racial composition. The same cannot be said for D. To the contrary, D can and sometimes does take high values when two groups are not residentially separated and in fact live together in the same neighborhoods and experience quantitatively similar residential outcomes on area racial composition. Thus, high values on D cannot and do not reliably signal the presence of group residential separation and neighborhood racial polarization.

I view the issue of whether groups are separated and live apart in different neighborhoods or live together in the same areas and share neighborhood outcomes as fundamental to segregation research. The following two quotes from Massey and Denton's (1988) landmark methodological study are consistent with this view. Speaking of segregation in broad terms they state "At a general level, residential segregation is *the degree to which two or more groups live separately from one another, in different parts of the urban environment*." (1988:282, emphasis added). Speaking more specifically of the dimension of uneven distribution they state "Evenness is minimized and segregation maximized when no minority and majority members share a common area of residence" (1988:284).

These statements resonate with prevailing substantive intuitions about residential segregation. Researchers and broader audiences alike presume that high scores on segregation signal that the groups in the comparison live apart from each other in different neighborhoods and thus do not share common fate based on area of

M. Fossett, *New Methods for Measuring and Analyzing Segregation*,
The Springer Series on Demographic Methods and Population Analysis 42,
DOI 10.1007/978-3-319-41304-4_7

residence. The separation index (S) provides a reliable signal on this count. The dissimilarity index (D) does not. D does not because it measures something different from whether groups live together or apart. Specifically, D provides a reliable signal regarding whether groups differ in their extent of being displaced from parity. Significantly, however, *group differences in being displaced from parity and group residential separation are not the same things and they do not necessarily correlate closely empirically*. Displacement and separation often do take high values together. But, importantly, group difference in displacement from parity can be high when group residential separation is low.

In this chapter I seek to clarify the differences between separation (S) and displacement (D) in more detail. I begin by noting that the issue has become more important in recent decades because conceptual distinctions between separation and displacement have come to take on greater practical significance in empirical analyses. The main reason for this is that the scope of segregation studies has expanded and the racial demography of US urban areas in the United States has become more complex. As a result, researchers are now frequently investigating segregation in situations where large differences between scores on separation and displacement are more common than was the case in an earlier era of segregation research.

I frame the substantive issues involved by introducing two terms. The first is "prototypical segregation" which is associated with a pattern of "concentrated displacement". The second is the opposite condition of "dispersed displacement" a pattern of segregation that is empirically common but largely unrecognized in the measurement literature.

In the pattern of "prototypical segregation" displacement from even distribution concentrates the populations of the two groups into homogeneous areas that differ by quantitatively large amounts on area racial composition. When such a pattern of "concentrated displacement" is present, group residential separation and area racial polarization as indicated by S will approach the maximum levels possible at a given level of displacement from parity as indicated by D. In the logical extreme where displacement is concentrated to the maximum possible extent, the value of S will equal the value of D. The pattern of "dispersed displacement" is at the opposite end of the spectrum. Under this pattern levels of group residential separation and area racial polarization are far below the maximum levels possible for a given level of displacement. In sum, under "prototypical segregation" involving concentrated displacement values of D and S correspond closely. Under dispersed displacement, values of D and S diverge by large amounts.

I next explore these issues in two extended technical discussions that clarify the basis for D-S congruence and divergence. In the first discussion I contrast how D and S respond differently to residential exchanges that promote integration and/or segregation and I describe how this can lead to D and S taking either similar or discrepant values. In the second discussion I introduce simple analytic models that reveal more precisely how displacement (D) and separation (S) can vary independently to produce residential patterns ranging from "prototypical segregation" to "dispersed displacement" at any given combination of displacement (D) and overall city racial composition (P). I then "exercise" the models to produce graphical results that reveal the nature and range of potential combinations of displacement (D) and group separation (S) by level of city racial composition.

I close the chapter by considering the question of whether displacement (D) and separation (S) should be seen as distinctly different dimensions of segregation. My discussion gives attention to three alternative views. One is the position suggested by Stearns and Logan (1986) which holds that group separation and area racial polarization should be seen as a distinctive dimension of segregation to be considered along with uneven distribution and exposure. Another view takes the position that group separation and area racial polarization can be seen as an important aspect of uneven distribution that may or may not be present when group distributions are displaced from even distribution. I also consider and dismiss a mistaken third view, sometimes suggested in the literature, that group separation and area polarization reflects exposure.

In the end I endorse a practical compromise. In my view it ultimately is not crucial whether one classifies group separation and area racial polarization as a distinct dimension of segregation or is a particular aspect of uneven distribution. What is crucial is for researchers to recognize that separation, displacement, and exposure all provide useful information and all three can and do vary independently in empirical analyses. This knowledge will help researchers choose measures that best serve their research interests. My view is that this will lead researchers to pay closer attention to group separation and area polarization as measured by S because S provides a reliable signal about the presence or absence of "prototypical segregation" which researchers and broad audiences alike find more interesting and compelling than "dispersed displacement".

7.1 The Increasing Practical Importance of the Distinction Between Displacement and Separation

Stearns and Logan (1986) argued that the distinction between D and S is important noting that the measures "are responsive to different aspects of changes in racial residential patterns" and can "lead to divergent, sometimes contradictory, results" (1986:125–126). To support their view they noted the example of Logan and Schneider (1984) who found that D and S gave different results regarding trends in White-Black segregation in suburban areas with S showing increasing segregation while D indicated declining segregation. Studies by Schnare (1980) and Smith (1991) also reported finding different patterns and trends in residential segregation when using D and S.

Coleman et al. (1982:177–179) had previously argued that D and S differ in ability to provide a reliable signal of when group have important differences in residential outcomes and noted that D can take high values when the two groups in the comparison have fundamentally similar distributions on residential outcomes. Zoloth (1976) made similar points in an earlier methodological study. Unfortunately, the findings and observations reported in these studies have had minimal impact on prevailing practices in segregation research. Empirical studies overwhelmingly use D over alternative measures and typically do not report whether findings are similar or different depending on whether alternative indices such as S are used. This suggests that researchers generally are not aware of two points. The first is that D and S can take highly discrepant scores and can move in different directions. The second is that whether scores for D and S align or diverge it has important implications about fundamental aspects of the nature of segregation.

Prevailing practices may have been more understandable and less consequential in an earlier era of segregation research. For many decades empirical studies focused primarily on White-Black segregation in large metropolitan areas where Black populations were substantial in size and typically were concentrated in large ghettos. The empirical analyses in the Chap. 6 showed that discrepancies between displacement (D) and separation (S) tend to be less dramatic when analysis is restricted to this particular subset of segregation comparisons. So, while D and S are not exactly interchangeable in these situations, displacement typically is highly concentrated. As a result the values of D and S tend to correlate closely and index choice may be less likely to lead to important practical differences in findings.

Times have changed. The racial and ethnic composition of cities in the United States has undergone dramatic demographic transformation. Additionally, the scope of segregation studies has expanded to consider segregation across a wider range of group comparisons and a wider range of community settings. *In these new circumstances of segregation research, researchers cannot safely assume that index choice does not matter*. To the contrary, nowadays the logical differences between displacement (D) and separation (S) routinely take on greater practical importance. Over the last four decades the Latino and Asian populations have grown rapidly and diffused from traditional settlement areas to wider distribution nationally. Consequently, segregation studies now examine a broader range of group comparisons beyond the earlier narrow focus on White-Black segregation and routinely give attention to White-Latino and White-Asian segregation. Additionally, the focus of research has expanded from beyond considering just large metropolitan areas where minority presence often is sizeable. Empirical studies now increasingly consider a broader range of communities including communities where minority population presence is relatively small. This is reflected, for example, in studies that examine White-Latino and White-Asian segregation in “new destination” communities where Latino and Asian populations are newly arrived and growing rapidly. Additionally, segregation studies nowadays investigate segregation over an increasingly wide range of settings including not only the largest metropolitan areas but also smaller metropolitan areas, micropolitan areas, noncore counties, and small towns.

All of these trends make the topic of this chapter more relevant to current and future segregation studies. The empirical analyses of White-Minority segregation across CBSAs reviewed Chap. 6 document that the correlation of D and S is weaker when examining White-Latino and especially White-Asian segregation, weaker when examining segregation in smaller communities, and weaker in communities where minorities are smaller in relative size. As a result, we should expect discrepancies between scores for D and S to be increasingly common and larger in size and for the discrepancies to carry increasing substantive importance. Accordingly, it is useful to understand the substantive issues that are relevant when D and S align and when D and S diverge. To serve this goal I now explore the notion of "prototypical segregation" and the contrast between "concentrated" and "dispersed" displacement.

7.2 Prototypical Segregation and Concentrated Versus Dispersed Displacement

I use the term "displacement" to refer to group differences in distribution across neighborhoods that are above or below "parity." Taking Whites as the reference group in an analysis of White-Black segregation, displacement is high when a large share or proportion of White population resides in "above-parity" areas (i.e., where $p_i < P$) and a similarly large share or proportion of the Black population resides in "below-parity" areas. Alternatively, displacement is high when Whites and Blacks differ on the proportion residing in "above-parity" areas or on the proportion residing in "below-parity" areas. These are all slightly different ways of describing the same arrangement and all result in the same values on displacement as measured by D.

Significantly, the notion of displacement from parity does not specify anything further about group residential distributions beyond the narrow confines of what was just stated. Displacement varies in extensiveness – the degree to which it involves large differences in group portions. But extensiveness of displacement does not carry specific implications for the quantitative magnitude of the differences in area racial composition between above-parity neighborhoods and below-parity neighborhoods. To the contrary, the magnitude of the differences involved can vary dramatically at a given level of displacement. The notion of displacement is captured well by the dissimilarity index (D) as its value directly registers majority-minority differences in proportions residing in "parity" or "above-parity" areas.[1] This quality of D was recognized by Duncan and Duncan (1955) who referred to D as the "displacement index."

I use the terms "group separation" and "neighborhood polarization" to refer to residential distributions that are characterized by groups living apart from each other such that members of <u>*both*</u> of the groups in the comparison are disproportionately located in areas where their own group predominates. Significantly, *displacement does not necessarily involve group separation*. Thus, D is not a valid proxy for group separation. Whether or not displacement involves separation depends on additional consideration; namely, whether displacement is "concentrated" or

[1] Alternatively, the value of D can be obtained from the Black-White difference in group proportions residing in "below- parity" areas.

"dispersed." Under concentrated displacement both groups reside apart from each other in racially homogeneous areas that differ markedly on racial composition. Under dispersed displacement, the groups reside together in areas that differ modestly on racial composition.

To clarify, at a given level of displacement, separation is maximized when displacement is *concentrated* in a way that maximizes same-group contact *for both groups*.[2] Conversely, group separation is minimized when displacement is *dispersed* in a way that produces a low level of same-group contact for at least one of the two groups. The notion of separation just outlined is captured well by S which registers the majority-minority difference in (pairwise) contact with the majority group.

7.2.1 Prototypical Segregation

I use the term "prototypical segregation" to refer to a residential pattern where group separation approaches the maximum that can occur at a given level of displacement. I characterize this pattern as prototypical because, without exception so far as I have been able to find, this is the pattern of segregation depicted when examples of high levels of segregation are introduced and reviewed in didactic discussions of residential segregation. For example, it is the kind of segregation pattern seen in didactic illustrations and discussions provided by Taeuber (1964), Taeuber and Taeuber (1965), Jaret (1995), and Iceland et al. (2000). It also is the kind of segregation pattern seen in familiar examples of high levels of segregation as observed for White-Black segregation in cities such as Chicago, Detroit, Cleveland, and Milwaukee and as observed for White-Latino segregation in Los Angeles. What is common in these situations of prototypical White-Minority segregation is this: White households are living in above-parity neighborhoods that are predominantly White in racial composition and, similarly, minority households are living apart from Whites in below-parity neighborhoods that are predominantly minority in racial composition. Accordingly, non-parity areas are "polarized" into areas that differ greatly on racial composition with Whites being concentrated in predominantly White areas and minorities being concentrated in predominantly minority areas typically forming enclaves, barrios, and ghettos.

Values of D and S correspond closely when the condition of prototypical segregation hold because displacement from parity is concentrated rather than dispersed. When prototypical segregation is pronounced, values of both D and S are high; displacement from parity is extensive for both groups and the populations residing in non-parity areas are concentrated into areas that are ethnically homogeneous. Because the two groups live apart in neighborhoods that are fundamentally different in terms of racial composition, residential redistribution that substantially reduces

[2] I place emphasis on "for both groups" because this distinguishes separation from simple same-group contact and isolation. Isolation is intrinsically affected by city racial composition and separation is not.

or eliminates displacement from even distribution also will bring about correspondingly large quantitative changes in neighborhood racial composition. This will in turn carry the potential to also bring about large changes in group differences on neighborhood outcomes that are correlated with area racial composition (e.g., social problems, amenities, services, etc.).

My strong sense is that broad audiences, most academics, and even many segregation researchers generally assume that the residential patterns associated with "prototypical" segregation will be present when scores on widely used segregation indices such as the dissimilarity index (D) are high. This assumption is mistaken. In fairness, however, it is easy to understand why this mistaken view is so widely held. Standard examples and didactic discussions encourage the assumption and little in the standard methodological literature cautions otherwise. That is,

> *Methodological discussions that present examples illustrating how residential segregation is captured by the segregation curve and the dissimilarity index (D) rarely, if ever – feature residential distributions with low group separation (S) resulting from dispersed displacement. Instead, they feature residential distributions with high levels of group separation resulting from concentrated displacement.*

As a result, the prevailing understanding of segregation measurement rests on a widely shared but incorrect assumption that high scores on popular segregation indices always signal the condition of prototypical segregation involving concentrated displacement and group residential separation. This is not the case. In particular, high values of the dissimilarity index (D), the most widely used segregation index, do not and intrinsically cannot provide a reliable signal about the presence of prototypical segregation.[3] In contrast, high values of the separation index (S) provide a certain indication that a high level of prototypical segregation is present.

The outcome of high displacement but with low separation – that is, high D and low S – occurs when residential distributions are characterized by "dispersed displacement." In the pattern of dispersed displacement, individuals residing in non-parity areas are not concentrated in areas where their group predominates. Instead, the residential distribution for at least one of the groups – usually the smaller of the two groups, which in White-Minority comparisons in US cities is typically, but not always, the non-White minority group – is dispersed widely and thinly across non-parity areas such that most members of the group live in "mixed" areas where their group is not the predominant presence. Indeed, it can be the case that few members of the group live in areas where their group is a majority presence and instead most members of the group live in areas where the other group in the comparison is the predominant group. As a result, under dispersed displacement the two groups in the comparison live together in areas with similar racial composition, not apart from each other in areas where racial composition is polarized.

---

[3] The same can be said for any index that ranks segregation comparisons consistent with the principle of segregation curve dominance. In addition to the dissimilarity index (D), this includes the gini index (G), the symmetric version of the Atkinson index ($A_{0.5}$), and the Hutchens square root index (R).

The contrasting notions of prototypical segregation and dispersed displacement can be clarified by comparing two logically possible but fundamentally different outcomes that can occur at a given level of displacement. One outcome is that all group members not living in parity areas reside in perfectly segregated, homogeneous areas. For example, in the case of White-Black segregation, Whites and Blacks not living in parity areas would reside in all-White and all-Black areas, respectively. I term this "maximally concentrated displacement." The other outcome is that all group members not residing in parity areas reside in areas that come as close to matching parity as is demographically feasible. I term this "maximally dispersed displacement."

Importantly, the values of D and S vary dramatically across these two logical possibilities. The value of D will necessarily be the same in both cases. In contrast, the value of S will vary across these two cases, potentially by a very large amount. For the level of displacement in question, the value of S will take its highest possible value, in which case it will equal the value of D, under maximally concentrated displacement. S will take its lowest possible value under maximally dispersed displacement. This leads to a broad rule of thumb for characterizing segregation patterns. At a given level of displacement, "prototypical segregation" holds when the value of S is relatively close to its highest possible value and "dispersed displacement" holds when the value of S is relatively close to its lowest possible value.

Under "prototypical segregation," D-S combinations are characterized by close agreement; their scores roughly correspond at low-low, medium-medium, high-high, and so forth. Under "dispersed displacement," D-S combinations are characterized by disagreement, sometimes very dramatic disagreement, with scores for D being much higher than scores for S. Figure 7.1 places combinations of D and S in four general categories based on a two-by-two classification of high and low outcomes on the dissimilarity index (D) and the separation index (S). The purpose of this simplified presentation is to focus attention of the fundamental differences between the logically possible combinations.

To begin I note that the D-S combination in the upper-left cell of the figure cannot occur. As I show below, displacement as measured by D sets the upper limit for group separation as measured by S. Accordingly, high values of group separation (S) always are accompanied by values of displacement (D) of equal or greater size. Consequently, a low-D, high-S combination is not logically possible. The lower-left cell (A) is labeled "Low Prototypical Segregation." It involves a low-level of group displacement from even distribution (D) and a corresponding low level of group separation (S). The upper-right cell (C) is labeled "High Prototypical Segregation." It involves a high level of group displacement from even distribution (D) and a corresponding high level of group separation (S). The lower-right cell (B) is labeled "Displacement without Separation." It involves a high level of displacement from even distribution (D) but with levels of group separation substantially below what is possible (S). Since this pattern involves dispersed rather than concentrated displacement, the alternative label of "Dispersed Displacement" also is appropriate.

Recall from discussion in earlier chapters that the dissimilarity index (D) can be characterized as summary index of group inequality in rank-order position on area

Value of S	Value of D	
	Low	High
High	This outcome cannot occur	(C) High Prototypical Segregation (Concentrated Displacement) Displacement from even distribution is extensive and it is concentrated so it involves maximal group separation and area racial polarization. The group difference in percentage attaining parity on area group proportion (p) is large *and* the group difference of means on (p) is large. Example generating process – implement as many "segregation-promoting" exchanges as possible without changing D·
Low	(A) Low Prototypical Segregation Displacement from even distribution is low and group separation and residential polarization also are low. The group difference in percentage attaining parity on area group proportion ($p \geq P$) is small *and* the group difference of means on (p) is small. Example generating process – quota allocation or random distribution.	(B) Displacement without Separation (Dispersed Displacement) Displacement from even distribution is extensive, but it is dispersed and involves minimal group separation and area racial polarization. The group difference in percentage attaining parity on area group proportion (p) is large, but the group difference of means on (p) is small. Example generating process – implement as many integration-promoting exchanges as possible changing D.

Fig. 7.1 Possible combinations of high and low values on displacement (D) and separation (S)

racial composition. Specifically, in the case of White-Minority segregation, D, like the gini index (G), reflects rank-order inequality on area proportion White (p).[4] Similarly, the separation index (S) can be characterized as a summary index of group inequality on the original or "raw" quantitative scores on area racial composition (p). With this in mind, the four cells in Fig. 7.1 can be described in the

[4]Thus, the value of G can be given as twice the value of the group difference in mean percentile scores on area group proportion (p). The value of D can be given in the same way based on collapsing scoring of area group proportion into two categories of "above parity" ($p > P$) or not.

following terms. The lower-left cell (A) "Low Prototypical Segregation" and the upper-right cell (C) "High Prototypical Segregation" both involve situations where group distributions on area proportion White (p) produce similar high levels of inequality in rank-order position (D) and quantitative difference (S). The lower-right cell (B) "Displacement without Separation" involves a high level of group inequality on rank order position on area proportion White (p) but a low level of group inequality on quantitative differences on area proportion White (p). The combination indicates that Whites are consistently ranked above Blacks on area proportion White – as indicated by the high value of D, but the quantitative differences involved are small and thus result in the low value of S. Thus, the rank-order differences on area proportion White do not translate into group separation because the two groups have similar distributions on area racial composition (p) and thus the two populations are living together, not apart from each other.

7.3 Clarifying the Logical Potential for D-S Concordance and Discordance – Analysis of Exchanges

Scores for D and S can diverge because they assess group differences in residential distribution in fundamentally different ways. D measures group differences on area proportion White (p) in a crude way; it assesses the group difference in relative distribution ***between*** two kinds of areas; those that are "above-parity" on area proportion White (p) and those that are "below-parity."[5] In contrast, S measures group difference in area proportion White (p) based on quantitative differences over the full distribution of values for area proportion White (p). Thus, where S registers all quantitative information about group differences on area proportion White (p), D instead collapses this information into a dichotomous rank-order scoring of "above P" or not. Thus, at any value of D, the value of S can vary by a considerable amount because, unlike D, S registers group differences in distribution on area proportion White (p) both within and across "non-parity" areas.

Methodological studies establish that the potential for scores of D and S to diverge traces to two technical differences between D and S. The first is a well-known technical deficiency with D. It is that D does not register all integration-promoting exchanges of White and Black households between two areas (Reardon and Firebaugh 2002).[6] The value of D changes only for a partial subset of integration-promoting exchanges – those that cause at least one of the two areas involved in the exchange to move from being above the value of proportion White for the city (P) to at or below P when the exchange is completed, or, alternatively, to move from being below P to at or above P. When integration-promoting exchanges involve

[5] The same quantitative result is obtained if the distinction is "at-or-above-parity" and "below-parity."

[6] The nature of integration-promoting and segregation-promoting exchange is discussed in more detail in a separate section below.

households from areas on the same side of the cut point (P) before and after the exchange, the value of D does not change. In contrast, S behaves as accepted principles of segregation measurement require; the value of S goes down when any integration-promoting exchange occurs and the value of S goes up when any segregation-promoting exchange occurs (Reardon and Firebaugh 2002).

This provides the initial basis for understanding how the value of S can move independently of the value of D. It is that, at any value of D, integration-promoting exchanges that involve areas on the same side of overall proportion White (P) before and after the exchange will cause the value of S to go down while the value of D remains fixed. Similarly, segregation-promoting exchanges that involve areas where proportion White (p) is on the same side of overall proportion White (P) before and after the exchange will cause the value of S to go up while the value of D remains fixed. Under accepted principles of segregation measurement the changes in values of S that take place while D is remaining constant are desirable; they occur because S is registering changes in uneven distribution within non-parity areas. In contrast, the non-responsiveness of D is undesirable; it occurs because D is insensitive to changes in uneven distribution that are taking place within non-parity areas.

There is a second basis for why the value of S can move independently of the value of D. It is that, even in cases where D does register the impact of an integration-promoting exchange, D has a "flat" or "uniform" response regardless of the impact of the exchange on group separation as it relates to the magnitude of the changes in area racial composition. In contrast, S responds differentially depending on the impact the exchange has on group separation by responding more strongly when the two areas involved in the exchange are more "polarized" based on being further apart on area proportion White (p). That is to say, all else equal, for any exchange producing a change in D, the impact on the value of D will be the same regardless of the magnitude of the difference on area proportion White (p) between the two areas in the exchange but the impact on the value of S will be larger when the difference is larger rather than smaller. This conforms to the substantively appealing property that exchanging White and Black households across all-White and all-Black areas reduces segregation more than exchanging White and Black households across areas that are nearly identical on area proportion White (p). The former exchange reduces group separation to a greater degree than the latter exchange because it has a larger impact on reducing area racial polarization and White-Black differences in distribution on area proportion White (p).

I review the formal basis for this conclusion in the next two sections. I motivate the discussion by trying to briefly give an intuitive sense of why the issue is important. At a given level of displacement from even distribution as measured by D households not residing in parity areas can be maximally segregated or minimally segregated under the exchange criterion. Under maximal segregation, all possible segregation-promoting exchanges that do not change the value D are implemented. The value of S will equal the value of D and White and Black households residing in non-parity areas will be separated into maximally polarized, homogeneous areas. Under minimal segregation, all possible integration-promoting exchanges that do not change the value of D are implemented. The value of S will be very low in comparison to the value of D because White and Black households residing in non-parity areas will live together in areas that are relatively similar on racial composition. The difference between the two extremes is unquestionably sociologically meaningful. So it is important to understand how D and S differ in their ability to reveal these two fundamentally different residential patterns.

7.3.1 Overview of D-S Differences in Responding to Integration-Promoting Exchanges

In this section I review how D and S respond to exchanges of White and Black households across areas. To begin, I note that uneven distribution emerges when two areas with the same racial composition – in the White-Black comparison, the same area proportion White (p) – exchange a White and Black household. The area receiving the White household and losing the Black household now has a higher proportion White and the area losing the White household and receiving the Black household now has a lower proportion White. Reversing the exchange restores even distribution. Accordingly, an "integration-promoting exchange" is one in which the White household in the exchange moves from an area where proportion White (p_i) is higher to an area where proportion White (p_j) is lower (i.e., $p_i > p_j$) *and* the Black household in the exchange moves from an area where proportion White (p_j) is lower to an area where proportion White (p_i) is higher (Reardon and Firebaugh 2002:38). Conversely, a "segregation-promoting exchange" is one in which the White household in the exchange moves from an area where proportion White (p_i) is lower to an area where proportion White (p_j) is higher (i.e., $p_i < p_j$) *and* the Black household in the exchange moves from an area where proportion White (p_j) is lower.

In the theory of segregation measurement, the "exchange" criterion holds that indices should register all integration-promoting and segregation-promoting exchanges by decreasing or increasing in value, respectively, when the exchange is completed (Reardon and Firebaugh 2002). The separation index (S) meets this criterion. The dissimilarity index (D) does not.

I note that it is reasonable to term segregation-promoting exchanges as "polarizing" and "concentrating" and it is similarly appropriate to term integration-promoting exchanges as "depolarizing," "deconcentrating", and "dispersing." A segregation-promoting exchange is "polarizing" because it moves the two areas involved in the exchange further apart on area proportion White since $|p_i - p_j|$ is larger after the exchange is completed. At the same time, the exchange is "concentrating" because pairwise same-group contact goes up for both Whites and Blacks in the affected areas. Since the residential distribution of Whites and Blacks in other areas is unchanged, the result of the exchange is greater overall area polarization, greater overall group concentration, greater overall group separation, and a higher value of S.

An integration-promoting exchange is "depolarizing" because it moves the two areas involved in the exchange closer together on area proportion White since $|p_i - p_j|$ is smaller after the exchange is completed. At the same time, the exchange is "deconcentrating" because pairwise same-group contact goes down for both Whites and Blacks in the affected areas. Again, since the residential distribution of Whites and Blacks in other areas is unchanged, the exchange reduces overall area polarization, reduces overall group concentration, reduces overall group separation, and lowers the value of S.

Based on this, it is clear that the underlying logic of the separation index (S) resonates well with the exchange criterion. In contrast, the underlying logic of the dissimilarity index (D) is often at odds with the criterion. D registers integration-promoting exchanges only in the circumstance that the racial composition of the two areas involved in the exchange are on opposite sides of P, proportion White for the city overall. Integrating-promoting exchanges that involve areas with racial compositions on the same side of P have no impact on D.

In addition to meeting the minimum requirements for satisfying the exchange criterion, the separation index (S) has additional properties that in my opinion are desirable for assessing whether groups live apart or together. I list them as follows.[7]

- All else equal, an integration-promoting exchange produces a larger reduction in S when the two areas involved in the exchange are more polarized.

  I term this the "polarization" property with polarization or dispersion being based on the initial size of $|p_i - p_j|$. Substantively, this is appealing because, assuming area size is constant, exchanges between more polarized areas reduces same group contact for larger fractions of the affected population.

  No surprisingly, D does not have this property.

- All else equal, an integration-promoting exchange produces a larger reduction in S when the two areas involved in the exchange are closer to one of the polarization boundaries of all-White or all-Black. That is, the reduction is larger when the minimum of the two values $|p_i - 1|$ and $|p_j - 0|$ is closer to 0.0.

  The substantive appeal of this characteristic is similar to that for the "polarization" property. Here again exchanges that involve areas that are nearer to the homogeneous "poles" of 0 and 1 reduce same group-contact for larger fractions of the affected population.

  D does not have this property.

- The "polarization" property holds throughout the full range of area proportion White (p). Thus, in contrast to D, integration-promoting exchanges have desirable impacts on reducing S regardless of whether the two areas involved in the

[7] I establish these properties by drawing on previous methodological discussions (e.g., Zoloth 1976; James and Taeuber 1985; Reardon and Firebaugh 2002) and by simulation analyses that systematically exercise the possible "event-space" of exchanges between areas in a model city.

exchange have racial composition on opposite sides of P – the racial composition of the city overall – or on the same side of P.

This is substantively attractive because it is nonsensical to limit the principle of exchanges to apply to exchanges on opposite sides of P (i.e., where $p_i > P > p_j$). It is possible to achieve integration by making only exchanges of this nature. But substantial integration also can be achieved with exchanges on the same side of P (i.e., where $p_i > p_j > P$ or $P > p_i > p_j$.

There is no substantive basis for ignoring the impact of integration-promoting exchanges involving areas with racial compositions on the same side of P.

7.3.2 *Examples of D-S Differences in Responding to Integration-Promoting Exchanges*

To illustrate selected points from the preceding discussion, I compare four integration-promoting exchanges for a hypothetical city that is populated by only White and Black households and has an overall proportion White of 0.50. For simplicity, I assume all areas are the same size and are populated with 100 households. Under these assumptions, relative impact of an exchange on S is strictly determined by the impact the exchange has on the White-Black difference in segregation-relevant average contact with Whites (p) for the 200 households residing in the two areas involved in the exchange.[8] For the purposes of this discussion I will designate this difference with the Greek letter lambda (λ) and express it in percentage form (instead of as proportions) for ease of presentation and discussion.

Figure 7.2 presents results for two pairs of hypothetical exchanges. The first panel summarizes results for a pair of integration-promoting exchanges that involve areas on opposite sides of P, one above parity and the other below parity. The second panel summarizes results for a pair of integration-promoting exchanges that involve two areas that are not above parity. I begin by discussing the pair of exchanges in the first panel. The first exchange shown involves two areas that are highly polarized on racial composition. The first area (Area 1) is an all-White area with 100 White and 0 Black households. The second area (Area 2) is all-Black area with 0 White and 100 Black households. The integration-promoting exchange moves a White household from Area 1 (higher p) to Area 2 (lower p) and a Black household from Area 2 (lower p) to Area 1 (higher p). Following the exchange, Area 1 has 99 White households and 1 Black household and Area 2 has 1 White household and 99 Black households.

The integration-promoting exchange could be imagined as two "pioneering" residential moves. For example, the exchange could involve the moves of a "pioneering" Black household and a "gentrifying" White household. The pioneering Black household leaves a predominantly Black neighborhood and moves to a pre-

[8] The racial composition of all other areas remains unchanged. So the any change in S derives solely from the impact of changes in the areas involved in the exchange.

Two Examples of Exchanges Involving Areas on Opposite Sides of P

	First Exchange		Second Exchange	
Area Population Distributions	White N	Black N	White N	Black N
Area 1 – Before Exchange	100	0	51	49
Area 2 – Before Exchange	0	100	49	51
Area 1 – After Exchange	99	1	50	50
Area 2 – After Exchange	1	99	50	50
Impact on Index	S	D	S	D
Initial White Mean (y·100)	100.00	100.00	50.02	51.00
Initial Black Mean (y·100)	0.00	0.00	49.98	49.00
λ Before Exchange (x100)	100.00	100.00	0.04	2.00
Final White Mean (y·100)	98.02	99.00	50.00	0.00
Final Black Mean (y·100)	1.98	1.00	50.00	0.00
λ After Exchange (x100)	96.04	98.00	0.00	0.00
λ Change (x100)	−3.96	−2.00	−0.04	−2.00

Two Examples of Exchanges Involving Below-Parity Areas

	Third Exchange		Fourth Exchange	
Area Population Distributions	White N	Black N	White N	Black N
Area 1 – Before Exchange	49	51	26	74
Area 2 – Before Exchange	1	99	24	76
Area 1 – After Exchange	48	52	25	75
Area 2 – After Exchange	2	98	25	75
Impact on Index	S	D	S	D
Initial White Mean (y·100)	48.04	0.00	25.04	0.00
Initial Black Mean (y·100)	17.32	0.00	24.99	0.00
λ Before Exchange	30.72	0.00	0.05	0.00
Final White Mean (y·100)	46.16	0.00	25.00	0.00
Final Black Mean (y·100)	17.95	0.00	25.00	0.00
λ After Exchange (x100)	28.21	0.00	0.00	0.00
λ Change (x100)	−2.51	0.00	−0.05	0.00

Fig. 7.2 Impacts of selected integration-promoting exchanges on the value of the separation index (S) and the dissimilarity index (D)

dominantly White neighborhood. The "gentrifying" White household leaves a predominantly White neighborhood and moves to a predominantly Black neighborhood.

In the difference of means framework, the impact of the exchange on an index score can be assessed by considering how segregation-relevant residential outcomes (y) change for the 200 households in the affected neighborhoods. For S, y is simply area proportion White ($y = p$) so average contact with Whites is initially 100.0 points for Whites and 0.0 points for Blacks. This yields a value of λ – the White-Black average difference for the population in the two areas – of 100.0 points. After the exchange, average contact with Whites falls to 98.02 points for Whites in the two areas and rises to 1.98 points for Blacks, producing a value of λ of 96.04 points. Thus, the exchange causes the average White-Black contact difference for the subset of affected households – quantified as λ – to fall by 3.96 points.

The second integration-promoting exchange involves White and Black households residing in two areas that differ only slightly on racial composition. Before the exchange the first area (Area 1) has 51 White and 49 Black households and the second area (Area 2) has 49 White and 51 Black households. After the exchange Area 1 and Area 2 both change to 50 White and 50 Black households thus bringing about integration. For the subset of households in the affected households, average contact with Whites is initially 50.02 points for Whites and 49.98 points for Blacks, producing a White-Black difference (λ) of 0.04 points. After the exchange, average contact with Whites falls to 50.0 points for Whites and rises to 50.00 points for Blacks, producing a White-Black difference (λ) of 0.00 points. Thus, the exchange causes the average White-Black contact difference for the subset of affected households (λ) to fall, but only by 0.04 points.

The larger reduction in λ for S in the first exchange compared to the second exchange −3.96 points versus −0.04 points – highlights a property of S discussed above and noted previously by Zoloth (1976), James and Taeuber (1985), and Reardon and Firebaugh (2002). The property is that S responds more strongly to integration-promoting exchanges between areas that are more polarized in terms of area racial composition (i.e., exchanges when $| p_i - p_j |$ is larger). The reduction in the first exchange is larger by 3.92 points than the reduction in the second exchange and in relative terms is 99 times larger.

I view this as sensible and desirable. In substantive terms the first exchange has a larger impact on reducing group separation because it does more to "deconcentrate" the group distributions across the two areas because it brings together White and Black households from areas that initially were at opposite extremes on area racial composition.

The first exchange reduces White's contact with Whites by a larger amount – 1.98 points compared to 0.02 points – while simultaneously increasing Black's contact with Whites by a larger amount – 1.98 points compared to 0.02 points. As a result, the first exchange reduces the White-Black difference in contact with Whites by a larger amount. In contrast, the second exchange has a small impact on reducing group separation because it brings people together from areas that initially were minimally different on area racial composition. Accordingly, the exchange has less impact on group separation as measured by S because the affected White and Black households were already living together.

The relative impact of these integration-producing exchanges on D can be assessed by calculating lambda (λ) in the same manner as just performed for S. The only difference is that segregation-relevant contact with Whites (y) is scored differently for D than for S. Specifically, contact with Whites (y) is scored 1 for "above parity" and 0 otherwise. For D, the relative impact of the exchanges on the value of D is the same for both of the exchanges. Specifically, the White-Black difference in average (scaled) contact with Whites for the affected households (λ) is reduced by two points under both scenarios. The reason for this is that in both cases a single White household changes from being scored 1 for "above parity" to being scored 0 for "not above parity." Similarly in both cases only a single Black household changes from being scored 0 for "not above parity" to being scored 1 for "above parity". The initial average on contact with Whites as measured by D is 100.0 for Whites and 0.0 for Blacks, producing an average White-Black difference of 100.0 for the population in the affected neighborhoods. After the exchange, the average on contact with Whites as measured by D is 99.0 for Whites and 1.0 for Blacks resulting in a difference of 98.0. The exchange thus reduces the value of λ by 2.0 points.

The second exchange also produces a reduction in the value of λ of 2.0 points. In this case, average contact with Whites as measured by D is initially 51.0 for Whites and 49.0 for Blacks, producing an average White-Black difference of 2.0 for the population in the affected neighborhoods. After the exchange, the average on contact with Whites as measured by D is 0.0 for Whites and 0.0 for Blacks resulting in a difference of 0.0 since now no one in either group lives in an "above-parity" area. The exchange thus reduces the value of λ by 2.0 points, a reduction identical to the amount in first exchange.

The "flat" or "fixed" response of the relative impact of λ on D can be seen as appropriate for the goal of assessing "displacement" conceived narrowly in terms of population fractions moving from one side of parity to the other. These fractions are the same for both exchange scenarios, so λ is the same for both scenarios. The fact that the two exchanges in question have fundamentally different effects on group separation and area polarization is not relevant to the narrow conception of displacement embodied in D.

The contrast of the flat response for D for these two exchanges and the variable response for S highlights how displacement and separation are distinct and can vary independently. This point is further established by considering the pair of integration-promoting exchanges summarized in the second panel of Fig. 7.2. The most important difference between this pair of exchanges and the pair summarized in the top panel is that both of the areas in the bottom panel are "below-parity" on area proportion White.

The third exchange depicted involves one area (Area 1) with 49 White and 51 Black households and a second area (Area 2) with 1 White and 99 Black households. Both are "below-parity" areas. The integration-promoting exchange involved moves a White household from Area 1 (higher p) to Area 2 (lower p) and a Black household from Area 2 (lower p) to Area 1 (higher p). Following the exchange Area 1 changes to 48 White and 52 Black households and Area 2 changes to 2 White and 98 Black households.

This exchange involves two areas that differ substantially on racial composition with values on area proportion White of 49.0 and 1.0, respectively. The impact on S can be assessed as before by examining the value of λ – the White-Black difference on segregation-relevant contact with Whites for the population in the affected areas. Initially, average contact with Whites as measured for S is 48.04 for Whites and 17.32 for Blacks yielding a value of λ of 30.72. After the exchange, average contact with Whites as measured for S is 46.16 for Whites and 17.95 for Blacks yielding a value of λ of 2.21. Thus, under this exchange scenario the White-Black contact difference (λ) for the subset of affected households is reduced by 2.51 points.

The fourth exchange involves two areas that together have the same number of White and Black households as in the two areas in the third exchange; 50 White and 150 Black households, respectively. The initial distribution is less polarized than in the previous example. One area (Area 1) begins with 26 White and 74 Black households and a second area (Area 2) that begins with 24 White and 76 Black households. The integration-promoting exchange involves moving one White household from the area of higher p to the area with lower p and moving one Black moving from the area of lower p to the area of higher p. Following the exchange, Area 1 and Area 2 both change to having 25 White and 75 Black households. As with the third exchange, this exchange involves two areas that are "below-parity". Here, however, the two areas initially are very similar on racial composition with area proportion White at 0.26 and 0.24, respectively. As a result, the White-Black contact difference (λ) for the affected households changes by a very small amount. Initially, average contact with Whites as measured by S is 25.04 for Whites and 24.99 for Blacks yielding a value of λ of 0.05. After the exchange, average contact with Whites as measured by S is 25.00 for Whites and 25.00 for Blacks yielding a value of λ of 0.0. Thus, the exchange reduces the White-Black contact difference (λ) for the subset of affected households by 0.05 points.

There are two key findings. One is that both exchanges produce reductions in S whereas we will soon see that neither exchange produces a reduction in D. Another key finding is that the impact on reducing S is much larger in the third exchange than in the fourth exchange. The reduction in the third exchange is 2.46 points larger than in the fourth exchange and in relative terms is almost 50 times larger. Again, considered in relation to the goal of assessing whether groups live together or apart, it is substantively sensible that the third exchange has a bigger relative impact on S than the fourth exchange. As previously seen in the first exchange, the third exchange does more to "deconcentrate" the group distributions. The third exchange reduces White's contact with Whites by a larger amount than the fourth exchange – 1.88 points compared to 0.04 points, respectively – while simultaneously increasing Black's contact with Whites by a larger amount – 0.63 points compared to 0.01 points, respectively.

In substantive terms, the third exchange could be imagined to reflect a "middle-stage" integrating sequence where a pioneering Black household leaves a predominantly Black neighborhood and moves to diverse (50/50) neighborhood and a gentrifying White household leaves a 50/50 area and moves to a predominantly Black area. In contrast, the fourth exchange is a small-impact integrating exchange. Like the second exchange reviewed earlier, the two areas involved are near-identical in terms of racial composition before the exchange so on balance the households affected by the exchange experience minimal changes in neighborhood outcomes and very small reductions in pairwise same-group contact.

The response of D in the third and fourth exchanges in the second panel is easy to summarize. D does not change in either case because all households in both groups reside in areas that are "below-parity" both before and after the exchanges. So again, D has a flat response of no change while S registers a decline in both exchanges. The response by S varies from the response by D in two ways. First, S responds to both integrating moves while D does not. Second, S responds more strongly to the third exchange which clearly reduces group separation and area racial polarization by a larger amount.

One implication from the comparison of how S and D are affected by these two exchanges is readily obvious. It is that the value of D can remain fixed while the value of S can run higher or lower depending on whether integrating moves involving areas that are not above parity reduce polarization or whether segregating moves increase polarization. I discuss this more carefully in the next section.

7.3.3 Implications of Analysis of Example Exchanges

A couple of important implications follow from these examples of how D and S respond to exchanges. I start first with integration-promoting exchanges where both areas involved in the exchange are on opposite sides of P. In these exchanges D has a "flat" response to all integration-promoting exchanges; its value declines by the same amount in all cases. In contrast, S will respond more strongly when the exchange is between more polarized areas (and therefore more distant from parity) and S will respond less strongly when the exchange is between areas of similar racial composition (and therefore closer to parity). This leads to the following conclusion.

> *Values of D and S can be similar or they can diverge depending on whether displacement from uneven distribution arises from segregation-promoting exchanges that produce maximally polarized areas (higher S, closer to D) or minimally polarized areas (lower S, further from D) on opposite sides of P.*

The second important implication concerns integration-promoting exchanges where both areas involved in the exchange are on the same side of P. D again has a "flat" response. It does not change. In contrast, S will always respond and S will respond more strongly when the exchange is between more polarized areas and S will respond less strongly when the exchange is between areas of similar racial composition. This leads to the following conclusions.

> *Values of D and S can be similar or they can diverge depending on whether displacement from uneven distribution arises from segregation-promoting exchanges that produce maximally polarized areas (higher S, closer to D) or minimally polarized areas (lower S, further from D) on the same side of P.*

Stated another way, S will take higher values when the population residing in non-parity areas is concentrated to form racially polarized areas and S will take lower values when the population residing in non-parity areas is dispersed widely to form areas that are similar on racial composition instead of being polarized.

The practical consequence for D-S comparisons is this. At a given level of displacement as measured by the dissimilarity index (D), the value of the separation index (S) can vary independently and by substantial amounts depending on whether group distributions both ***between*** "above-parity" areas and "other" areas and ***within*** "non-parity" areas tend toward maximum area racial polarization or minimum area racial polarization. The former concentrates both groups in homogeneous areas and maximizes same-group contact and group separation. The latter disperses both groups across less homogeneous areas and minimizes same-group contact and group separation.

Ultimately, as I show below, this leads to the following conclusion about the relationship between D and S. At a given level of displacement (D), the value of the separation index (S) can vary substantially depending on whether group distributions within "non-parity" areas tend toward concentration or dispersion. When concentration within non-parity areas is at its maximum, the value of S will equal the value of D. But when concentration is at its minimum – that is, when groups are maximally dispersed across non-parity areas, the value of S will be lower, sometimes much lower, than the value of D.

Intuitively, one can get to these two alternative outcomes via simple steps as follows. At a given level of displacement, implement as many segregation-promoting exchanges as possible ***within*** non-parity areas. If such exchanges can be made, group residential distributions will shift toward the pattern of "prototypical segregation" and the value of S will increase. The value of D will not change so the D-S disparity will decrease. Ultimately, the value of S will rise until it reaches the value of D and D-S disparity will be zero.

Alternatively, implement as many integration-promoting exchanges as possible *within* non-parity areas. If such exchanges can be made, group residential distributions will shift toward the pattern of "dispersed displacement" and the value of S will decrease. The value of D will not change so D-S disparity will increase. Ultimately, S will fall until it reaches its minimum possible level and the D-S disparity reaches its maximum. At the conclusion of the process, S will take a value substantially below the value of D.

7.4 Clarifying the Potential for D-S Concordance and Discordance – Analytic Models

I further clarify the potential for both D-S agreement and disagreement by reviewing a series of analytic exercises that illustrate how group separation and area polarization (S) can vary independently from the level of displacement as measured by dissimilarity (D) while holding city-level racial composition (P) constant. To keep the exercises simple and easier to follow, I limit the hypothetical city to only three kinds of neighborhoods designated as Areas 1, 2, and 3 with the following characteristics.

Area_1 is "Above parity" (i.e., disproportionately White with $p_1 > P$ and $q_1 < Q$)
Area_2 is at "Parity" (i.e., exactly average on proportion White with $p_2 = P$ and $q_2 = Q$)
Area_3 is "Below parity" (i.e., disproportionately Black with $p_3 < P$ and $q_3 > Q$)

The model can be extended to allow for more variation in area racial composition, but this provides no benefit for present purposes.

I first note that, at a given level of displacement from even distribution as registered by D, S will take its maximum value of $S = D$ when the population residing in non-parity areas is maximally concentrated. This occurs when non-parity areas are either all-White or all-Black and thus are perfectly "polarized" as either 1.0 or 0.0 on area proportion White (p_i). This result can be produced by a "Maximum Concentration" or "Maximum S" algorithm involving three steps as follows.

1. Set the share of Whites in Area_1 to D (i.e., $s_{W1} = w_1 / W = D$). Proportion White in the area will be 1.0 (i.e., $p_1 = 1.0$).
2. Set the share of Blacks in Area_3 to D (i.e., $s_{B3} = b_3 / B = D$). Proportion White will be 0.0 (i.e., $p_3 = 0.0$).
3. Place remaining Whites and Blacks in Area_2. Area share scores for Whites and Blacks will be $s_{W2} = s_{B2} = (1 - D)$ and proportion White for the area will be at parity (i.e., $p_2 = P$).

The resulting group distributions will produce a distinctive "four-point" segregation curve that Duncan and Duncan (1955: Figure 5) termed a "William's model" segregation curve. In this distribution, S takes its maximum possible value (S_{Max}) under the prevailing level of displacement from even distribution with $S_{Max} = D$.

This establishes that logical upper bound on separation (S) is the level of displacement (D). In addition, since D can vary independently of racial composition (P) and S can always match D, this result also establishes that group separation (S) can vary independently of city racial composition (P). This finding lays to rest any claim that the value of S is inherently dependent on city racial composition. S can match D when displacement is concentrated. Whether displacement is concentrated or not depends on sociological dynamics governing population distribution, not the inherent nature of S.

The next issue to take up is whether D and S can vary independently. This is relatively easy to establish as S will take a lower value than D when groups residing in non-parity areas are dispersed rather than concentrated.[9] When groups residing in non-parity areas are concentrated, higher values of S result and in the situation of complete concentration S reaches a maximum value of D. When groups residing in non-parity areas are dispersed widely, values of S will be substantially lower than values of D. When groups are exactly equal in size, a relatively uncommon but logically possible situation, the value of S can fall to at least D^2. In cases where groups are unequal in size, values of S can fall well below D^2 and in some circumstances S can potentially fall to very low values.[10]

A variety of algorithms will produce patterns of dispersed displacement from even distribution that yield low values of S while maintaining a specified value of D. In a more detailed discussion of this issue (Fossett 2015) I review a progression of algorithms. For present purposes, I introduce an algorithm that produces the lowest levels of S I have been able to obtain under the three-area scenario under discussion. This "Minimum S" (S_{Min}) algorithm actually uses just two areas, one area that is "above parity" and one that is "below parity".

The algorithm to obtain S_{Min} involves two variations which I term here Model A1 and Model A2. Each version will produce the lower value of S over some ranges of city racial composition (P) as follows.

$$\text{if } (P > 0.5) \;\; S_{MinA1} = S_{Min} \leq S_{Min\,A2}$$
$$\text{if } (P = 0.5) \;\; S_{Min\,A1} = S_{Min} = S_{Min\,A2}$$
$$\text{if } (P < 0.5) \;\; S_{Min\,A1} \geq S_{Min} = S_{Min\,A2}$$

Accordingly, one can obtain the value of S_{Min} by assigning the value of S generated by Model A1 when $P \geq 0.5$ and the value of S generated by Model A2 when $P \leq 0.5$
.

Both versions of the algorithm proceed to an intermediate step with one homogeneous area and one mixed area. The A1 version of the algorithm begins as follows.

A1 Step 1. For $Area_1$, set the group share for Whites (s_{W1}) to D and the group share for Blacks (s_{B1}) to 0.0.
A1 Step 2. For $Area_3$, set the group share of Whites (s_{W3}) to $1-D$ and the group share for Blacks (s_{B3}) to 1.0.

This produces an "above-parity" area ($Area_1$) that is all-White and a "below-parity" area ($Area_3$) that is mixed White and Black.

Similarly, the A2 version of the algorithm begins as follows.

[9] The exceptions are when D is close to boundary values of 0 and 1.0. Under these conditions, scores for all popular measures of uneven distribution will agree.

[10] I offer this conclusion based on exercising the models discussed here over the full "event space" of possible combinations of D and P. In all instances where $0 < D < 1$, I obtained values of S below the value of D^2.

A2 Step 1. For $Area_1$, set the group share for Whites (s_{W1}) to 1.0 and the group share for Blacks (s_{B1}) to $1-D$.

A2 Step 2. For $Area_3$, set the group share for Whites (s_{W3}) to 0.0 and the group share for Blacks (s_{B3}) to D.

This produces an "above-parity" area ($Area_1$) that is mixed White and Black and a "below-parity" area ($Area_3$) is all-Black.

For most logically possible combinations of D and P the value of S can be reduced even further by transferring an optimal amount (X) of equal shares of Whites and Blacks from the "mixed" area to the homogeneous area to reduce concentration (increase dispersion). For Model A1 these transfers move equal group shares (X) of Whites and Blacks from $Area_3$, which is mixed, to $Area_1$, which is all-White. For Model A2, these transfers move equal group shares (X) of Whites and Blacks from $Area_1$, which is mixed, to $Area_3$, which is all-Black.

The value of D is unaffected when equal group shares are transferred from one area to another. But the transfers can have substantial impacts on the value of S. A wide range of alternative group transfer share values are logically possible subject to the restriction that the transfers cannot produce area group share values below 0.0 or above 1.0. The task is to find the optimal value (X) that will reduce the value of S to S_{Min}, the lowest possible value under the three-area model under consideration. One strategy is to conduct a numerical search over the feasible values of X. I developed algorithms that implemented this approach and used them to establish benchmarks for what is possible. Using this approach I found I could obtain the same result for S_{Min} regardless of whether starting from the residential distributions created at the intermediate steps of Model A1 or Model A2.

With additional exploration I discovered that the same residential distributions and resulting value of S_{Min} can be obtained using a direct analytic solution. This solution involves modifying the transfer of equal group shares so it brings the share of total population (i.e., Whites and Blacks combined) in "above-parity" and "below-parity" areas as close to 0.5 as possible. This is accomplished as follows. First, identify the range of logically possible share transfer values (X) that will maintain the value of D. These will range from a minimum of 0.0 to a maximum of (1-D). Next calculate the value of $|s_{T3}-0.5|$, the unsigned difference between the total population share in $Area_3$ and 0.5. Tentatively adopt this as the share amount (X) to be transferred. If the value of X is larger than the maximum feasible value $(1-D)$, set the group transfer share value (X) to $(1-D)$. In other words, set X to the minimum of $|s_{T3}-0.5|$ and $(1-D)$. Next implement the transfer of the identified group share amounts (X) from the mixed area to the homogeneous area.

Thus, when $P \geq 0.5$, use the A1 algorithm with these additional steps.

A1 Step 3. Set the optimal share (X) of Whites and Blacks to transfer from the mixed area ($Area_3$) to the all-White area ($Area_1$) as the minimum of $|s_{T3}-0.5|$ and $(1-D)$.

A1 Step 4. Implement the transfer, thus increasing s_{W1} to $D+X$ and s_{B1} to X and reducing s_{W3} to $1-D-X$ and s_{B3} to $1-X$.

When $P \leq 0.5$, use the A2 algorithm with these additional steps.

A2 Step 3. Set the optimal share (X) of Whites and Blacks to transfer from the mixed area ($Area_1$) to the all-Black area ($Area_3$) as the minimum of $|s_{T3} - 0.5|$ and $(1 - D)$.

A2 Step 4. Implement the transfer, thus reducing s_{W1} to $1 - X$ and s_{B1} to $1 - D - X$ and increasing s_{W3} to X and s_{B3} to $D + X$.

7.4.1 Examples of Calculating Values of S_{Min} Given Values of D and P

Figure 7.3 provides a summary listing of formulas for calculating terms relating to group residential distributions under the "Maximum S" and "Minimum S" analytic models just introduced. I establish the basis for the formulas in a more detailed review of analytic models for group separation (Fossett 2015). The formulas in Fig. 7.3 establish how, in the context of the three-area analytic model considered here, algorithms for dispersed and concentrated displacement will generate group residential distributions producing lower and higher values of group separation (S) under a given combination of fixed values for displacement (D) and city racial composition (P). As best I have been able to determine, the formulas in Fig. 7.3 establish the logically possible range for S under a given combination of D and P by yielding the minimum possible value for S (S_{Min}) under dispersed displacement and the maximum possible value for S (S_{Max}) under concentrated displacement.

In this section I review examples to illustrate how values of S_{Min} and S_{Max} can be calculated for a given combination of displacement (D) and city racial composition (P). The value of S under the "Maximum S" algorithm can be obtained by using the formulas in Fig. 7.3 to first establish the values of relevant component terms – area group share distributions (s_{Wi} and s_{Bi}) and area group proportions (p_i) – used in computing formulas for S and then carry through the calculations to obtain S.

Consideration of the two general computing formulas for S given below (as well as earlier) reveals that the "parity area" in the three-area analytic model under consideration can be ignored because calculations for this area yield values of zero (0) and have no impact on the value of S.

$$S = \Sigma s_{Ti} (p_i - P)^2 / PQ, \text{ and}$$

$$S = \Sigma s_{Wi} \cdot p_i - \Sigma s_{Bi} \cdot p_i .$$

The value of S thus results from the calculations for the "above parity" and "below parity" areas and can be given as either

$$S = (s_{W1} \cdot p_1 + s_{W3} \cdot p_3) - (s_{B1} \cdot p_1 + s_{B3} \cdot p_3), \quad \text{or}$$

	Area 1 Above Parity ($p_i > P$)	Area 2 Parity ($p_i = P$)	Area 3 Below Parity ($p_i < P$)
	S_{Max}, S = D under Maximum Concentration Model		
White Share (s_{Wi})	D	1−D	---
Black Share (s_{Bi})	---	1−D	D
Total Share (s_{Ti})	PD	1−D	QD
Prop. White (p_i)	1	P	0
	$S_{Min\ A1}$, S under Dispersed Displacement Model A1		
White Share (s_{Wi})	D+X	---	1−D−X
Black Share (s_{Bi})	X	---	1−X
Total Share (s_{Ti})	PD+X	---	1−PD−X
p_i	P(D+X)/(PD+X)	---	P(1-D-X)/(1-PD-X)
	$S_{Min\ A2}$, S under Dispersed Displacement Model A2		
White Share (s_{Wi})	1−X	---	X
Black Share (s_{Bi})	1−D−X	---	D+X
Total Share (s_{Ti})	P+Q(1−D)−X	---	QD+X
Prop. White (p_i)	P(1-X)/(1-QD-X)	---	PX/(QD+X)

Fig. 7.3 Summary of formulas for group residential distributions by level of dissimilarity (D) and racial composition (P) under selected algorithms for producing concentrated and dispersed displacement from even distribution (Notes: Per discussion in text, $X = \min(|s_{T3} - 0.5|, (1 - D))$ where S_{T3} is $(1 - PD)$ under Model A1 and QD under Model A2, respectively)

$$S = (s_{W1} - s_{B1})p_1 + (s_{W3} - s_{B3})p_3.$$

Taking the example combination of displacement as measured by the dissimilarity index (D) set to 60 and pairwise city proportion White (P) set to 0.90, the resulting value of S under the "Maximum S" Model can be obtained by first establishing the values of relevant component terms and then carrying through the computations. The relevant component terms for the non-parity areas can be obtained as follows.

$$s_{W1} = D = 0.60$$

$$s_{W3} = 0.0$$

$$s_{B1} = 0.0$$

$$s_{B3} = D = 0.60$$

$$(s_{W1} - s_{B1}) = D - 0.0 = D = 0.60$$

$$(s_{W3} - s_{B3}) = 0.0 - D = -D = -0.60$$

$$p_1 = 1.0$$

$$p_3 = 0.0$$

The following calculations now demonstrate that $S_{Max} = D$.

$$\begin{aligned} S &= (s_{W1} \cdot p_1 + s_{W3} \cdot p_3) - (s_{B1} \cdot p_1 + s_{B3} \cdot p_3) \\ &= (0.60 \cdot 1.0 + 0.0 \cdot 0.0) - (0.0 \cdot 1.0 + 0.60 \cdot 0.0) = 0.60 \end{aligned}$$

$$S = (s_{W1} - s_{B1})p_1 + (s_{W3} - s_{B3})p_3 = 0.60 \cdot 1.0 + -0.60 \cdot 0.0 = 0.60$$

This expression reveals something interesting and important. It is this.

> *The value of P is not directly involved in the formulas for the component terms. This indicates that the value of S_{Max} is unaffected by city racial composition. Accordingly, under concentrated displacement, S can equal D for any city racial composition.*

The calculations for "Minimum S" (S_{Min}) under dispersed displacement are more involved. Model A1 applies when city racial composition (P) is ≥ 0.50 and thus would be the relevant model for most White-Minority comparisons in US cities. Model A1 also is relevant for the example just considered where D is 60 and pairwise city proportion White (P) is 0.90. The value of S under Model A1 can be obtained by first establishing the values of relevant component terms and then carrying through subsequent calculations. The relevant component terms can be obtained as follows.

$$\begin{aligned} X &= \min(|(1-PD)-0.5|,(1-D)) = \min(|(1-0.90 \cdot 0.60)-0.5|,(1-0.60)) \\ &= \min(|(1-0.54)-0.5|,0.40) = \min(|0.46-0.5|,0.40) = \min(0.04,0.40) \\ &= 0.04 \end{aligned}$$

$$s_{W1} = D + X = 0.60 + 0.04 = 0.64$$

$$s_{W3} = (1 - D - X) = 1 - 0.60 - 0.04 = 0.36$$

$$s_{B1} = X = 0.04$$

$$s_{B3} = 1 - X = 1 - 0.04 = 0.96$$

$$(s_{W1} - s_{B1}) = (D + X) - X = D = 0.60$$

$$(s_{W3} - s_{B3}) = (1 - D - X) - (1 - X) = -D = -0.60$$

$$p_1 = P(D+X)/(PD+X) = 0.90(0.60+0.04)/(0.90\cdot 0.60+0.04)$$
$$= 0.576/0.58 = 0.9930$$

$$p_3 = P(1-D-X)/(1-PD-X) = 0.90(1-0.60-0.04)/(1-0.90\cdot 0.60-0.04)$$
$$= (0.90\cdot 0.36)/(1-0.58) = 0.324/0.42 = 0.7714$$

Note that ($s_{W1} - s_{B1}$) resolves to D and ($s_{W3} - s_{B3}$) resolves to –D. As a result, the expression

$$S = (s_{W1} - s_{B1})p_1 + (s_{W3} - s_{B3})p_3$$

can be restated in the following convenient computing formula.

$$S = D(p_1 - p_3)$$

The following calculations illustrate that any of the three expressions can be used to obtain $S_{Min} = 0.1330$ under Model A1.

$$S = (s_{W1}\cdot p_1 + s_{W3}\cdot p_3) - (s_{B1}\cdot p_1 + s_{B3}\cdot p_3)$$
$$= (0.64\cdot 0.9930 + 0.36\cdot 0.7714) - (0.04\cdot 0.9930 + 0.96\cdot 0.7714)$$
$$= (0.6355 + 0.2777) - (0.0397 + 0.7405) = 0.9132 - 0.7802 = 0.1330$$

$$S = (s_{W1} - s_{B1})p_1 + (s_{W3} - s_{B3})p_3 = (0.64 - 0.04)0.9930 + (0.36 - 0.96)0.7714$$
$$= 0.60\cdot 0.9930 - 0.60\cdot 0.7714 = 0.5958 - 0.4628 = 0.1330$$

$$S = D\cdot(p_1 - p_3) = 0.60\cdot(0.9930 - 0.7714) = 0.60\cdot 0.2216 = 0.1330$$

Model A2 applies when the city racial composition (P) is ≤ 0.50. Typically this model is not relevant for most White-Minority comparisons in US cities. But it is occasionally relevant, perhaps most often for White-Latino comparisons in the border region of the southwestern United States. As with Model A1, the value of S can be obtained from any of the following three equivalent expressions.

$$S = (s_{W1}\cdot p_1 + s_{W3}\cdot p_3) - (s_{B1}\cdot p_1 + s_{B3}\cdot p_3)$$

$$S = (s_{W1} - s_{B1})p_1 + (s_{W3} - s_{B3})p_3$$

$$S = D(p_1 - p_3)$$

Thus, for example, if D is 60 and pairwise city proportion White (P) is 0.30 (similar to the value of P for many White-Latino comparison in Texas border region cities) the resulting value of S under Model A2 can be obtained by first establishing the values of relevant component terms and then carrying through computations. The relevant component terms are as follows.

$$X = \min\left(|\left(1 - QD\right) - 0.5|, \left(1 - D\right)\right)$$
$$= \min\left(|\left(1 - 0.70 \cdot 0.60\right) - 0.5|, (1 - 0.60)\right)$$
$$= \min\left(|\left(1 - 0.42\right) - 0.5|, 0.40\right) = \min\left(|0.58 - 0.5|, 0.40\right) = \min\left(0.08, 0.40\right)$$
$$= 0.08$$

$$s_{W1} = 1 - X = 1 - 0.08 = 0.92$$

$$s_{W3} = X = 0.08$$

$$s_{B1} = 1 - D - X = 1 - 0.60 - 0.08 = 0.32$$

$$s_{B3} = D + X = 0.60 + 0.08 = 0.68$$

$$\left(s_{W1} - s_{B1}\right) = \left(1 - X\right) - \left(1 - D - X\right) = \left(1 - 1\right) + D + \left(X - X\right) = D = 0.60$$

$$\left(s_{W3} - s_{B3}\right) = X - \left(D + X\right) = -D = -0.60$$

$$p_1 = P\left(1 - X\right) / \left(1 - QD - X\right)$$
$$= 0.30\left(1 - 0.08\right) / \left(1 - 0.70 \cdot 0.60 - 0.08\right) = 0.30 \cdot 0.92 / \left(1 - 0.42 - 0.08\right)$$
$$= 0.276 / 0.50 = 0.552$$

$$p_3 = PX / \left(QD + X\right) = 0.30 \cdot 0.08 / \left(0.70 \cdot 0.60 + 0.08\right) = 0.024 / 0.50 = 0.048$$

The following calculations illustrate that $S_{Min} = 0.3024$ under Model A2 can be obtained using any one of the following three expressions.

$$S = \left(s_{W1} \cdot p_1 + s_{W3} \cdot p_3\right) - \left(s_{B1} \cdot p_1 + s_{B3} \cdot p_3\right)$$
$$= \left(0.92 \cdot 0.552 + 0.08 \cdot 0.048\right) - \left(0.32 \cdot 0.552 + 0.68 \cdot 0.048\right)$$
$$= \left(0.5078 + 0.0038\right) - \left(0.1766 + 0.0326\right) = 0.5116 - 0.2092) = 0.3024$$

$$S = \left(s_{W1} - s_{B1}\right) p_1 + \left(s_{W3} - s_{B3}\right) p_3$$
$$= \left(0.92 - 0.32\right) 0.552 + \left(0.08 - 0.68\right) 0.048$$
$$= \left(0.60\right) 0.552 + \left(-0.60\right) 0.048$$
$$= 0.3312 - 0.0288 = 0.3024$$

$$S = D \cdot \left(p_1 - p_3\right)$$
$$= 0.60 \cdot \left(0.552 - 0.0480\right) = 0.60 \cdot 0.5040 = 0.3024$$

7.4.2 *Examining D, S_{Max}, and S_{Min} over Varying Combinations of D and P*

The models for obtaining maximum and minimum values of the separation index (S) just reviewed provide a basis for establishing the potential for D and S to vary across varying combinations of the level of displacement from even distribution as measured by the dissimilarity index (D) and the racial composition of the city (P). I used these models to compute values of S_{Max} and S_{Min} over possible combinations of D ranging from 0 to 100 with P ranging from 1 to 99. Results from these calculations are depicted graphically in Figs. 7.4 and 7.5 which depict the upper and lower bounds of the relationship between D and S at selected values for city racial composition (P). Figure 7.4 depicts the relationship by plotting values of S_{Max} and S_{Min} against values of D. Figure 7.5 depicts the relationship by plotting values of D against values of S_{Min}.

I comment first on the diagonal line on Fig. 7.4. This results from plotting values of S_{Max} against the value of D over all values of D and all values of P. The diagonal documents that S will equal D at any combination of values for D and P when displacement from parity involves concentration of both groups in racially polarized areas wherein Whites in non-parity areas live apart from Blacks in areas that are

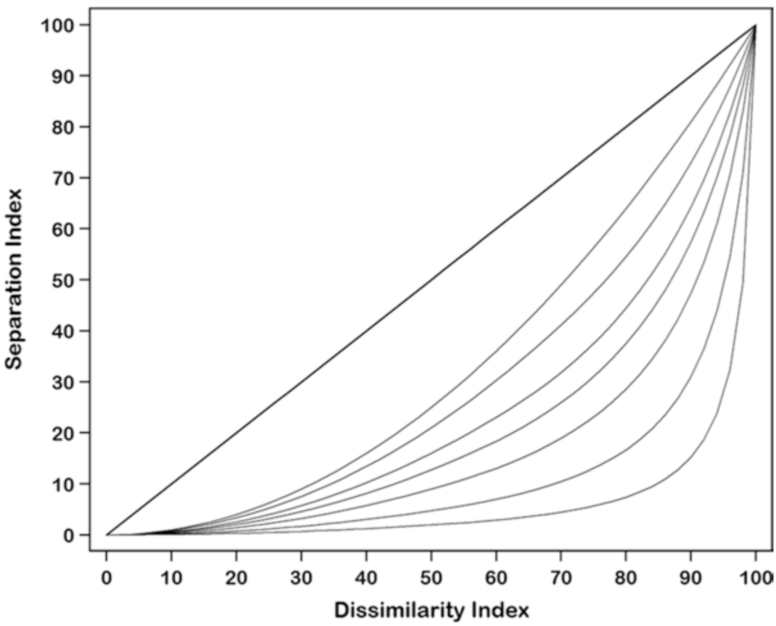

Fig. 7.4 Maximum and minimum values of the separation index (S) by values of the dissimilarity index (D) for selected values of city percent White (P) under a three-area analytic model (Notes: Maximum and minimum values of S under three-area analytic exercise. See text for disscussion of analytic model. Curves are plotted for values of percent White (P) of 50, 70, 80, 90, 95. and 98)

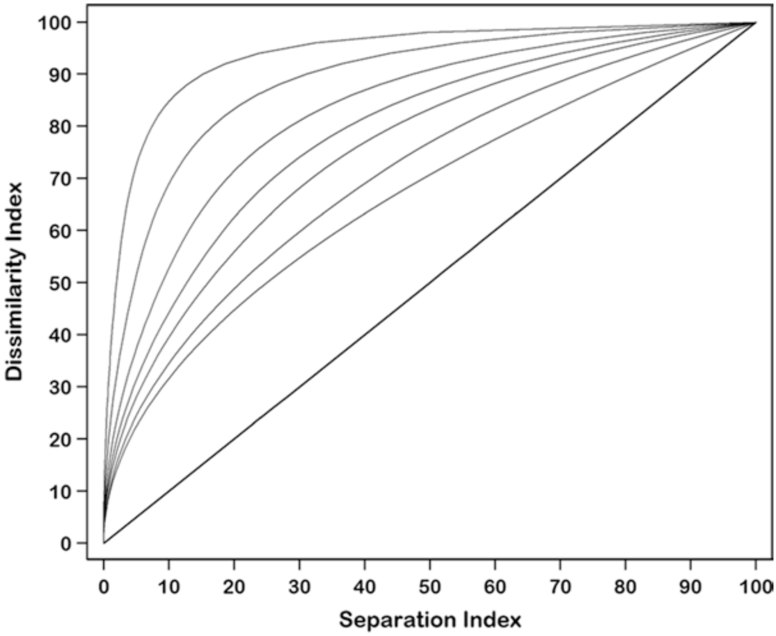

Fig. 7.5 Maximum and minimum values of the dissimilarity index (D) by values of the separation index (S) for selected values of city percent White (P) under a three-area analytic model (Notes: Maximum and minimum values of S under three-area analytic exercise. See text for discussion of analytic model. Curves are plotted for values of percent White (P) of 50, 70, 80, 90, 95, and 98)

all-White and Blacks in non-parity areas live apart from Whites in areas that are all-Black. The diagonal in the figure thus serves as a reference line indicating the maximum degree to which groups can be residentially separated at a given level of displacement from even distribution.

The graph in Fig. 7.4 also plots the values of S_{Min} against the value of D over values of D ranging from 0 to 100 and at selected values of P ranging from 2 to 50. Note that it is not necessary to plot the same relationships for values of P above 50 they are identical to the relationships already shown for values of $1-P$ already shown. Thus, for example, the curve obtained when $P = 98$ is identical to the curve obtained when $P = 2$. Importantly, all of the curves fall below the diagonal and thus visually depict the fact that S can take a lower value than D at any combination of values for D and P when group displacement from even distribution is dispersed in a way that maximizes group residential mixing instead of being concentrated in a way that maximizes group residential separation. The set of curves also makes it clear that the maximum possible difference between D and S is conditioned by city racial (P). This is visually indicated by the fact that different curves result for each value of P.

The maximum possible size of the D-S difference is smallest when the two groups in the comparison are equal in size (i.e., $P = Q = 0.5$). Intuitively, this is because the maximum departure of S from D occurs when one group is dispersed widely across areas where it is over-represented, thus resulting in small departures of p_i from P in these areas. This is demographically more feasible when one group is small in comparison to the other and it is less feasible when groups are equal in size. Elsewhere I establish that the $D - S_{Min}$ relationship when groups are equal in size is $S = D^2$ (Fossett 2015). This relationship is reflected in the curve that is closest to the diagonal. This curve documents that the absolute and relative magnitude of the possible D-S difference can be substantial even when it is at its minimum. The D-S difference when groups are equal in size reaches a maximum of 25 points when D is 50 and it is 20 points or more when D is in the range 28–72. In relative terms, the value of S can be up to 20 % lower than the value of D when D is 80; up to 30 % lower when D is 70; up to 40 % lower when D is 60; up to 50 % lower when D is 50; and so on.

The $D - S_{Min}$ curves plotted at selected values of P depart further from the diagonal as the racial composition of the city becomes progressively more imbalanced. Since most White-Minority segregation comparisons in empirical studies involve groups that differ greatly in size, these curves are highly relevant. They document that potential D-S differences can be very large in both absolute and relative terms under combinations of D and P that are common in "real world" settings. When P is 85, the $D - S_{Min}$ difference exceeds 25 when D is in the range of 30–93 and it exceeds 40 when D is in the range of 56–83. In relative terms, the value of S can be up to 50 % lower than the value D when $D \leq 82$ and 70 % lower or more when $D \leq 58$. The potential D-S differences are even more dramatic when P is 95 or higher. For example, when P is 95, the $D - S_{Min}$ difference exceeds 25 when D is in the range of 28–98 and it exceeds 40 when D is in the range of 44–96. In relative terms, the value of S can be up to 50 % lower than the value D when $D \leq 94$ and 70 % lower or more when $D \leq 84$.

Importantly, group size differentials of this magnitude are common in empirical studies of segregation in US cities. For example, they are typical of White-Asian comparisons in most cities and they are typical of White-Latino comparisons in the "new destination" communities of the Midwest, South, and Northeast. The potential for D-S differences to be very large in these situations is clearly revealed in Fig. 7.4. The patterns seen here provide compelling evidence that the prevailing practice of examining only D in empirical studies of segregation should be reconsidered. The curves in the figure document that the level of group separation and area racial polarization as measured by S can vary widely across cities that are identical in terms of group displacement from even distribution (D) and relative group size (P).

Figure 7.5 makes the same point but from the vantage point of the separation index (S) instead of the dissimilarity index (D). Here the diagonal depicts the values of D plotted by S when displacement from even distribution is maximally concentrated (S_{Max}). The curves in the figure depict the values of D plotted by S when displacement from even distribution is maximally dispersed. The implication of these curves is straightforward. If one is interested in group separation as measured by S, D is an unreliable indicator because D can take very high values when groups are not residentially separated. This occurs when group displacement from even distribution is extensive but the group populations are dispersed across non-parity areas in a way that minimizes group concentration and maximizes group mixing and co-residence.

7.4.3 Implications of Findings from Analytic Models for S_{Max} and S_{Min}

The preceding discussion establishes that scores for D and S can differ depending on three factors. The first is whether displacement of groups from even distribution is present and is substantial. All else equal, the potential for D and S to differ is greatest when D is high (e.g., at or above 60) but less than its maximum of 100.[11] The second factor is whether the group displacement from even distribution in question is concentrated or dispersed. When displacement is maximally concentrated, $S = S_{Max} = D$; when displacement is maximally dispersed (minimally concentrated), $S = S_{Min} \leq D^2$. The third factor is the relative sizes of the groups in the segregation comparison. All else equal, the maximum possible difference between D and S is larger when groups are unequal in size. Accordingly, the logical possibility for a large $D - S$ difference is greatest under the following conditions: (1) displacement from even distribution is extensive (i.e., D is high), (2) displacement is maximally dispersed, and (3) the groups are highly unequal in relative size (e.g., $|P - Q| > 90$). Analysis of empirical segregation patterns presented in Chap. 8 will document examples of such situations and establish that large D-S discrepancies are not just logically possible, they can and do occur with some regularity in empirical studies.

7.5 Is Separation a Distinct Dimension of Segregation?

I conclude this chapter by considering the issue of whether group separation and area racial polarization as measured by S should be viewed as a distinct dimension of segregation. Stearns and Logan (1986) argued that D and S tap different aspects of group differences in residential distribution and noted that D and S can differ both in overall value and in direction of change. On this basis they argued that S is a distinct dimension of segregation and should be routinely examined in empirical studies. The core of their position is that, unlike D, S registers whether or not groups live apart due to *both* groups being concentrated in homogeneous areas, a residential pattern of compelling substantive interest to researchers.

[11] When displacement reaches its maximum possible level, $S = D = 100$ and the D-S difference is necessarily zero. Similarly, if there is no displacement from even distribution, $S = D = 0$ and the D-S difference is zero.

The view Stearns and Logan advocate runs counter to most methodological studies which view S as one among many alternative measures of uneven distribution including the gini index (G), the dissimilarity index (D), the Theil entropy index (H), and the Atkinson index (A) represented here by the closely related Hutchens square root index (R) (Zoloth 1976; James and Taeuber 1985; White 1986; Reardon and Firebaugh 2002).[12] These various alternative indices all differ from each other in at least the narrow sense that they can yield different numerical scores when applied to the same residential distributions. So the question arises, when does one measure become different enough from the alternatives that it should be considered a distinctive dimension of segregation?

One basis for grouping indices together is similarity of computing formulas – the operational implementations of the conceptions of segregation embodied in the indices. On this basis one can argue that the separation index (S) is a measure of even distribution based on the close similarity of one of its computing formulas with a computing formula for the index of dissimilarity (D).

$$\mathrm{D} = 100 \cdot \left(1/2\mathrm{TPQ}\right) \cdot \Sigma \mathrm{t_i} \mid \mathrm{p_i} - \mathrm{P} \mid$$

$$\mathrm{S} = 100 \cdot \left(1/\mathrm{TPQ}\right) \cdot \Sigma \mathrm{t_i} \left(\mathrm{p_i} - \mathrm{P}\right)^2$$

The view can also be supported by noting the close similarity of the following computing formulas for the separation index (S) and the Hutchens square root index (R) which empirically is closely related to D as well as to Atkinson's A.[13]

$$\mathrm{R} = 100 \cdot \left[1 - \left(1/\mathrm{T}\right) \cdot \Sigma \sqrt{p_i q_i / PQ}\right]$$

$$\mathrm{S} = 100 \cdot \left[1 - \left(1/\mathrm{T}\right) \cdot \Sigma \mathrm{p_i q_i} / \mathrm{PQ}\right]$$

Similarity of computing formulas for measures of uneven distribution also can be summarized in another, more abstract way. S is like G, D, R, and H, in that all of these indices can be described in the following way. The value of each of these indices registers the population weighted average of quantitative scoring of the deviations of area pairwise racial composition (p_i) from the pairwise racial composition of the city (P) overall, normalized to the range 0–1 where 0 indicates no

---

[12] I make two qualifications. First, technically, Massey and Denton (1988) classified S as an exposure measure, but they noted others classify it as a measure of uneven distribution. Second, James and Taeuber (1985), Massey and Denton (1988), and White (1986) include the Atkinson index (A) as a measure of uneven distribution. But I instead list the Hutchens square root index (R) which is a superior and closely related substitute for the Atkinson index.

[13] D and R both rank segregation comparisons in accord with the principle of segregation dominance. Using the data set for analyses reported in this chapter the simple linear correlation of D and R is extremely high (0.962) and the correlation is even higher when allowing for nonlinearity (the correlation of D with the square root of R is 0.984).

deviations and 1 indicates that deviations have reached the maximum possible result.[14]

Finally, S fares well when it is reviewed on non-controversial technical criteria suggested for measures of uneven distribution. Ironically, it fares much better than D, the most widely used measure of uneven distribution (Reardon and Firebaugh 2002).

From the points just reviewed there is a clear case for grouping S with other measures of uneven distribution. But there is room for further discussion on both conceptual and practical grounds. On conceptual grounds, the theory of segregation measurement can be described as "incomplete." This means that the generally accepted criteria for evaluating measures of uneven distribution are compatible with a variety of measures each of which embodies a unique, albeit implicit, conception of uneven distribution. For now, however, the ambiguity of the situation is not likely to be eliminated. Some criteria for measuring even distribution such as the exchange principle discussed earlier in this chapter, have been endorsed widely (e.g., James and Taeuber 1985; White 1986; Reardon and Firebaugh 2002). But other criteria that would reduce ambiguity in measurement have been offered but not widely accepted.

In particular, the criterion of "composition invariance" offered by James and Taeuber (1985) is seen as controversial so too is Taeuber and James' (1982) criticism of the separation index (termed V in their discussion) based on related concerns. This principle has the practical consequence of requiring indices to order segregation comparisons in agreement with the principle of "segregation curve dominance."[15] Two widely used indices – the separation index (S) and the Theil entropy index (H) – do not satisfy this criterion. However, the criterion itself is controversial. Some have explicitly and forcefully rejected it (e.g., Coleman et al. 1982; White 1986). Others note the criterion has been suggested but do not endorse it (e.g., Reardon and Firebaugh 2002). The "revealed consensus" in the empirical literature has been that researchers ignore the criteria and use H and S when they find these indices to be useful for meeting the needs of a their study.[16]

So where do things stand? If one accepts the principles of "composition invariance" and "segregation curve dominance" as integral and essential to the measurement of uneven distribution, the separation index (S) and also the Theil index (H) cannot be considered measures of uneven distribution. Under this circumstance,

[14] Thus, the index scores are normalized to the range 0-1 by dividing the average deviation scores by the maximum value the average can take under complete segregation.

[15] Even if this principle is accepted, segregation measurement theory is still technically incomplete because the principle is silent on how segregation comparisons should be ranked when segregation curves cross, as they sometimes do. This is less important on practical grounds as indices that satisfy the principle of segregation curve dominance tend to correlate at very high levels.

[16] Subordinating measurement principles to researcher needs is typical, not uncommon, as the most widely used index, D, does not satisfy the non-controversial principles of transfers and exchanges.

Stearns and Logan (1986) would then be correct in arguing that S taps a distinct dimension of segregation.[17]

Personally, I am comfortable with this position. It would reduce ambiguity in the current relatively flexible notion of uneven distribution by distinguishing between indices that measure displacement and indices that measure separation. Displacement would be compatible with the geometric interpretation of the gini index (G) in relation to the segregation curve and the closely related vertical distance and volume of movement interpretations of the dissimilarity index (D). Displacement also would be compatible with notions of group difference on rank-order position on area racial composition. G would then stand as an attractive index of displacement as it satisfies the principle of exchanges and responds to all directional changes in rank-order differences between groups and thus supports interpretation as the "net difference" in group rank order advantage noted by Lieberson (1976). D would then stand as a crude version of G that may be useful due to its simplicity and ease of calculation even though it does not satisfy the principle of changes and responds only to directional changes in rank-order distribution above and-below P.

Two other measures – the symmetric version of Atkinson index (A) and the Hutchens square root index (R) – also could be categorized as measures of displacement. So far as I am aware, they do not offer the specific geometric interpretation of displacement that is available for G and D. But they are like G and D in satisfying the criterion of segregation curve dominance and their values correlate very closely with values of D and G in empirical analyses. Hutchens (2004) makes the case that R has attractive options for certain kinds of analysis based on being "additively decomposable" where G and D are not.

Separation registers differences in group distribution that are not registered by displacement as measured by G, D, and R. Separation assesses group differences in *quantitative* position, instead of rank-order position, on area racial composition. A formal distinction can be made between displacement and separation by adopting a "polarization" criterion to supplement the "exchange criterion." The current exchange criterion is minimal; it requires only that an index register an integration-promoting exchange. A polarization criterion supplement would additionally require the following.

> *All else equal, exchanges involving more polarized areas and resulting in larger average reductions in same-group contact should have greater impact on an index than exchanges involving less polarized areas and resulting in smaller average reductions in same-group contact.*

Specifically, the principle would require the impact of the exchange on the index score to increase as the value of $|p_i - p_j|$ increases. Thus, in the example exchanges discussed earlier in this chapter, S will respond more strongly to an exchange

[17] The issue is more complicated than my statement suggests. The dissimilarity index (D) does not satisfy the principle of transfers – a principle that does enjoy consensus support – yet methodological reviews typically characterize D as a valid measure of uneven distribution on the grounds that the practical consequences of violating the principle of transfers are not sufficient to justify disallowing the measure.

between highly polarized areas such as an exchange between one area with 100 Whites and 0 Blacks and another area with 0 Whites and 100 Blacks and less strongly to an exchange between minimally polarized areas such as an exchange between one area with 51 Whites and 49 Blacks and another area with 49 Whites and 51 Blacks. In contrast, as demonstrated in examples reviewed earlier and established more carefully elsewhere (e.g., James and Taeuber 1985; Reardon and Firebaugh 2002), D will treat these exchanges as identical in impact.

G responds in a more complicated way that ultimately is similar in nature to D. G will potentially treat these exchanges differently, but not based on the quantitative magnitude of the level polarization; that is, not in proportion to the value of $| p_i - p_j |$. Instead, since G assesses group differences in rank order position, it will treat these exchanges differently when they differ in terms of the share of the combined group populations residing in areas with values on racial composition (p) that fall in between the values on racial composition for the two areas involved in the exchange. Specifically, G would be reduced by a larger amount when the exchange causes the moving White and Black households to cross over a larger "intermediate" population; that is a larger share of the combined group populations residing in "intermediate" areas where area proportion White (p) is larger than that for the area receiving the White household (p_j) and smaller than that for the area sending the White household (p_i). This property of G has little practical consequence for overcoming insensitivity to polarization because, if the quantitative difference between the two areas (i.e., $| p_i - p_j |$) is small, polarization is small and G, like D, can respond strongly to group differences in distribution across areas that are similar in terms of area racial composition.

One potential benefit of adopting a strong conceptual distinction between displacement and separation is that it would reduce ambiguity in segregation measurement. It would make something clear both to researchers and also to consumers of segregation research. Namely, it would clarify that

> *Segregation indices that rank segregation comparisons in terms of the segregation curve are poor choices for measuring group residential separation and area racial polarization.*

Similarly, it would signal that

> *Segregation indices that measure group residential separation and area racial polarization are poor choices for measuring group displacement from even distribution.*

It appears, however, that prevailing practices in empirical research place greater priority on practical concerns such as flexibility and ease of use rather than conformity to technical measurement criteria. When approaching segregation guided by these priorities, which some might view as appropriate since key aspects of segregation measurement theory are unresolved, one could argue that D and S both measure uneven distribution construed broadly. However, even when one adopts this view, it is important to recognize and acknowledge the following.

> *D and S are sufficiently different in behavior that the choice between them has potentially important consequences for empirical findings that should not be overlooked.*

Once this point is acknowledged, the responsibility falls to researchers to first determine whether index choice matters for the findings obtained in any given empirical analysis and, when it does, to then report this and note the implications it may carry.

All indices of uneven distribution register group differences in residential distributions differently. But some differences are negligible on practical grounds while others are potentially more important. In the case of D and S, the differences are especially likely to have important practical consequences for findings in empirical studies because they are at opposite ends of a continuum in how indices respond to group difference in distribution on area group proportion (p). Specifically, as a crude form of G, D is sensitive to rank order differences without regard to the quantitative magnitude of the differences involved while S is sensitive to quantitative differences that are large in size and is only weakly responsive to rank order differences that involve small quantitative differences.[18] Understanding this difference helps clarify the nature of segregation patterns when D and S yield different results. Because D is sensitive to group differences in rank position on area group proportion (p), D can take high values even when the group differences on p are small in quantitative magnitude but are extensive. In contrast, S takes high values only when group differences on area group proportion (p) are quantitatively large, and will take low values when group differences in rank position on p are extensive but the quantitative differences involved are small.

Whether one sees this practical difference between D and S as justifying the conclusion that they measure distinctly different dimensions of segregation is a matter of judgment. I take the position that, at the very least, it is important to note that the two measures are similar in measuring group differences on area group proportions (p_i) and give researchers the option of assigning priority to rank-order differences or to quantitative differences. The choice between the two options is important because rank-order differences can be high even when groups live together in areas that differ by small amounts on area racial composition and quantitative differences can only be high when groups live apart in areas that differ substantially on area racial composition.

Once this point is "on the table", the choice between indices becomes sharply defined. If one adopts the separation index (S) one is choosing to focus on quantitative differences between group residential outcomes on racial composition and the question of whether groups live together or apart. When S takes high values, it also necessarily implies the presence of substantial differences in rank order position on racial composition as values of D cannot fall below values of S. This clearly fits well with prevailing, albeit usually implicit, notions regarding what I term "prototypical" segregation wherein rank-order and quantitative differences track each other closely. The contrasting possibility is when group differences in rank-order position on area racial composition (p_i) are widespread, but they are small in magnitude resulting in a high-D, low-S outcome. This possibility of this outcome is not widely recognized.

[18] As noted earlier, G registers group differences in rank position on area group proportion (p) regardless of the quantitative magnitude of the differences involved. D is a crude version of G and behaves in a similar way.

Perhaps because of this, compelling arguments for why one would prioritize this result over assessments of prototypical segregation have not been articulated in the literature.

My own sense of the matter is that researchers should always examine S because it registers an aspect of residential distributions that is sociologically compelling and clearly relevant for the concerns that motivate researchers to assess uneven distribution in the first place. For example, Taeuber, a leading segregation researcher whose efforts popularized the use of the dissimilarity index, motivated one of his influential studies of White-Black differences in residential distribution by stating that "[r]esidential segregation of whites and nonwhites effects their *separation* in schools, hospitals, libraries, parks, stores, and other institutions" (1964:42; emphasis added).

The distinction between separation and "mere" displacement is important because residential separation is a logical prerequisite for groups to have fundamentally different neighborhood outcomes and life chances based on area of residence. To the extent that residential outcomes and life chances are liked to area of residence, groups will tend to have similar residential outcomes and life chances when the two populations live together.[19] All else equal, populations that reside together share the same physical and built environment whether despoiled and blighted or scenic and well kept; they likewise share the same neighborhood amenities such as roads, sidewalks, air and water quality; they have the same neighbors; they share the same neighborhood institutions, businesses, and public services; they have the same public schools; they have the same exposure to noise, crime and social problems; and so on.

Alternatively, as Stearns and Logan pointed out, polarization of neighborhoods into White and minority areas makes minority households concentrated in minority areas vulnerable to discriminatory practices such as formal and informal redlining for loans and insurance coverage for homes and businesses that can undermine property values and inhibit private and public investment. Similarly, area racial polarization puts minority areas and minority households at risk of disadvantage in neighborhood outcomes resulting from differential siting of less desirable public institutions such as prisons, half-way houses, low-income housing developments, waste management facilities, etc., and similarly at risk for inequality in quantity and/or quality of schools, parks, libraries, government services, roads and other public infrastructure, and so on (Stearns and Logan 1986:127–128).

[19] Of course, residential outcomes and life chances can differ substantially for groups that live together when stratification processes are tied to group-membership independently of area of residence. The Jim Crow South is an example where groups could live together at the neighborhood level but have fundamentally different life chances based on group membership. For example, in the extreme, Whites and Blacks living together in the same neighborhood – and sometimes even on the same block and residential property – went to different schools and used different public amenities such as water fountains, restrooms, and swimming pools. Even in this circumstance, however, many public goods aspects of neighborhoods – such as desirable amenities, roads, exposure to natural and man-made hazards, etc. – are shared equally when groups reside together.

In the ideal, research motivated by the kind of concerns just noted would assess group disparities on the relevant neighborhood characteristics directly. But, unfortunately, the requisite data are not available in comprehensive form. The next best option is to determine whether group residential separation creates the logical potential for disparities to be pronounced. The separation index (S) is directly relevant for this concern. Measures of displacement, D in particular, are not reliable substitutes.

References

Coleman, J. S., Hoffer, T., & Kilgore, S. (1982). Achievement and segregation in secondary schools: A further look at public and private school differences. *Sociology of Education, 55*, 162–182.

Duncan, O. D., & Duncan, B. (1955). A methodological analysis of segregation indices. *American Sociological Review, 20*, 210–217.

Fossett, M. (2015, October 14–16). *A methodological comparison of the dissimilarity and separation indices: Is displacement without separation sociologically or demographically important?* Paper presented at the annual meeting of the Southern Demographic Association, San Antonio, TX.

Hutchens, R. (2004). One measure of segregation. *International Economic Review, 45*, 555–578.

Iceland, J., Weinberg, D. H., & Steinmetz, E. (2000). *2002. U.S. Census Bureau, series CENSR-3, racial and ethnic residential segregation in the United States*. Washington, DC: U.S. Government Printing Office.

James, D., & Taeuber, K. (1985). Measures of segregation. *Sociological Methodology, 13*, 1–32.

Jaret, C. (1995). *Contemporary racial and ethnic relations*. New York: HarperCollins.

Lieberson, S. (1976). Rank-Sum comparisons between groups. *Sociological Methodology, 3*, 276–291.

Logan, J. R., & Schneider, M. (1984). Racial segregation and racial change in American suburbs, 1970–1980. *American Journal of Sociology, 89*, 874–888.

Massey, D. S., & Denton, N. A. (1988). The dimensions of residential segregation. *Social Forces, 67*, 281–309.

Reardon, S. F., & Firebaugh, G. (2002). Measures of multi-group segregation. *Sociological Methodology, 31*, 33–67.

Schnare, A. B. (1980). Trends in residential segregation by race, 1960–1970. *Journal of Urban Economics, 7*, 293–301.

Smith, R. A. (1991). The measurement of segregation change through integration and deconcentration, 1970–1980. *Urban Affairs Quarterly, 26*, 477–496.

Stearns, L. B., & Logan, J. (1986). Measuring segregation: Three dimensions, three measures. *Urban Affairs Quarterly, 22*, 124–150.

Taeuber, K. (1964). Negro residential segregation: Trends and measurement. *Social Problems, 12*, 42–50.

Taeuber, K., & James, D. R. (1982). Racial segregation among public and private schools. *Sociology of Education, 55*, 133–143.

Taeuber, K., & Taeuber, A. (1965). *Negroes in cities: Racial segregation and neighborhood change*. Chicago: Aldine Publishing Company.

White, M. J. (1986). Segregation and diversity: Measures of population distribution. *Population Index, 65*, 198–221.
Zoloth, B. S. (1976). Alternative measures of school segregation. *Land Economics, 52*, 278–298.