Journal of King Saud University – Computer and Information Sciences 31 (2019) 459–468

Contents lists available at ScienceDirect

Journal of King Saud University – Computer and Information Sciences

journal homepage: www.sciencedirect.com

Group decision making using neutrosophic soft matrix: An algorithmic approach

Sujit Das [a,*], Saurabh Kumar [b], Samarjit Kar [c], Tandra Pal [b]

[a] Dept. of Computer Science and Engineering, Dr. B. C. Roy Engineering College, Durgapur-6, India
[b] Dept. of Computer Science and Engineering, National Institute of Technology, Durgapur-9, India
[c] Dept. of Mathematics, National Institute of Technology, Durgapur-9, India

ARTICLE INFO

Article history:
Received 10 September 2016
Revised 9 April 2017
Accepted 1 May 2017
Available online 5 May 2017

Keywords:
Group decision making
Neutrosophic soft matrix
Choice matrix
Relative weight

ABSTRACT

This article proposes an algorithmic approach for group decision making (GDM) problems using neutrosophic soft matrix (NSM) and relative weights of experts. NSM is the matrix representation of neutrosophic soft sets (NSSs), where NSS is the combination of neutrosophic set and soft set. We propose a new idea for assigning relative weights to the experts based on cardinalities of NSSs. The relative weight is assigned to each of the experts based on their preferred attributes and opinions, which reduces the chance of unfairness in the decision making process. Firstly we introduce choice matrix and combined choice matrix using neutrosophic sets. Multiplying combined choice matrices with the individual NSMs, this study develops product NSMs, which are aggregated to find out the collective NSM. Then neutrosophic cross-entropy measure is used to rank the alternatives and for selecting the most desirable one (s). This study also provides a comparative analysis of the proposed weight based approach with the normal procedure, where weight is not considered. Finally, a case study illustrates the applicability of the proposed approach.

1. Introduction

Fuzzy set (Zadeh, 1965a) uses membership degree $\mu_A(x) \in [0, 1]$ to find the belongingness of an element to a set. When $\mu_A(x)$ itself becomes uncertain, then it is hard to define by a crisp value for it. This was solved by using interval-valued fuzzy sets (IVFSs) in Turksen (1986). In some real life applications, one has to consider not only the truth membership supported by the evidence but also the falsity membership against the evidence, which is beyond the scope of fuzzy sets and IVFSs. Intuitionistic fuzzy set (IFS) (Atanassov, 1986) was introduced as a generalization of fuzzy sets to consider both truth membership and falsity membership. Later IFS was extended to the interval-valued intuitionistic fuzzy sets (IVIFSs) (Atanassov, 1989) for generalization purpose. A bibliometric analysis on fuzzy decision-related research and a scientometric review on IFS studies can be respectively found in Liu and Liao (2017) and Yu and Liao (2016). Due to some restrictions on truth and falsity membership values, fuzzy sets and its extensions can only handle uncertain information but not the indeterminate and inconsistent information, which may exist in reality. For example, when we ask the opinion of an expert about certain statement, he/she may inform that the possibility of the statement to be true is between 0.5 and 0.7 and to be false is between 0.2 and 0.4 and the degree where he/she is not sure be between 0.1 and 0.3. Consider another example, where 10 voters are participating in a voting process. In time t_1, three vote "yes", two vote "no" and five are undecided. In neutrosophic notation, it is expressed as (0.3, 0.5, 0.2). In time t_2, three vote "yes", two vote "no", two give up and three are undecided, then it can be expressed as (0.3, 0.3, 0.2), which is beyond the scope of the intuitionistic fuzzy set. This type of situation is well managed by the neutrosophic set (NS), where indeterminacy is quantified explicitly and truth, indeterminacy, and falsity membership are independent to each other. NS provides a more reasonable mathematical framework to deal with indeterminate and inconsistent information. During the last decade, the concept of NS and interval neutrosophic set (INS) have been used in various applications such as medical diagnosis, database,

* Corresponding author.
E-mail addresses: sujit_cse@yahoo.com (S. Das), saurabh14oct@gmail.com (S. Kumar), kar_s_k@yahoo.com (S. Kar), tandra.pal@gmail.com (T. Pal).

Peer review under responsibility of King Saud University.

Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.jksuci.2017.05.001
1319-1578/

topology, image processing (Guo and Sengur, 2014), and decision making problems (Ye, 2013, 2014a,b; Broumi and Smarandache, 2013b; Peng et al., 2014; Zhang et al., 2014; Ye, 2013).

Smarandache (Smarandache, 1999, 2003) first introduced neutrosophy as a branch of philosophy which studies the origin, nature, and scope of neutralities. Neutrosophic set is an important tool which generalizes the concept of the classical set, fuzzy set, interval-valued fuzzy set, intuitionistic fuzzy set, interval-valued intuitionistic fuzzy set, paraconsistent set, dialetheist set, paradoxist set, and tautological set (Smarandache, 1999). Smarandache (1999) defined indeterminacy explicitly and stated that truth, indeterminacy, and falsity-membership are independent and lies within $]0^-, 1^+[$, which is the non-standard unit interval and an extension of the standard interval $[0, 1]$. Wang et al. (2010) proposed single valued neutrosophic sets (SVNSs) from scientific and engineering points of view. SVNS is an instance of the neutrosophic set which was developed considering the difficulty of applying NS in real life problems. Entropy measure of SVNS was introduced by Majumdar and Samant (2014). As an extension of the cross entropy of fuzzy sets, Ye (2014a) defined cross entropy measure of SVNSs called single valued neutrosophic cross entropy. Then the author presented a multi criteria decision making method based on the proposed single valued neutrosophic cross entropy. Ye (2014b) also introduced the concept of simplified neutrosophic sets (SNSs) and proposed an MCDM method using the aggregation operators of SNSs. Peng et al. (2015) defined some operations of simplified neutrosophic numbers (SNNs) and developed a comparison method using the related research of intuitionistic fuzzy numbers. Based on these operations and the comparison method, the authors developed some SNN aggregation operators and applied them in multi criteria group decision making (MCGDM) problems. Zhang et al. (2014) defined some new operations on INSs and developed aggregation operators for interval neutrosophic number. Broumi and Smarandache (2013b) discussed the correlation coefficient of INSs. Ye (2013) proposed the similarity measures between INSs based on the Hamming and Euclidean distances and developed a multi criteria decision making method based on the similarity degree. Ye (2016) also introduced new exponential operational laws of INSs, where the bases are crisp values or interval numbers and the exponents are interval neutrosophic numbers (INNs). The author proposed a couple of aggregation operators. i.e., interval neutrosophic weighted exponential aggregation (INWEA) operator and a dual interval neutrosophic weighted exponential aggregation (DINWEA) operator based on these exponential operational laws. Finally he applied INWEA and DINWEA operators in decision making problems. Zhao et al. (2015) studied that many types of incomplete or complete information can be expressed as interval valued neutrosophic sets (IVNSs) and proposed improved aggregation operation rules for IVNSs. They also extended the generalized weighted aggregation (GWA) operator in the context of IVNSs. Peng et al. (2014) presented a new outranking approach for multi criteria decision making (MCDM) problems in the context of a simplified neutrosophic environment based on ELECTRE method. Combining neutrosophic set with other mathematical models, a number of research works have been published. Maji (2012) introduced neutrosophic soft set (NSS) as a combination of NS and soft set (Molodtsov, 1999) and presented a neutrosophic soft set theoretic approach for a multi-observer object recognition problem. Combining generalized neutrosophic set (Salama, 2012) with soft set (Molodtsov, 1999), Broumi (2013) introduced the concept of generalized neutrosophic soft set. Broumi and Smarandache (2013a) developed intuitionistic neutrosophic soft set by combining intuitionistic neutrosophic set and soft set. Deli (2014) developed interval-valued neutrosophic soft set which is a combination of interval-valued neutrosophic set and soft set. Broumi et al. (2014a) defined neutrosophic parameterized soft set and studied some of its properties. They introduced neutrosophic parameterized aggregation operator and applied it in decision making problem. Broumi et al. (2014b), in 2014, extended generalized neutrosophic soft set and proposed the idea of generalized interval neutrosophic soft set. Deli and Broumi (2014) introduced the concept of neutrosophic soft matrix and redefined some operations of neutrosophic soft set given by Maji (2012). Deli et al. (2014) introduced the concept of neutrosophic soft multi-set theory and studied their properties and operations. Rivieccio (2008) presented a critical introduction to neutrosophic logics. The author defined suitable neutrosophic propositional connectives and discussed the relationship between neutrosophic logics and other well-known frameworks. Deli et al. (2015) introduced bipolar neutrosophic set and studied some of its operations. To compare the bipolar neutrosophic sets, they studied score functions and accuracy functions. Deli et al. (2016) proposed interval valued bipolar fuzzy weighted neutrosophic set (IVBFWN-set) as a generalization of fuzzy set, bipolar fuzzy set, neutrosophic set and bipolar neutrosophic set. Liu and Luo (2017) developed a series of power aggregation operators on simplified neutrosophic set (SNS) called simplified neutrosophic number power weighted averaging (SNNPWA) operator, simplified neutrosophic number power weighted geometric (SNNPWG) operator, simplified neutrosophic number power ordered weighted averaging (SNNPOWA) operator and simplified neutrosophic number power ordered weighted geometric (SNNPOWG) operator. Additionally, using the developed aggregation operators, they presented a multi attribute group decision making (MAGDM) approach within the framework of SNSs. In Ye (2015), Ye proposed a neutrosophic number tool for group decision making problems with indeterminate information under a

Table 1
Some significant contributions in neutrosophic set.

Authors	Type of study	Tools and approaches
Smarandache (1999, 2003)	Proposed concept	Introduced neutrosophy and defined indeterminacy explicitly and stated that truth, indeterminacy, and falsity-membership are independent
Wang et al. (2010)	Proposed concept	Introduced SVNS as an instance of the neutrosophic set
Maji (2012)	Proposed concept	Introduced NSS and presented a neutrosophic soft set theoretic approach
Ye (2013)	Developed approach	Proposed similarity measures between INSs
Broumi and Smarandache (2013)	Proposed concept	Developed intuitionistic neutrosophic soft set
Majumdar and Samant (2014)	Proposed concept	Introduced entropy measure of SVNS
Ye (2014a,b)	Proposed concept	Proposed cross-entropy and correlation coefficients of SVNSs
Zhang et al. (2014)	Developed approach	Developed aggregation operators for interval neutrosophic number
Broumi et al. (2014a)	Proposed concept	Defined neutrosophic parameterized soft set
Peng et al. (2014)	Proposed approach	Presented a new outranking approach for MCDM problem
Peng et al. (2015)	Proposed concept	Developed SNN aggregation operators
Zhao et al. (2015)	Proposed concept	Improved aggregation operation rules for IVNSs
Ye (2016)	Proposed concept	Proposed a couple of aggregation operators
Liu and Luo (2017)	Proposed concept	Developed a series of power aggregation operators on simplified neutrosophic set

neutrosophic number environment. Some significant contributions in neutrosophic sets are given below in Table 1.

As per our knowledge, there are no articles published in neutrosophic sets and its hybridizations, where experts' relative weights have been considered. Experts opinions are vital for any decision making process. When experts prescribe their preferences using soft sets, they provide their opinions only about their known set of attributes/parameters. As the domains of expertise of different experts are different, they may be interested in different subset of attributes and remain silent about the rest of attributes. More specifically, one expert may provide opinion about more number of attributes; whereas other expert may be interested in less number of attributes. In another way, one expert may be more confident on his/her opinion than the other on the same set of attributes. For this type of environment, equal weights assignment to different experts may lead to improper and biased solution. Preceding situation motivated us to develop the relative weight assignment procedure, where more confident opinions are given more importance. Suppose Mr. X and Mr. Y provide their opinions about an attribute using neutrosophic set. According to Mr. X, the opinion is (0.3, 0.4, 0.6) and opinion of Mr. Y is (0.9, 0.1, 0.2). Here more importance will be given to Mr. Y as he/she is more confident, since the truth membership value for his/her is close to 1 which is more than that of Mr. X. In this article, we propose an algorithmic approach for GDM using NSMs and relative weights of experts. Initially experts provide their opinions using NSMs, which are normalized by the relative weights of the corresponding experts. The relative weights are assigned to individual experts based on their information, which reduces the chances of biasness. The proposed approach focuses on the choice parameters/attributes of various experts to find out the neutrosophic choice matrix (NCM) and combined NCM for individual decision maker/expert. In the process, the combined NCMs are multiplied with the normalized NSMs to obtain the product NSMs, which are aggregated to produce the resultant NSM. Then cross-entropy measure is applied on the alternatives of the resultant NSM to rank the alternatives. The case study is related to investment in business sectors, where two cases have been considered. Case I shows the final outcome without assigning any weight and Case II shows the result by assigning weights.

The rest of the article is organized as follows. Section 2 provides the basic notions and backgrounds of neutrosophic sets, NSSs, NSMs, and cross-entropy measure of neutrosophic sets. Section 3 proposes NCM, combined NCM, and some operations on NSMs. Proposed algorithmic approach is presented in section 4 followed by case study in Section 5. Then a brief discussion on the results is given in Section 6. Finally, we have concluded in Section 7.

2. Preliminaries

This section briefly describes neutrosophic set (NS), neutrosophic soft set (NSS), neutrosophic soft matrix (NSM), and cross-entropy measure of neutrosophic sets.

2.1. Neutrosophic set, NSS, and NSM

Definition 1 Smarandache (1999). Let U be an universe of discourse. The neutrosophic set A in U is expressed by $A = \{\langle x : T_{A(x)}, I_{A(x)}, F_{A(x)}\rangle, x \in U\}$, where the characteristic functions $T, I, F : U \rightarrow]-0, 1^+[$ respectively define the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element $x \in U$ to the set A with the condition: $-0 \leqslant T_{A(x)} + I_{A(x)} + F_{A(x)} \leqslant 3^+$.

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]-0, 1^+[$. For convenient application in real life problems, we take the interval $[0, 1]$ instead of $]-0, 1^+[$. For a given element $x \in U$, the triplet $(T_{A(x)}, I_{A(x)}, F_{A(x)})$ is usually called neutrosophic value (NV) or neutrosophic number (NN).

Soft set was introduced by Molodtsov (1999) as a generic mathematical tool for dealing with uncertain problems which cannot be handled using traditional mathematical tools. There are many theories, such as theory of probability, theory of fuzzy sets (Zadeh, 1965a), theory of neutrosophic sets (Smarandache, 1999, 2003), etc., which can be considered to deal with uncertainties. But all these theories have their inherent difficulties due to the inadequacy of the parameterization tool. Soft set is free from such difficulties which can be used for approximate description of objects without any restriction. As a definite outcome, soft set theory has emerged as a convenient and easily applicable tool in practice. Neutrosophic set is combined with soft set to get the advantages of both the neutrosophic set and soft set.

Definition 2 Maji (2012). Let U be a universe of discourse and E be a set of parameters. Let NS (U) denotes the set of all neutrosophic subsets of U and $A \subset E$. A pair $(N_{\{A\}}, E)$ is called a NSS over U, where $N_{\{A\}}$ is a mapping given by $N_{\{A\}} : E \rightarrow NS(U)$.

Example 1.. Let $U = \{c_1, c_2, c_3\} = \{\text{celerio, xcent, eon}\}$ be the set of 3 models of cars, $E = \{e_1, e_2, e_3, e_4, e_5\}$ = {speed, comfort, durability, costly, branded} be the set of parameters considered for a car, and $A = \{e_1, e_2, e_3, e_5\} \subset E$. Let

$$N_{\{A\}}(e_1) = \{c_1/(0.6, 0.4, 0.8), c_2/(0.4, 0.6, 0.8)\},$$
$$N_{\{A\}}(e_2) = \{c_2/(0.4, 0.9, 0.7), c_3/(0.2, 0.3, 0.6)\},$$
$$N_{\{A\}}(e_3) = \{c_1/(0.5, 0.3, 0.8), c_2/(0.1, 0.6, 0.3), c_3/(0.4, 0.1, 0.7)\},$$
$$N_{\{A\}}(e_5) = \{c_1/(0.7, 0.1, 0.3), c_3/(0.5, 0.4, 0.7)\}$$

Here $N_{\{A\}}(e_1) = \{c_1/(0.6, 0.4, 0.8), c_2/(0.4, 0.6, 0.8)\}$ implies the association of objects (cars) c_1, c_2 {celerio, xcent} with the parameter e_1 (speed). Parameter e_1 is associated with the object c_1 using the degree of membership 0.6, degree of indeterminacy 0.4, and degree of non-membership 0.8. This example shows that parameter e_5 (branded) is not associated with any other cars, i.e., when the cars are being considered, the parameter *branded* has no significance particularly in this example.

Then the NSS $(N_{\{A\}}, E)$ is given by

$$(N_{\{A\}}, E) = \left\{ \begin{array}{l} (e_1, \{c_1/(0.6, 0.4, 0.8), c_2/(0.4, 0.6, 0.8)\}), \\ (e_2, \{c_2/(0.4, 0.9, 0.7), c_3/(0.2, 0.3, 0.6)\}), \\ (e_3, \{c_1/(0.5, 0.3, 0.8), c_2/(0.1, 0.6, 0.3), c_3/(0.4, 0.1, 0.7)\}) \\ (e_4, \{\varnothing\}), \\ (e_5, \{c_1/(0.7, 0.1, 0.3), c_3/(0.5, 0.4, 0.7)\} \end{array} \right\}.$$

Definition 3 Deli and Broumi (2014). Let $(N_{\{A\}}, E)$ be a NSS over the initial universe U. Let E be a set of parameters and $A \subseteq E$. Then a subset of $U \times E$ is uniquely defined by the relation $\{(x, e) : e \in A, x \in N_{\{A\}}(e)\}$ and denoted by $R_A = (N_{\{A\}}, E)$. Now the relation R_A is characterized by the truth function $T_A : U \times E \rightarrow [0, 1]$, indeterminacy $I_A : U \times E \rightarrow [0, 1]$, and the falsity function $F_A : U \times E \rightarrow [0, 1]$. [$T_A(x, e)$ is the truth value, $I_A(x, e)$ is the indeterminacy value, and $F_A(x, e)$ is the falsity value of the object x associated with the parameter e.] R_A is represented as $R_A = \{((T_A(x,e), I_A(x,e), F_A(x,e)) : 0 \leqslant T_A + I_A + F_A \leqslant 3, (x, e) \in U \times E\}$.

Now if the set of universe $U = \{x_1, x_2, \cdots, x_m\}$ and the set of parameters $E = \{e_1, e_2, \ldots, e_n\}$, then R_A can be represented by a matrix as follows:

$$R_A = (a_{ij})_{m\times n} = \begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{pmatrix},$$

where $a_{ij} = (T_A(x_i, e_j), I_A(x_i, e_j), F_A(x_i, e_j))$.

The above matrix is called a neutrosophic soft matrix (NSM) of order m × n corresponding to the neutrosophic set $(N_{\{A\}}, E)$ over U.

Example 2. The matrix representation of the NSS, described above in Example 1, is shown in Table 2.

2.2. Cross-entropy of neutrosophic sets

Entropy is used to measure the degree of fuzziness or uncertain information in fuzzy set theory. The fuzzy entropy was first introduced by Zadeh (1965b, 1968) to quantify the amount of fuzziness. De Luca and Termini (1972) introduced some axioms to describe the degree of fuzziness of a fuzzy set based on Shannon's function. Shannon (1949) defined an information theory in 1949 which introduced cross-entropy. A measure of fuzzy cross-entropy between fuzzy sets was proposed by Shang and Jiang (1997) which is used to measure the discrimination information between two fuzzy sets. Zhang and Jiang (2008) defined vague cross-entropy measure between vague sets. Ye (2011) extended the idea of fuzzy cross-entropy to interval-valued intuitionistic fuzzy sets. Hesitant fuzzy linguistic entropy and cross-entropy measures were proposed by Gou (2017). Ye also proposed cross-entropy measure in Ye (2014a) for single valued neutrosophic set as an extension of the fuzzy cross-entropy.

Suppose $\widehat{A} = (\widehat{A}(x_1), \widehat{A}(x_2), \ldots, \widehat{A}(x_n))$ and $\widehat{B} = (\widehat{B}(x_1), \widehat{B}(x_2), \ldots, \widehat{B}(x_n))$ be two fuzzy sets in the universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$. Then the fuzzy cross-entropy between $\widehat{A}$ and $\widehat{B}$ is defined as

$$H(\widehat{A}, \widehat{B}) = \sum_{i=1}^{n}\left(\widehat{A}(x_i)\log_2\frac{\widehat{A}(x_i)}{\frac{1}{2}(\widehat{A}(x_i)+\widehat{B}(x_i))} + (1-\widehat{A}(x_i))\log_2\frac{1-\widehat{A}(x_i)}{1-\frac{1}{2}(\widehat{A}(x_i)+\widehat{B}(x_i))}\right), \quad (1)$$

which describes the degree of discrimination of $\widehat{A}$ from $\widehat{B}$. Since $H(\widehat{A}, \widehat{B})$ is not symmetric with respect to its arguments, a symmetric discrimination information measure was proposed in Shang and Jiang (1997) as $I(\widehat{A}, \widehat{B}) = H(\widehat{A}, \widehat{B}) + H(\widehat{B}, \widehat{A})$, where $I(\widehat{A}, \widehat{B}) \geqslant 0$ and $I(\widehat{A}, \widehat{B}) = 0$ if and only of $\widehat{A} = \widehat{B}$. The cross-entropy and symmetric discrimination information measures between two fuzzy sets have been extended to the context of SVNSs. Let $A = \{\langle x_i, T_A(x_i), I_A(x_i), F_A(x_i)\rangle x_i \in X\}$ and $B = \{\langle x_i, T_B(x_i), I_B(x_i), F_B(x_i)\rangle x_i \in X\}$ be two SVNSs and $X = \{x_1, x_2, \ldots, x_n\}$ be the universe of discourse, where $T_A(x_i), I_A(x_i), F_A(x_i)$, $T_B(x_i), I_B(x_i), F_B(x_i) \in [0,1]$. Now considering truth-membership, indeterminacy-membership, and falsity-membership, the amount of information discrimination of $T_A(x_i)$ from $T_B(x_i)$, $I_A(x_i)$ from $I_B(x_i)$, and $F_A(x_i)$ from $F_B(x_i)$ are respectively computed using (2), (3), and (4) given below.

$$E^T(A,B) = \sum_{i=1}^{n}\left[T_A(x_i)\log_2\frac{T_A(x_i)}{\frac{1}{2}(T_A(x_i)+T_B(x_i))} + (1-T_A(x_i))\log_2\frac{1-T_A(x_i)}{1-\frac{1}{2}(T_A(x_i)+T_B(x_i))}\right] \quad (2)$$

$$E^I(A,B) = \sum_{i=1}^{n}\left[I_A(x_i)\log_2\frac{I_A(x_i)}{\frac{1}{2}(I_A(x_i)+I_B(x_i))} + (1-I_A(x_i))\log_2\frac{1-I_A(x_i)}{1-\frac{1}{2}(I_A(x_i)+I_B(x_i))}\right] \quad (3)$$

$$E^F(A,B) = \sum_{i=1}^{n}\left[F_A(x_i)\log_2\frac{F_A(x_i)}{\frac{1}{2}(F_A(x_i)+F_B(x_i))} + (1-F_A(x_i))\log_2\frac{1-F_A(x_i)}{1-\frac{1}{2}(F_A(x_i)+F_B(x_i))}\right] \quad (4)$$

The single valued neutrosophic cross-entropy measure between A and B is computed as the sum of three amounts:

$$\begin{aligned} E(A,B) = &\sum_{i=1}^{n}\left[T_A(x_i)\log_2\frac{T_A(x_i)}{\frac{1}{2}(T_A(x_i)+T_B(x_i))} + (1-T_A(x_i))\log_2\frac{1-T_A(x_i)}{1-\frac{1}{2}(T_A(x_i)+T_B(x_i))}\right] \\ &+ \sum_{i=1}^{n}\left[I_A(x_i)\log_2\frac{I_A(x_i)}{\frac{1}{2}(I_A(x_i)+I_B(x_i))} + (1-I_A(x_i))\log_2\frac{1-T_A(x_i)}{1-\frac{1}{2}(T_A(x_i)+T_B(x_i))}\right] \\ &+ \sum_{i=1}^{n}\left[F_A(x_i)\log_2\frac{F_A(x_i)}{\frac{1}{2}(F_A(x_i)+F_B(x_i))} + (1-F_A(x_i))\log_2\frac{1-F_A(x_i)}{1-\frac{1}{2}(F_A(x_i)+F_B(x_i))}\right] \end{aligned} \quad (5)$$

$E(A,B)$ is also a discrimination degree between A and B, where $E(A,B) \geqslant 0$. Here $E(A,B) = 0$ if and only if $T_A(x_i) = T_B(x_i), I_A(x_i) = I_B(x_i), F_A(x_i) = F_B(x_i)$. Since $E(A,B)$ is not symmetric, so a symmetric discrimination information measure for SVNSs is obtained as

$$D(A,B) = E(A,B) + E(B,A). \quad (6)$$

$$\begin{aligned} D_i(A^*, A_i) = &\sum_{j=1}^{n}\left[\log_2\frac{1}{\frac{1}{2}(1+T_{ij})} + \log_2\frac{1}{1-\frac{1}{2}(I_{ij})} + \log_2\frac{1}{1-\frac{1}{2}(F_{ij})}\right] \\ &+ \sum_{j=1}^{n}\left[T_{ij}\log_2\frac{T_{ij}}{\frac{1}{2}(1+T_{ij})} + (1-T_{ij})\log_2\frac{1-T_{ij}}{1-\frac{1}{2}(1+T_{ij})}\right] \\ &+ \sum_{j=1}^{n}\left[I_{ij} + (1-I_{ij})\log_2\frac{1-I_{ij}}{1-\frac{1}{2}(I_{ij})}\right] \\ &+ \sum_{j=1}^{n}\left[F_{ij} + (1-F_{ij})\log_2\frac{1-F_{ij}}{1-\frac{1}{2}(F_{ij})}\right]. \end{aligned} \quad (7)$$

Table 2
Tabular representation of NSS $(N_{\{A\}}, E)$.

U/E	e_1	e_2	e_3	e_4	e_5
c_1	(0.6,0.4,0.8)	0	(0.5,0.3,0.8)	0	(0.7,0.1,0.3)
c_2	(0.4,0.6,0.8)	(0.4,0.9,0.7)	(0.1,0.6,0.3)	0	0
c_3	0	(0.2,0.3,0.6)	(0.4,0.1,0.7)	0	(0.5,0.4,0.7)

Using (5) and (6), Ye (2014a) defined weighted neutrosophic cross-entropy $D_i(A^*, A_i)$ between an alternative A_i and the ideal alternative A^* as given in (7). Here the smaller the value of $D_i(A^*, A_i)$, the better will be the alternative A_i and the alternative A_i will be close to the ideal alternative A^*. The concept of ideal alternative is used in decision making environment for the purpose of identifying the best alternative in the decision set. Normally the ideal alternative does not exist in real world, but it provides theoretical framework for evaluating the alternatives.

The ideal alternative A^* in neutrosophic concept is defined as $A^* = \langle T^*, I^* F^* \rangle = \langle 1, 0, 0 \rangle$.

3. NCM, combined NCM, and basic operations on NSMs

This section presents NCM, combined NCM, and operations on NSMs, such as addition, complement, and product of NSMs with the combined NCMs.

3.1. NCM and combined NCM

This sub-section defines NCM and combined NCM with necessary examples.

Definition 4. NCM is a square matrix whose rows and columns both indicates parameters. If ξ is a NCM, then its element $\xi(i,j)$ is defined as

$\xi(i,j)$ = (1, 0.5, 0) when *i*th and *j*th both parameters are the choice parameters of decision maker

=(0, 0.5, 1) when at least one of the *i*th or *j*th parameters be not under choice of the decision maker.

Example 3. Let U and E are same as in Example 1. Suppose Mr. X is interested for buying a car based on the attributes $A_X = \{e_3, e_4, e_5\} \subset E$. Then the NCM for Mr. X can be represented as

$$\xi_X(i,j) = e_X \overset{e_X}{\begin{pmatrix} \{(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1)\} \\ \{(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1)\} \\ \{(0,0.5,1),(0,0.5,1),(1,0.5,0),(1,0.5,0),(1,0.5,0)\} \\ \{(0,0.5,1),(0,0.5,1),(1,0.5,0),(1,0.5,0),(1,0.5,0)\} \\ \{(0,0.5,1),(0,0.5,1),(1,0.5,0),(1,0.5,0),(1,0.5,0)\} \end{pmatrix}}$$

In combined NCM, denoted by ξ^c, rows indicate choice parameters of single decision maker and columns indicate combined choice parameters (obtained by the intersection of parameter sets) of the remaining decision makers. It is noted that in NCM ξ, both row and column indicate the attributes of same decision maker.

Example 4. Suppose Mr. X and Mr. Y want to buy a car as per their combined opinion. Attributes preferred by of Mr. X are mentioned in Example 3. Let Mr. Y shows his/her preference in $A_Y = \{e_1, e_3\} \subset E$. Combined NCM ξ^c_X, for Mr. X is given below.

$$\xi^c_X(i,j) = e_X \overset{e_Y}{\begin{pmatrix} \{(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1)\} \\ \{(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1)\} \\ \{(0,0.5,1),(0,0.5,1),(1,0.5,0),(0,0.5,1),(0,0.5,1)\} \\ \{(0,0.5,1),(0,0.5,1),(1,0.5,0),(0,0.5,1),(0,0.5,1)\} \\ \{(0,0.5,1),(0,0.5,1),(1,0.5,0),(0,0.5,1),(0,0.5,1)\} \end{pmatrix}}$$

3.2. Addition of NSMs

Two NSMs, $(N,A)_{m\times n} = [a_{ij}]_{m\times n}$ and $(N,B)_{m\times n} = [b_{ij}]_{m\times n}$ are said to be conformable for addition, if they have the same order. The addition of two NSMs matrices $[a_{ij}]_{m\times n}$ and $[b_{ij}]_{m\times n}$ is defined by $[c_{ij}]_{m\times n} = [a_{ij}]_{m\times n} \oplus [b_{ij}]_{m\times n}$, where $[c_{ij}]_{m\times n}$is also the NSM of order $m \times n$ and $(c_{ij}) = (\max\{T_{a_{ij}}, T_{b_{ij}}\}, avg\{I_{a_{ij}}, I_{b_{ij}}\}, \min\{F_{a_{ij}}, F_{b_{ij}}\})\ \forall i,j$.

Example 5. Suppose NSMs $(a_{ij})_{m\times n}$ and $(b_{ij})_{m\times n}$ are given below.

$$(a_{ij})_{m\times n} = \begin{pmatrix} (0.6,0.5,0.3),(0,0.5,1),(0.6,0.8,0.3),(0,0.8,1),(0,0.5,1) \\ (0.8,0.9,0.3),(0,0.6,1),(0.8,0.6,0.3),(0,0.2,1),(0,0.7,1) \\ (0.8,0.5,0.4),(0,0.5,1),(0.8,0.5,0.4),(0,0.5,1),(0,0.2,1) \end{pmatrix}$$

$$(b_{ij})_{m\times n} = \begin{pmatrix} (0.4,0.5,0.7),(1,0.5,1),(0.4,0.2,0.7),(1,0.2,0),(1,0.5,0) \\ (0.2,0.1,0.7),(1,0.4,0),(0.2,0.4,0.7),(1,0.8,0),(1,0.3,0) \\ (0.2,0.5,0.6),(1,0.5,0),(0.2,0.5,0.6),(1,0.5,0),(1,0.8,0) \end{pmatrix}$$

Then the addition of these two NSMs gives $(c_{ij})_{m\times n}$, where

$$(c_{ij})_{m\times n} = (a_{ij})_{m\times n} \oplus (b_{ij})_{m\times n} = \begin{pmatrix} (0.6,0.5,0.3),(1,0.5,1),(0.6,0.5,0.3),(1,0.5,0),(1,0.5,0) \\ (0.8,0.5,0.3),(1,0.5,0),(0.8,0.5,0.3),(1,0.5,0),(1,0.5,0) \\ (0.8,0.5,0.4),(1,0.5,0),(0.8,0.5,0.4),(1,0.5,0),(1,0.5,0) \end{pmatrix}.$$

3.3. Product of NSM and combined NCM

If the number of columns of NSM (N,A) is equal to the number of rows of the combined NCM ξ^c, then (N,A) and ξ^c are said to be conformable for the product $\{(N,A) \otimes \xi^c\}$ and the product $\{(N,A) \otimes \xi^c\}$ becomes an NSM, which is denoted by (N,P). If $(N,A) = (a_{ij})_{m\times n}$ and $\xi^c = (\beta_{jk})_{n\times p}$, then $(N,P) = (N,A) \otimes \xi^C = (c_{ik})_{m\times p}$, where

$$c_{ik} = (\max_{j=1}^{n} \min\{T_{a_{ij}}, T_{\beta_{jk}}\}, \max_{j=1}^{n} \min\{I_{a_{ij}}, I_{\beta_{jk}}\}, \min_{j=1}^{n} \max\{F_{a_{ij}}, F_{\beta_{jk}}\}).$$

Example 6. Let NSM and combined NCM of Mr. X are respectively defined as given below.

$$(N,A)_X = \begin{pmatrix} \{(0,0,0),(0,0,0),(0.3,0.5,0.6),(0.6,0.7,0.3),(0.3,0.6,0.7)\} \\ \{(0,0,0),(0,0,0),(0.7,0.6,0.3),(0.8,0.2,0.7),(0.7,0.2,0.3)\} \\ \{(0,0,0),(0,0,0),(0.7,0.8,0.4),(0.6,0.1,0.7),(0.8,0.1,0.4)\} \end{pmatrix}$$

$$\xi^c_X(i,j) = \begin{pmatrix} \{(0.0.5,1),(0.0.5,1),(0.0.5,1),(0.0.5,1),(0.0.5,1)\} \\ \{(0.0.5,1),(0.0.5,1),(0.0.5,1),(0.0.5,1),(0.0.5,1)\} \\ \{(0.0.5,1),(0.0.5,1),(1,0.5,0),(0.0.5,1),(0.0.5,1)\} \\ \{(0.0.5,1),(0.0.5,1),(1,0.5,0),(0.0.5,1),(0.0.5,1)\} \\ \{(0.0.5,1),(0.0.5,1),(1,0.5,0),(0.0.5,1),(0.0.5,1)\} \end{pmatrix}$$

Then the product

$$(N,P) = (N,A)_X \otimes \xi^C(i,j)_{\{X\}} = \begin{pmatrix} \{(0,0.5,1),(0,0.5,1),(0.6,0.5,0.3),(0,0.5,1),(0,0.5,1)\} \\ \{(0,0.5,1),(0,0.5,1),(0.8,0.5,0.3),(0,0.5,1),(0,0.5,1)\} \\ \{(0,0.5,1),(0,0.5,1),(0.7,0.5,0.4),(0,0.5,1),(0,0.5,1)\} \end{pmatrix}$$

3.4. Complement of NSM

Complement of an NSM $(N,A) = (a_{ij})_{m\times n}$ is denoted by $(N^C,A) = (a^c_{ij})_{m\times n}$, where $(a_{ij})_{m\times n}$ is the matrix representation of the NSS $(N_{\{A\}}, E)$. $(a^c_{ij})_{m\times n}$ is the matrix representation of the NSS$(N^c_{\{\neg A\}}, E)$ and defined as $a^c_{ij} = (T^c_{a_{ij}}, I^c_{a_{ij}}, F^c_{a_{ij}}) = (1 - T_{a_{ij}}, 1 - I_{a_{ij}}, 1 - F_{a_{ij}})$.

Example 7. Let NSM $(a_{ij})_{m\times n}$ is given below.

$$(N,A)=\begin{pmatrix}(0.6,0.5,0.3),(0,0.5,1),(0.6,0.8,0.3),(0,0.8,1),(0,0.5,1)\\(0.8,0.9,0.3),(0,0.6,1),(0.8,0.6,0.3),(0,0.2,1),(0,0.7,1)\\(0.8,0.5,0.4),(0,0.5,1),(0.8,0.5,0.4),(0,0.5,1),(0,0.2,1)\end{pmatrix}$$

Then the complement (N^C,A) is given by

$$(N^C,A)=\begin{pmatrix}(0.4,0.5,0.7),(1,0.5,1),(0.4,0.2,0.7),(1,0.2,0),(1,0.5,0)\\(0.2,0.1,0.7),(1,0.4,0),(0.2,0.4,0.7),(1,0.8,0),(1,0.3,0)\\(0.2,0.5,0.6),(1,0.5,0),(0.2,0.5,0.6),(1,0.5,0),(1,0.8,0)\end{pmatrix}.$$

3.5. Cardinality of NSM

More cardinality is assigned to the NSM, which gives more emphasis to the given opinion on the set of attributes irrespective of the number of attributes focused by the NSM. The cardinal set of neutrosophic soft set $(N_{\{A\}},E)$, denoted by $(cN_{\{A\}},E)$ and defined by $(cN_{\{A\}},E)=\{(T_{cN_{\{A\}}}(x),I_{cN_{\{A\}}}(x),F_{cN_{\{A\}}}(x)),x\in E\}$, is a neutrosophic soft set over E. The membership $T_{cN_{\{A\}}}(x)$, indeterminacy $I_{cN_{\{A\}}}(x)$ and falsity membership value $F_{cN_{\{A\}}}(x)$ are respectively defined by

$$T_{cN_{\{A\}}}(x)=\sum_{\substack{x\in E\\o\in U}}T_{N_{\{A\}}}(x)/|E_n|,I_{cN_{\{A\}}}(x)=\sum_{\substack{x\in E\\o\in U}}|0.5-I_{N_{\{A\}}}(x)|/|E_n|,$$

and

$$F_{cN_{\{A\}}}(x)=\sum_{\substack{x\in E\\o\in U}}F_{N_{\{A\}}}(x)/|E_n|,$$

where

$E_n=\{i|T_{N_{\{A\}}}(x),I_{N_{\{A\}}}(x),F_{N_{\{A\}}}(x)\neq 0\,\forall x\in E\}.$

Cardinal score of a neutrosophic soft set corresponding to the cardinal set $(cN_{\{A\}},E)$ is defined as

$S(cN_{\{A\}})=(T_{cN_{\{A\}}}(x)+I_{cN_{\{A\}}}(x)-F_{cN_{\{A\}}}(x))/|U|.$

Example 8. Let two experts Mr. X and Mr. Y provide their opinions using their NSMs as given below (see Tables 3 and 4).

Cardinality score of Mr. X is given below.

$$S(cN_{\{A\}})=(T_{cN_{\{A\}}}(x)+I_{cN_{\{A\}}}(x)-F_{cN_{\{A\}}}(x))/|U|$$

$$T_{cN_{\{A\}}}(x)=\sum_{\substack{x\in E\\o\in U}}T_{N_{\{A\}}}(x)/|E_n|=(0.3+0.4+0.8)/3=0.5$$

$$I_{cN_{\{A\}}}(x)=\sum_{\substack{x\in E\\o\in U}}|0.5-I_{N_{\{A\}}}(x)|/|E_n| =|0.5-0.2|+|0.5-0.5|+|0.5-0.2|/3=0.2$$

Table 3
NSM for Mr. X.

U/E	e_1	e_2	e_3
c_1	(0.3,0.2,0.4)	(0.4,0.5,0.4)	(0.8,0.2,0.1))

Table 4
NSM for Mr. Y.

U/E	e_1	e_2	e_3
c_1	(1,0.5,0)	(0,0,0)	(0,0,0)

$$F_{cN_{\{A\}}}(x)=\sum_{\substack{x\in E\\o\in U}}F_{N_{\{A\}}}(x)/|E_n|=(0.4+0.4+0.1)/3=0.3$$

$$S^X(cN_{\{A\}})=(T_{cN_{\{A\}}}(x)+I_{cN_{\{A\}}}(x)-F_{cN_{\{A\}}}(x))/|U| =(0.5+0.2-0.3)/1=0.4$$

Cardinality score of Mr. Y is

$$T_{cN_{\{A\}}}(x)=\sum_{\substack{x\in E\\o\in U}}T_{N_{\{A\}}}(x)/|E_n|=1/1=1,I_{cN_{\{A\}}}(x) =\sum_{\substack{x\in E\\o\in U}}|0.5-I_{N_{\{A\}}}(x)|/|E_n|=|0.5-0.5|=0,$$

$$F_{cN_{\{A\}}}(x)=\sum_{\substack{x\in E\\o\in U}}F_{N_{\{A\}}}(x)/|E_n|=0$$

$$S^Y(cN_{\{A\}})=(T_{cN_{\{A\}}}(x)+I_{cN_{\{A\}}}(x)-F_{cN_{\{A\}}}(x))/|U|=1/1=1.$$

4. An approach for GDM using NSM

In this section, we propose an algorithmic approach based on NSM, cardinality score of NSM, and NCM of experts to solve the multiple attribute group decision making (MAGDM) problems. Suppose that $U=\{o_1,o_2,\ldots,o_m\}$ be a discrete set of alternatives, $E=\{e_1,e_2,\ldots,e_n\}$ be the set of attributes or parameters, and $D=\{d_1,d_2,\ldots,d_k\}$ be the set of decision makers or experts. NSM R_A^l denotes the decision information provided by decision maker d_l, $l=1,2,\ldots,k$. The neutrosophic value a_{ij}^l, which is in the form of neutrosophic number represents the evaluation of alternative o_i, $i=1,2,\ldots,m$ for the attribute e_j, $j=1,2,\ldots,n$ given by decision maker d_l, $l=1,2,\ldots,k$.

The steps of the proposed algorithm for solving GDM problems using NSM are presented below.

Step 1: NCM $\xi_l(i,j)$ and combined NCM $\xi_l^c(i,j)$ of each of the decision makers $d_l, l=1,2,\ldots,k$ are computed in the context of NSS based on their choice parameters or attributes.

Step 2: Cardinal score $S^l(cN_{\{A\}})$ for NSM R_A^l is computed, where $l=1,2,\ldots,k$.

Step 3: Cardinal score $S^l(cN_{\{A\}})$ is multiplied with the corresponding NSM $R_A^l, l=1,2,\ldots,k$ to produce the normalized NSM. Let $[\widehat{A}^l_{(ij)}]_{m\times n}$ be an NSM and h is the cardinal score, then the normalized NSM, denoted by N^l_{NSM}, is defined by $N^l_{NSM}[\widehat{A}_{(ij)}]=[h*\widehat{A}^l_{(ij)}]_{m\times n}\ \forall i,j,k.$

Step 4: Product of normalized NSM N^l_{NSM} and combined choice matrix $\xi_l^c(i,j)$ for each decision maker d_l is calculated as given in Section 3.3 and denoted by $P^l_{NSM}, l=1,2,\ldots,k.$

Step 5: Aggregation of the product NSMs $P^l_{NSM}\,\forall l$ is done as defined in Section 3.2, which produces the resultant NSM denoted by (R_{NSM}).

Step 6: Neutrosophic cross-entropy, discussed above in Section 2.2, between the ideal alternative and the ith alternative $o_i\,\forall i$ is computed to rank the alternatives.

Step 7: Alternative(s) having lowest cross-entropy value is selected as the most desirable one(s).

In this approach, NCM shows the impact of choice parameters of individual experts in decision making process, whereas combined NCM is used to provide the impact of choice parameters of individual experts with respect to other experts. Basically NCM and combined NCM have been used to provide more importance to the choice parameters of different experts. Here cardinal score is used to assign relative weights to the experts. When an expert is confident about his/her opinion, this approach assigns more weight, i.e., more cardinal score to the corresponding NSM. By normalizing the NSMs, we provide more importance to the NSMs of confident experts and less importance to the NSMs of less confident experts. Then normalized NSMs are associated with the combined NCMs with the hope that the resultant NSM will focus on both of the expert's confident and choice parameters. Finally, we compute neutrosophic cross-entropy between our alternatives and the ideal alternative to find the ranking.

5. Case study

In order to demonstrate the application of the proposed method, we cite an example about the investment for three possible business sectors. Let three experts Mr. John, Mr. Smith, and Mr. Peter, the members of a set $D = \{d_1, d_2, d_3\}$ jointly want to select a business sector for investment. Their proposed business sectors are travel agencies, hotel, and restaurant, given by $U = \{o_1, o_2, o_3\}$. These business sectors have a set of common attributes: first time Investment, risk factor, profit, place of running business, and quality of services, given by $E = \{e_1, e_2, e_3, e_4, e_5\}$. Among three experts, Mr. John is interested in profit, place of running business, and quality of services, i.e., (e_3, e_4, e_5). Mr. Smith shows his interest in first time investment and profit, i.e., (e_1, e_3) and Mr. Peter is interested in first time investment, profit, and quality of services, i.e., (e_1, e_3, e_5). Opinions of Mr. John, Mr. Smith, and Mr. Peter are represented in the three different NSMs, R_A^J, R_A^S, and R_A^P respectively, which are given below.

$$R_A^J = \begin{bmatrix} \{(0,0,0),(0,0,0),(0.3,0.5,0.6),(0.6,0.7,0.3),(0.3,0.6,0.7)\} \\ \{(0,0,0),(0,0,0),(0.7,0.6,0.3),(0.8,0.2,0.7),(0.7,0.2,0.3)\} \\ \{(0,0,0),(0,0,0),(0.7,0.8,0.4),(0.6,0.1,0.2),(0.8,0.1,0.4)\} \end{bmatrix}$$

$$R_A^S = \begin{bmatrix} \{(0.6,0.4,0.8),(0,0,0),(0.3,0.5,0.6),(0,0,0),(0,0,0)\} \\ \{(0.7,0.5,0.6),(0,0,0),(0.7,0.6,0.3),(0,0,0),(0,0,0)\} \\ \{(0.8,0.3,0.4),(0,0,0),(0.7,0.8,0.4),(0,0,0),(0,0,0)\} \end{bmatrix}$$

$$R_A^P = \begin{bmatrix} \{(0.6,0.4,0.8),(0,0,0),(0.3,0.5,0.6),(0,0,0),(0.3,0.6,0.7)\} \\ \{(0.7,0.5,0.6),(0,0,0),(0.7,0.6,0.3),(0,0,0),(0.7,0.2,0.3)\} \\ \{(0.8,0.3,0.4),(0,0,0),(0.7,0.8,0.4),(0,0,0),(0.8,0.1,0.4)\} \end{bmatrix}$$

In this case study, we consider two cases. In the first case, we use non-normalized NSM and normalized NSM in the second case.

Case I. In this case, we use non-normalized NSM as input.

[Step 1]: Neutrosophic choice matrices for the experts and their corresponding combined choice matrices are given below.

$$\xi_J(i,j) = \begin{bmatrix} \{(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1)\} \\ \{(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1)\} \\ \{(0,0.5,1),(0,0.5,1),(1,0.5,0),(1,0.5,0),(1,0.5,0)\} \\ \{(0,0.5,1),(0,0.5,1),(1,0.5,0),(1,0.5,0),(1,0.5,0)\} \\ \{(0,0.5,1),(0,0.5,1),(1,0.5,0),(1,0.5,0),(1,0.5,0)\} \end{bmatrix}$$

$$\xi_S(i,j) = \begin{bmatrix} \{(1,0.5,0),(0,0.5,1),(1,0.5,0),(0,0.5,1),(0,0.5,1)\} \\ \{(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1)\} \\ \{(1,0.5,0),(0,0.5,1),(1,0.5,0),(0,0.5,1),(0,0.5,1)\} \\ \{(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1)\} \\ \{(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1)\} \end{bmatrix}$$

$$\xi_P(i,j) = \begin{bmatrix} \{(1,0.5,0),(0,0.5,1),(1,0.5,0),(0,0.5,1),(1,0.5,0)\} \\ \{(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1)\} \\ \{(1,0.5,0),(0,0.5,1),(1,0.5,0),(0,0.5,1),(1,0.5,0)\} \\ \{(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1)\} \\ \{(1,0.5,0),(0,0.5,1),(1,0.5,0),(0,0.5,1),(1,0.5,0)\} \end{bmatrix}$$

$$\xi_J^c(i,j) = \begin{bmatrix} \{(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1)\} \\ \{(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1)\} \\ \{(1,0.5,0),(0,0.5,1),(1,0.5,0),(0,0.5,1),(0,0.5,1)\} \\ \{(1,0.5,0),(0,0.5,1),(1,0.5,0),(0,0.5,1),(0,0.5,1)\} \\ \{(1,0.5,0),(0,0.5,1),(1,0.5,0),(0,0.5,1),(0,0.5,1)\} \end{bmatrix}$$

$$\xi_S^c(i,j) = \begin{bmatrix} \{(0,0.5,1),(0,0.5,1),(1,0.5,0),(0,0.5,1),(1,0.5,0)\} \\ \{(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1)\} \\ \{(0,0.5,1),(0,0.5,1),(1,0.5,0),(0,0.5,1),(1,0.5,0)\} \\ \{(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1)\} \\ \{(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1)\} \end{bmatrix}$$

$$\xi_P^c(i,j) = \begin{bmatrix} \{(0,0.5,1),(0,0.5,1),(1,0.5,0),(0,0.5,1),(0,0.5,1)\} \\ \{(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1)\} \\ \{(0,0.5,1),(0,0.5,1),(1,0.5,0),(0,0.5,1),(0,0.5,1)\} \\ \{(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1)\} \\ \{(0,0.5,1),(0,0.5,1),(1,0.5,0),(0,0.5,1),(0,0.5,1)\} \end{bmatrix}$$

[Step 2 & Step 3]: These steps are not applied in Case 1.

[Step 4]: Product of NSM R_A^l and combined choice matrix $\xi_l^c(i,j)$ for each decision maker $d_l, l = 1, 2, 3$ are

$$P_{NSM}^J = R_A^J \otimes \xi_J^c(i,j)$$

$$= \begin{bmatrix} \{(0,0,0),(0,0,0),(0.3,0.5,0.6),(0.6,0.7,0.3),(0.3,0.6,0.7)\} \\ \{(0,0,0),(0,0,0),(0.7,0.6,0.3),(0.8,0.2,0.7),(0.7,0.2,0.3)\} \\ \{(0,0,0),(0,0,0),(0.7,0.8,0.4),(0.6,0.1,0.2),(0.8,0.1,0.4)\} \end{bmatrix} \otimes$$

$$\begin{bmatrix} \{(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1)\} \\ \{(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1)\} \\ \{(1,0.5,0),(0.0.5,1),(1,0.5,0),(0.0.5,1),(0,0.5,1)\} \\ \{(1,0.5,0),(0.0.5,1),(1,0.5,0),(0.0.5,1),(0,0.5,1)\} \\ \{(1,0.5,0),(0.0.5,1),(1,0.5,0),(0.0.5,1),(0,0.5,1)\} \end{bmatrix}$$

$$= \begin{bmatrix} (0.6,0.5,0.3),(0,0.5,1),(0.6,0.5,0.3),(0,0.5,1),(0,0.5,1) \\ (0.8,0.5,0.3),(0,0.5,1),(0.8,0.5,0.3),(0,0.5,1),(0,0.5,1) \\ (0.8,0.5,0.2),(0,0.5,1),(0.8,0.5,0.2),(0,0.5,1),(0,0.5,1) \end{bmatrix}$$

$$P^S_{NSM} = R^S_A \otimes \xi^c_S(i,j) = \begin{bmatrix} \{(0.6,0.4,0.8),(0,0,0),(0.3,0.5,0.6),(0,0,0),(0,0,0)\} \\ \{(0.7,0.5,0.6),(0,0,0),(0.7,0.6,0.3),(0,0,0),(0,0,0)\} \\ \{(0.8,0.3,0.4),(0,0,0),(0.7,0.8,0.4),(0,0,0),(0,0,0)\} \end{bmatrix} \otimes \begin{bmatrix} \{(0,0.5,1),(0,0.5,1),(1,0.5,0),(0,0.5,1),(1,0.5,0)\} \\ \{(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1)\} \\ \{(0,0.5,1),(0,0.5,1),(1,0.5,0),(0,0.5,1),(1,0.5,0)\} \\ \{(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1)\} \\ \{(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1)\} \end{bmatrix}$$

$$= \begin{bmatrix} (0,0.5,1),(0,0.5,1),(0.6,0.5,0.6),(0,0.5,1),(0,0.5,1) \\ (0,0.5,1),(0,0.5,1),(0.7,0.5,0.3),(0,0.5,1),(0,0.5,1) \\ (0,0.5,1),(0,0.5,1),(0.8,0.5,0.4),(0,0.5,1),(0,0.5,1) \end{bmatrix}$$

$$P^P_{NSM} = R^P_A \otimes \xi^c_P(i,j) = \begin{bmatrix} \{(0.6,0.4,0.8),(0,0,0),(0.3,0.5,0.6),(0,0,0),(0.3,0.6,0.7)\} \\ \{(0.7,0.5,0.6),(0,0,0),(0.7,0.6,0.3),(0,0,0),(0.7,0.2,0.3)\} \\ \{(0.8,0.3,0.4),(0,0,0),(0.7,0.8,0.4),(0,0,0),(0.8,0.1,0.4)\} \end{bmatrix} \otimes \begin{bmatrix} \{(0,0.5,1),(0,0.5,1),(1,0.5,0),(0,0.5,1),(0,0.5,1)\} \\ \{(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1)\} \\ \{(0,0.5,1),(0,0.5,1),(1,0.5,0),(0,0.5,1),(0,0.5,1)\} \\ \{(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1)\} \\ \{(0,0.5,1),(0,0.5,1),(1,0.5,0),(0,0.5,1),(0,0.5,1)\} \end{bmatrix}$$

$$= \begin{bmatrix} (0,0.5,1),(0,0.5,1),(0.6,0.5,0.6),(0,0.5,1),(0,0.5,1) \\ (0,0.5,1),(0,0.5,1),(0.7,0.5,0.3),(0,0.5,1),(0,0.5,1) \\ (0,0.5,1),(0,0.5,1),(0.8,0.5,0.4),(0,0.5,1),(0,0.5,1) \end{bmatrix}$$

[Step 5]: Aggregation of the product NSMs, $P^l_{NSM}\,\forall l$, is R_{NSM}, given below.

$$R_{NSM} = \begin{bmatrix} (0.6,0.5,0.3),(0,0.5,1),(0.6,0.5,0.3),(0,0.5,1),(0,0.5,1) \\ (0.8,0.5,0.3),(0,0.5,1),(0.8,0.5,0.3),(0,0.5,1),(0,0.5,1) \\ (0.8,0.5,0.2),(0,0.5,1),(0.8,0.5,0.2),(0,0.5,1),(0,0.5,1) \end{bmatrix} \oplus$$

$$\begin{bmatrix} (0,0.5,1),(0,0.5,1),(0.6,0.5,0.6),(0,0.5,1),(0,0.5,1) \\ (0,0.5,1),(0,0.5,1),(0.7,0.5,0.3),(0,0.5,1),(0,0.5,1) \\ (0,0.5,1),(0,0.5,1),(0.8,0.5,0.4),(0,0.5,1),(0,0.5,1) \end{bmatrix} \oplus$$

$$\begin{bmatrix} (0,0.5,1),(0,0.5,1),(0.6,0.5,0.6),(0,0.5,1),(0,0.5,1) \\ (0,0.5,1),(0,0.5,1),(0.7,0.5,0.3),(0,0.5,1),(0,0.5,1) \\ (0,0.5,1),(0,0.5,1),(0.8,0.5,0.4),(0,0.5,1),(0,0.5,1) \end{bmatrix}$$

$$= \begin{bmatrix} (0.6,0.5,0.3),(0,0.5,1),(0.6,0.5,0.3),(0,0.5,1),(0,0.5,1) \\ (0.8,0.5,0.3),(0,0.5,1),(0.8,0.5,0.3),(0,0.5,1),(0,0.5,1) \\ (0.8,0.5,0.2),(0,0.5,1),(0.8,0.5,0.2),(0,0.5,1),(0,0.5,1) \end{bmatrix}$$

[Step 6]: According to the formula defined in section 2.2, the cross-entropy values of $(A^*,o_1),(A^*,o_2)$, and (A^*,o_3) can be calculated as $D_1(A^*,o_1) = 16.7353$, $D_2(A^*,o_2) = 16.2216$, and $D_3(A^*,o_3) = 15.9770$

Since $D_3(A^*,o_3)$ has least cross-entropy value, alternative o_3, i.e., restaurant business sector is selected as the collective decision of all the three decision makers.

Case II. It uses normalized NSM as input.

[Step 1]: This step is similar as in case 1.

[Step 2]: It calculates the cardinal scores $S^J(cN_{\{A\}})$, $S^S(cN_{\{A\}})$, and $S^P(cN_{\{A\}})$ respectively for the NSMs R^J_A, R^S_A, and R^P_A.

$$S^J(cN_{\{A\}}) = (T_{cN_{\{A\}}}(x) + I_{cN_{\{A\}}}(x) - F_{cN_{\{A\}}}(x))/(|E_n| * |U|)$$

$$T_{cN_{\{A\}}}(x) = \sum_{\substack{x\in E\\ o\in U}} T_{N_{\{A\}}}(x)/|E_n| = (0.3+0.6+0.3+0.7+0.8+0.7+0.7+0.6+0.8)/3 = 5.5/3$$

$$I_{cN_{\{A\}}}(x) = \sum_{\substack{x\in E\\ o\in U}} |0.5 - I_{N_{\{A\}}}(x)|/|E_n| = \left(\begin{matrix} |0.5-0.5| + |0.5-0.7| + |0.5-0.6| + |0.5-0.6| + |0.5-0.2| \\ +|0.5-0.2| + |0.5-0.8| + |0.5-0.1| + |0.5-0.1| \end{matrix} \right) \Big/ 3 = (0+.2+.1+.1+.3+.3+.3+.4+.4)/3 = 2.1/3$$

$$F_{cN\{A\}}(x) = \sum_{\substack{x\in E\\ o\in U}} F_{N\{A\}}(x)|E_n| = (0.6+0.3+0.7+0.3+0.7+0.3+0.4+0.2(*0.7) + 0.4)/3 = 3.9/3$$

$$S^J(c\hat{N}_{\{A\}}) = S^J(cN_{\{A\}}) = ((5.5)+(2.1)-3.9)/3)/3 = 0.411$$

Similarly, $S^S(cN_{\{A\}}) = 0.23, S^P(cN_{\{A\}}) = 0.28$

[Step 3]: Normalized NSMs for the decision makers, John, Smith and Peter are respectively

$$N^J_{NSM}[a_{(ij)}] = \left[h * a^J_{(ij)}\right]_{3\times5} = (0.41) \times \begin{bmatrix} \{(0,0,0),(0,0,0),(0.3,0.5,0.6),(0.6,0.7,0.3),(0.3,0.6,0.7)\} \\ \{(0,0,0),(0,0,0),(0.7,0.6,0.3),(0.8,0.2,0.7),(0.7,0.2,0.3)\} \\ \{(0,0,0),(0,0,0),(0.7,0.8,0.4),(0.6,0.1,0.7),(0.8,0.1,0.4)\} \end{bmatrix}$$

$$= \begin{bmatrix} \{(0,0,0),(0,0,0),(0.123,0.205,0.246),(0.246,0.287,0.123),(0.123,0.246,0.287)\} \\ \{(0,0,0),(0,0,0),(0.287,0.246,0.123),(0.328,0.082,0.287),(0.287,0.082,0.123)\} \\ \{(0,0,0),(0,0,0),(0.287,0.328,0.164),(0.246,0.041,0.287),(0.328,0.041,0.164)\} \end{bmatrix}$$

$$N^S_{NSM}[a_{(ij)}] = \left[h * a^S_{(ij)}\right]_{3\times5} = (0.23) \times \begin{bmatrix} \{(0.6,0.4,0.8),(0,0,0),(0.3,0.5,0.6),(0,0,0),(0,0,0)\} \\ \{(0.7,0.5,0.6),(0,0,0),(0.7,0.6,0.3),(0,0,0),(0,0,0)\} \\ \{(0.8,0.3,0.4),(0,0,0),(0.7,0.8,0.4),(0,0,0),(0,0,0)\} \end{bmatrix}$$

$$= \begin{bmatrix} \{(0.138,0.092,0.184),(0,0,0),(0.069,0.115,0.138),(0,0,0),(0,0,0)\} \\ \{(0.161,0.115,0.138),(0,0,0),(0.161,0.138,0.069),(0,0,0),(0,0,0)\} \\ \{(0.184,0.069,0.092),(0,0,0),(0.161,0.184,0.092),(0,0,0),(0,0,0)\} \end{bmatrix}$$

$$N^P_{NSM}[a_{(ij)}] = \left[h * a^P_{(ij)}\right]_{3\times5} = (0.28) \times \begin{bmatrix} \{(0.6,0.4,0.8),(0,0,0),(0.3,0.5,0.6),(0,0,0),(0.3,0.6,0.7)\} \\ \{(0.7,0.5,0.6),(0,0,0),(0.7,0.6,0.3),(0,0,0),(0.7,0.2,0.3)\} \\ \{(0.8,0.3,0.4),(0,0,0),(0.7,0.8,0.4),(0,0,0),(0.8,0.1,0.4)\} \end{bmatrix}$$

$$= \begin{bmatrix} \{(0.168,0.112,0.224),(0,0,0),(0.084,0.140,0..168),(0,0,0),(0.084,0.168,0.196)\} \\ \{(0.196,0.149,0.168),(0,0,0),(0.196,0.168,0.084),(0,0,0),(0.196,0.056,0.084)\} \\ \{(0.224,0.084,0.112),(0,0,0),(0.196,0.224,0.112),(0,0,0),(0.224,0.028,0.112)\} \end{bmatrix}$$

[Step 4]: Product of normalized NSM N^l_{NSM} and combined choice matrix $\xi^c_l(i,j)$ for each decision maker d_l is computed as follows.

$$P^J_{NSM} = N^J_{NSM} \otimes \xi^c_J(i,j) = \begin{bmatrix} \{(0,0,0),(0,0,0),(0.123,0.205,0.246),(0.246,0.287,0.123),(0.123,0.246,0.287)\} \\ \{(0,0,0),(0,0,0),(0.287,0.246,0.123),(0.328,0.082,0.287),(0.287,0.082,0.123)\} \\ \{(0,0,0),(0,0,0),(0.287,0.328,0.164),(0.246,0.041,0.287),(0.328,0.041,0.164)\} \end{bmatrix} \otimes \begin{bmatrix} \{(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1)\} \\ \{(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1)\} \\ \{(1,0.5,0),(0,0.5,1),(1,0.5,0),(0,0.5,1),(0,0.5,1)\} \\ \{(1,0.5,0),(0,0.5,1),(1,0.5,0),(0,0.5,1),(0,0.5,1)\} \\ \{(1,0.5,0),(0,0.5,1),(1,0.5,0),(0,0.5,1),(0,0.5,1)\} \end{bmatrix}$$

$$= \begin{bmatrix} \{(0.246,0.287,0.123),(0,0.287,1),(0.210,0.287,0.105),(0,0.287,1),(0,0.287,1)\} \\ \{(0.328,0.246,0.123),(0,0.246,1),(0.328,0.246,0.123),(0,0.246,1),(0,0.246,1)\} \\ \{(0.328,0.328,0.164),(0,0.328,1),(0.328,0.328,0.164),(0,0.328,1),(0,0.328,1)\} \end{bmatrix}$$

$$P^S_{NSM} = N^S_{NSM} \otimes \xi^c_S(i,j) = \begin{bmatrix} \{(0.138,0.092,0.184),(0,0,0),(0.069,0.115,0.138),(0,0,0),(0,0,0)\} \\ \{(0.161,0.115,0.138),(0,0,0),(0.161,0.138,0.069),(0,0,0),(0,0,0)\} \\ \{(0.184,0.069,0.092),(0,0,0),(0.161,0.184,0.092),(0,0,0),(0,0,0)\} \end{bmatrix} \otimes \begin{bmatrix} \{(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1)\} \\ \{(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1)\} \\ \{(1,0.5,0),(0,0.5,1),(1,0.5,0),(0,0.5,1),(0,0.5,1)\} \\ \{(1,0.5,0),(0,0.5,1),(1,0.5,0),(0,0.5,1),(0,0.5,1)\} \\ \{(1,0.5,0),(0,0.5,1),(1,0.5,0),(0,0.5,1),(0,0.5,1)\} \end{bmatrix}$$

$$= \begin{bmatrix} \{(0.138,0.115,0),(0,0.138,1),(0.138,0.115,0),(0,.138,1),(0,0.138,1)\} \\ \{(0.161,0.138,0),(0,0.138,1),(0.161,0.138,0),(0,0.138,1),(0,0.138,1)\} \\ \{(0.184,0.184,0),(0,0.184,1),(0.184,0.184,0),(0,.184,1),(0,0.184,1)\} \end{bmatrix}$$

$$P^P_{NSM} = N^P_{NSM} \otimes \xi^c_P(i,j) =$$
$$\begin{bmatrix} \{(0.168,0.112,0.224),(0,0,0),(0.084,0.140,0.168),(0,0,0),(0.084,0.168,0.196)\} \\ \{(0.196,0.149,0.168),(0,0,0),(0.196,0.168,0.084),(0,0,0),(0.196,0.056,0.084)\} \\ \{(0.224,0.084,0.112),(0,0,0),(0.196,0.224,0.112),(0,0,0),(0.224,0.028,0.112)\} \end{bmatrix} \otimes$$
$$\begin{bmatrix} \{(1,0.5,0),(0,0.5,1),(1,0.5,0),(0,0.5,1),(1,0.5,0)\} \\ \{(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1)\} \\ \{(1,0.5,0),(0,0.5,1),(1,0.5,0),(0,0.5,1),(1,0.5,0)\} \\ \{(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1),(0,0.5,1)\} \\ \{(1,0.5,0),(0,0.5,1),(1,0.5,0),(0,0.5,1),(1,0.5,0)\} \end{bmatrix}$$
$$= \begin{bmatrix} \{(0.168,0.168,0.168),(0,0.168,1),(0.168,0.168,0.168),(0,0.168,1),(0.168,0.168,0.168)\} \\ \{(0.196,0.168,0.084),(0,0.168,1),(0.196,0.168,0.084),(0,0.168,1),(0.196,0.168,0.084)\} \\ \{(0.224,0.224,0.112),(0,0.224,1),(0.224,0.224,0.112),(0,0.224,1),(0.224,0.224,0.112)\} \end{bmatrix}$$

[Step 5]: Aggregation of the product NSMs, $P^l_{NSM}\,\forall l$ is denoted by R_{NSM}.

$$R_{NSM} = \begin{bmatrix} \{(0.246,0.287,0.123),(0,0.287,1),(0.210,0.287,0.105),(0,0.287,1),(0,0.287,1)\} \\ \{(0.328,0.246,0.123),(0,0.246,1),(0.328,0.246,0.123),(0,0.246,1),(0,0.246,1)\} \\ \{(0.328,0.328,0.164),(0,0.328,1),(0.328,0.328,0.164),(0,0.328,1),(0,0.328,1)\} \end{bmatrix} \oplus$$
$$\begin{bmatrix} \{(0.138,0.115,0),(0,0.138,1),(0.138,0.115,0),(0,0.138,1),(0,0.138,1)\} \\ \{(0.161,0.138,0),(0,0.138,1),(0.161,0.138,0),(0,0.138,1),(0,0.138,1)\} \\ \{(0.184,0.184,0),(0,0.184,1),(0.184,0.184,0),(0,0.184,1),(0,0.184,1)\} \end{bmatrix} \oplus$$
$$\begin{bmatrix} \{(0.168,0.168,0.168),(0,0.168,1),(0.168,0.168,0.168),(0,0.168,1),(0.168,0.168,0.168)\} \\ \{(0.196,0.168,0.084),(0,0.168,1),(0.196,0.168,0.084),(0,0.168,1),(0.196,0.168,0.084)\} \\ \{(0.224,0.224,0.112),(0,0.224,1),(0.224,0.224,0.112),(0,0.224,1),(0.224,0.224,0.112)\} \end{bmatrix}$$
$$= \begin{bmatrix} \{(0.246,0.19,0),(0,0.19,1),(0.246,0.19,0),(0,0.19,1),(0,0.19,1)\} \\ \{(0.328,0.184,0),(0,0.184,1),(0.328,0.184,0),(0,0.184,1),(0.196,0.184,1)\} \\ \{(0.328,0.245,0),(0,0.245,1),(0.328,0.245,0),(0,0.245,1),(0.224,0.245,1)\} \end{bmatrix}$$

[Step 6]: Cross-entropy values of $(A*, o_1), (A*, o_2)$, and $(A*, o_3)$ are

$$D_1(A*, o_1) = 15.235,\ D_2(A*, o_2) = 14.075,\ D_3(A*, o_3) = 14.366$$

Since $D_2(A*, o_2)$ has least cross-entropy value, alternative o_2, i.e., hotel business sector will be selected as the collective decision of John, Smith, and Peter.

6. Discussion on the results

Expert's opinion exhibit a vital role in GDM (Das et al., 2015; 2017). This study proposes a decision making methodology using NSM, which is used to represent the opinion of individual decision maker. The relative weight assigned to an expert is based on the expert's prescribed opinion and is computed by deriving the cardinal score of the corresponding NSM. The more the cardinal score, the more important is the opinion. When an expert is more confident about her opinion, more cardinal score is assigned to that expert. Due to lack of information or limited domain knowledge, experts often prefer to express their opinions only for a subset of attributes instead of the entire attribute set. Often it is also found that experts are confident about a few attributes among the subset of attributes. In that case, cardinal score will be more where an expert is confident about her opinion irrespective of the number of attributes provided. This is similar to our real life situations. It also removes the biasness which might be imposed by different experts and as a result adds more credibility to the final decision. In (Das and Kar, 2014; Das et al., 2014), authors considered to assign more cardinal score when an expert provides his/her opinion about more number of attributes, which is also practical in real life environment. More specifically, which case should be considered for assigning higher cardinal score, may be case dependent. When experts' opinions about the selected set of attributes are quite significant and no attributes can be ignored, then one can consider the relative weight assigning procedure as proposed in Das and Kar (2014), Das et al. (2014). But when opinion about some of the selected attributes are not significance and can be ignored, then the approach proposed here can be used. Moreover, the key aspect of the proposed algorithm is that it does not use any knowledgebase for finding the distances and similarity measures of the individual experts. Rather more importance is given on the parameter selection of experts by finding initial choice matrices and then combined choice matrices. Among the two cases used here in case study, Case I does not use the relative weight, while Case II uses it. As per collective opinion of a group of experts, the order of selection of the business sectors is given in Table 5. The result shows different ordering in Case II as we have considered the experts' relative weights. Case I produces the final outcome as o_3, i.e., restaurant business sector while Case II produces o_2, i.e., hotel business sector as per the collective opinion.

Table 5
Ordering of business sectors in different cases.

Case I	$o_3 > o_2 > o_1$
Case II	$o_2 > o_3 > o_1$

7. Conclusion

This article has proposed an algorithmic approach for solving GDM problems using NSM and relative weight of experts. Firstly, we have presented NSM and discussed some of its relevant operations. Next we have proposed the relative weight assigning procedure of experts using cardinal score in the context of neutrosophic environment. The proposed algorithm is based on combined choice matrix, product NSM, cardinal score, and cross entropy measure of NSMs, which yields the collective opinion of a group of decision makers. The case study is related to the selection of a business sector for investment purpose, where a set of three experts suggests their opinions about a common set of attributes. Future scope of this research work could be to investigate the application of robustness in GDM in the framework of neutrosophic set. Also researchers might focus on various properties of NSMs and then apply them to suitable uncertain decision making problems.

Acknowledgment

The authors are very grateful to the Editor-in-Chief and the anonymous referees, for their constructive comments and suggestions that led to an improved version of this paper.

References

Atanassov, K., 1986. Intuitionistic fuzzy sets. Fuzzy Sets Syst. 20, 87–96.
Atanassov, K., 1989. More on intuitionistic fuzzy sets. Fuzzy Sets Syst. 33, 37–46.
Broumi, S., 2013. Generalized neutrosophic soft set, international journal of computer science. Eng. Inf. Tech. (IJCSEIT) 3 (2), 17–30.
Broumi, S., Smarandache, F., 2013a. Intuitionistic neutrosophic soft set. J. Inf. Comput. Sci. 8 (2), 130–140.
Broumi, S., Smarandache, F., 2013b. Correlation coefficient of interval neutrosophic set. Appl. Mech. Mater. 436, 511–517.
Broumi, S., Deli, I., Smarandache, F., 2014a. Neutrosophic Parameterized soft set theory and its decision Making. Italian J. Pure Appl. Math. 32, 503–514.
Broumi, S., Sahin, R., Smarandache, F., 2014b. Generalized interval neutrosophic soft set and its decision making problem. J. New Results Sci. 7, 29–47.
Das, S., Kar, S., 2014. Group decision making in medical system: an intuitionistic fuzzy soft set approach. Appl. Soft Comput. 24, 196–211.
Das, S., Kar, S., Pal, T., 2014. Group decision making using interval-valued intuitionistic fuzzy soft matrix and confident weight of experts. J. Artif. Intell. Soft Comput. Res. 4 (1), 57–77.
Das, S., Kumari, P., Verma, A.K., 2015. Triangular fuzzy soft set and its application in MADM. Int. J. Comput. Syst. Eng. 2 (2), 85–93.
Das, S., Kar, S., Pal, T., 2017. Robust decision making using intuitionistic fuzzy numbers. Granular Comput. 2, 41–54. http://dx.doi.org/10.1007/s41066-016-0024-3.
De Luca, A., Termini, S., 1972. A definition of non probabilistic entropy in the setting of fuzzy sets theory. Inf. Control 20, 301–312.
Deli, I. 2014. Interval-valued Neutrosophic soft sets and its decision making, arXiv:1402.3130v3 [math.GM], pp. 1–24.
Deli, I., Broumi, S. 2014. Neutrosophic soft sets and neutrosophic soft matrices based on decision making, arXiv:1404.0673v1 [math.GM], pp. 1–28.
Deli, I., Broumi, S., Ali, M., 2014. Neutrosophic soft multi-set theory and its decision making. Neutrosophic Sets Syst. 5, 65–76.

Deli, I., Ali, M., Smarandache, F., 2015. Bipolar neutrosophic sets and their application based on multi-criteria decision making problems. Int. Conf. Adv. Mechatronic Syst. (ICAMechS). http://dx.doi.org/10.1109/ICAMechS.2015.7287068.

Deli, I., Şubaş, Y., Smarandache, F., 2016. Interval valued bipolar fuzzy weighted neutrosophic sets and their application. In: IEEE International Conference on Fuzzy Systems (FUZZ-IEEE). http://dx.doi.org/10.1109/FUZZ-IEEE.2016.7738002.

Gou, X., Xu, Z.-S., Liao, H., 2017. Hesitant fuzzy linguistic entropy and cross-entropy measures and alternative queuing method for multiple criteria decision making. Inf. Sci. 388–389, 225–246.

Guo, Y., Sengur, A., 2014. A novel image segmentation algorithm based on neutrosophic similarity clustering. Appl. Soft Comput. 25, 391–398.

Liu, W., Liao, H., 2017. A bibliometric analysis of fuzzy decision research during 1970–2015. Int. J. Fuzzy Syst. 19 (1), 1–14.

Liu, C.F., Luo, Y.S., 2017. Power aggregation operators of simplified neutrosophic sets and their use in multi-attribute group decision making. IEEE/CAA J. Automatica Sinica, 1–10. http://dx.doi.org/10.1109/JAS.2017.7510424.

Maji, P.K., 2012. A neutrosophic soft set approach to a decision making problem. Ann. Fuzzy Math. Inf. 3 (2), 313–319.

Majumdar, P., Samant, S.K., 2014. On similarity and entropy of neutrosophic sets. J. Intell. Fuzzy Syst. 26 (3), 1245–1252.

Molodtsov, D.A., 1999. Soft set theory-first results. Comput. Math. Appl. 37, 19–31.

Peng, J., Wang, J., Zhang, H., Chen, X., 2014. An outranking approach for multi-criteria decision-making problems with simplified neutrosophic sets. Appl. Soft Comput. 25, 336–346.

Peng, J., Wang, J., Wang, J., Zhang, H., Chen, X., 2015. Simplified neutrosophic sets and their applications in multi-criteria group decision-making problems. Int. J. Syst. Sci. http://dx.doi.org/10.1080/00207721.2014.994050.

Rivieccio, U., 2008. Neutrosophic logics: prospects and problems. Fuzzy Sets Syst. 159, 1860–1868.

Salama, A.A., 2012. Generalized neutrosophic set and generalized neutrosophic topological spaces. Comput. Sci. Eng. 2 (7), 129–132. http://dx.doi.org/10.5923/j.computer.20120207.01.

Shang, X.G., Jiang, W.S., 1997. A note on fuzzy information measures. Pattern Recognit. Lett. 18, 425–432.

Shannon, C., 1949. The Mathematical Theory of Communication. The University of Illinois Press, Urbana.

Smarandache, F., 1999. A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic. American Research Press, Rehoboth.

Smarandache, F. 2003. A unifying field in logics: neutrosophic logic. Neutrosophy, neutrosophic set, neutrosophic probability and statistics, third ed., Xiquan, Phoenix.

Turksen, I., 1986. Interval valued fuzzy sets based on normal forms. Fuzzy Sets Syst. 20, 191–210.

Wang, H., Smarandache, F., Zhang, Y.Q., Sunderraman, R., 2010. Single valued neutrosophic sets. Multispace Multistruct. 4, 410–413.

Ye, J., 2011. Fuzzy cross entropy of interval-valued intuitionistic fuzzy sets and its optimal decision-making method based on the weights of alternatives. Expert Syst. Appl. 38, 6179–6183.

Ye, J., 2013. Similarity measures between interval neutrosophic sets and their applications in multi criteria decision making. J. Intell. Fuzzy Syst. http://dx.doi.org/10.3233/IFS-120724.

Ye, J., 2014a. Single valued neutrosophic cross-entropy for multi criteria decision making problems. Appl. Math. Model. 38, 1170–1175.

Ye, J., 2014b. A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets. J. Intell. Fuzzy Syst. 26 (5), 2459–2466.

Ye, J., 2015. Multiple-attribute group decision-making method under a neutrosophic number environment. J. Intell. Syst. 25 (3), 377–386. http://dx.doi.org/10.1515/jisys-2014-0149.

Ye, J., 2016. Exponential operations and aggregation operators of interval neutrosophic sets and their decision making methods. SpringerPlus 5, 1488. http://dx.doi.org/10.1186/s40064-016-3143-z.

Yu, D., Liao, H., 2016. Visualization and quantitative research on intuitionistic fuzzy studies. J. Intell. Fuzzy Syst. 30 (6), 3653–3663.

Zadeh, L.A., 1965a. Fuzzy Sets. Inform. Control 8, 338–353.

Zadeh, L.A., 1965b. Fuzzy sets and systems. Proceedings of the Symposium on Systems Theory. Polytechnic Institute of Brooklyn, NY, pp. 29–37.

Zadeh, L., 1968. Probability measures of fuzzy events. J. Math. Anal. Appl. 23, 421–427.

Zhang, Q.S., Jiang, S.Y., 2008. A note on information entropy measures for vague sets. Inf. Sci. 178, 4184–4191.

Zhang, H., Wang, J., Chen, X., 2014. Interval Neutrosophic Sets and Their Application in Multi criteria Decision Making Problems. Sci. World J. http://dx.doi.org/10.1155/2014/645953 645953.

Zhao, A., Du, J., Guan, H., 2015. Interval valued neutrosophic sets and multi-attribute decision-making based on generalized weighted aggregation operator. J. Intell. Fuzzy Syst. 29 (6), 2697–2706.