Chapter 13

Market-Based Forecast Combination

13.1 Financial Markets

Markets can be a spectacularly effective method of information aggregation, as shown in both classic theoretical treatments of price systems as information aggregators (e.g., Koopmans (1957)) and similarly classic experimental work e.g., Plott (2000)). Hence one might suspect that markets would be useful for combining forecasts. In this section we explore that idea.

Markets and surveys are in certain respects opposite extremes. Markets are loved by economists (as market participants have real money on the line), and surveys are often unloved by economists (as survey participants typically have nothing on the line). That is, because market participants have real money on the line, markets may be more likely than otherwise to truthfully reveal traders' views, via their trading decisions). In any event, both market-based combined forecasts and survey-based combined forecasts are very different from forecasts from a simple single model.

Financial markets are naturally forward-looking, and market forecasts can sometimes be extracted from financial asset prices. There are many examples.

13.1.1 General Principles

Point Forecasts From Forward Markets

A classic example is using forward foreign exchange rates as forecasts of future spot exchange rates. Under risk neutrality, agents will always buy forward foreign exchange when it's "cheap" (under-priced) relative to their expectations of the future spot rate, and sell when it's "dear" (over-priced). Immediately then, we have that under risk neutrality:

$$F_t(t+h) = E_t(S_{t+h}),$$

where $F_t(t+h)$ is the h-day forward rate prevailing at time t and $E_t(S_{t+h})$ is the optimal (conditional mean) spot-rate forecast made at time t for time $t+h$.

Note well that in this example, and in financial markets more generally, typically only "risk neutral" forecasts are easy to extract from financial markets, the real-world usefulness of which remains an issue. That is, risk premia, which moreover are likely time-varying due to the time-varying financial-market volatility emphasized in Chapter 8, are always a potential issue.[1]

Point Forecasts From Futures Markets

Futures markets exist for many many things, trading contracts not only for standard financials (e.g., currencies, interest rates, ...), but also for myriad other things, including aspects of energy, agriculture, metals and other commodities – even weather, real estate, and stock market volatility!

Because futures are traded on exchanges, you can learn a lot about the contracts traded and currently-prevailing prices (and hence implied forecasts) by visiting exchanges' websites. (Some are listed at the end of this chapter.)

[1] But much of the evidence looks good, a point to which we will return in some detail.

Density Forecasts From Options Markets (Using Sets of Options)

We can infer market-based density forecasts of future spot price S by looking at currently-prevailing options prices across a (hopefully-wide) wide range of strike prices, k.

Event Probability Forecasts From Digital Options Markets

"Contingent claims," or "Arrow-Debreu securities," or "binary options," or "digital options" simply pay \$1 if a certain event occurs, and 0 otherwise. Hence we can infer the market's event probability assessment from the price at which the digital option sells. Digital options are now written on a variety of "underlyings," from the $S\&P$ 500, to the VIX, to the weather.

But digital options can be written on anything and traded by anyone (if only the regulators would stay away). Effectively they're just gambles, in a financial-market disguise.[2] This brings up the general idea of so-called "prediction markets," which have always been viewed as gambling markets (e.g., sports betting), to which we now turn.

Density Forecasts From Digital Options Markets (Using Sets of Digital Options)

Estimate sets of probabilities (i.e., a density or cdf) using sets of contracts.

13.1.2 More

Volatility Forecasts From Options Markets

Using no-arbitrage arguments (i.e., not even requiring risk neutrality), we can price options given a view on volatility, and conversely we can use market prices of options to infer the market volatility view. The famous Black-Scholes formula for pricing European options, although surely incorrect, as it

[2] Of course *all* financial markets are effectively casinos in significant part.

assumes that spot prices follow Gaussian random walks with constant volatility, nevertheless conveys all of the relevant lessons. We have

$$P_t = G(\sigma_t, i_t, S_t, k, \tau),$$

where P_t is a call price, σ_t is volatility, i_t is the risk-free rate corresponding to the remaining lifespan of the option, S_t is current spot price, k is strike price, and τ is time to maturity. Alternatively we can invert the equation and write

$$\sigma_t = G^{-1}(P_t, i_t, S_t, k, \tau).$$

This equation gives the current market view (forecast) of σ_t as a function of observed market price P_t.

Correlation Forecasts From Trios of Implied Volatilities

By a no-arbitrage argument (i.e., not even requiring risk neutrality), we have

$$cov(\Delta \ln Y/\$, \Delta \ln D/\$) = \frac{1}{2}\left(var(\Delta \ln Y/\$) + var(\Delta \ln D/\$) - var\left(\Delta \ln Y/D\right)\right).$$

To see why, note that in the absence of triangular arbitrage,

$$var(\Delta \ln(Y/D)) = var\left(\Delta \ln \frac{Y/\$}{D/\$}\right).$$

But

$$var\left(\Delta \ln \frac{Y/\$}{D/\$}\right) = var(\Delta \ln Y/\$) + var(\Delta \ln D/\$) - 2cov(\Delta \ln Y/\$, \Delta \ln D/\$),$$

so that

$$cov(\Delta \ln Y/\$, \Delta \ln D/\$) = \frac{1}{2}\left(var(\Delta \ln Y/\$) + var(\Delta \ln D/\$) - var\left(\Delta \ln \frac{Y/\$}{D/\$}\right)\right).$$

This means that, given exchange-rate volatility forecasts extracted from financial markets via options as discussed above, we can also produce market-

based covariance and correlation forecasts.

Skewness Forecasts From Risk Reversals

In a risk reversal, one buys/sells a call and sells/buys a put, both out of the money.

Inflation Forecasts From Indexed Bonds

The difference between yields on non-indexed vs. indexed bonds is an immediate risk-neutral forecast of inflation.

Inflation Forecasts from Bond Yields

Under risk neutrality, nominal government bond yields equal real yields plus expected inflation (the famous "Fisher equation"),

$$i_t(t+h) = r_t(t+h) + E_t(\pi_{t+h}),$$

where $i_t(t+h)$ is the nominal bond yield from time t to time $t+h$, $r_t(t+h)$ is the corresponding real yield, and $E_t(\pi_{t,t+h})$ the optimal (conditional mean) forecast of inflation between time t and time $t+h$. Hence under an assumption about the real rate one can extract expected inflation.

Bond Yield Forecasts From the Term Premium

Long rates always involve averages of expected future short rates. Under risk neutrality we get the famous Hicksian "expectations theory" of the yield curve, in which long rates are *precisely* averages of expected future short rates, so that borrowers are indifferent between issuing a long bond or issuing a short bond and sequentially rolling it over.[3]

[3]Put differently (for bond market aficionados), another way to state the expectations theory is that currently-prevailing forward interest rates should equal expected future short interest rates.

We have

$$i_t(t+h) = \frac{i_t(t+1) + E_t i_{t+1}(t+2) + ... + E_t i_{t+h-1}(t+h)}{h}.$$

This suggests that a useful predictive regression would relate changes in short yields to the currently-prevailing long-short spread ("term spread"), a so-called Campbell-Shiller regression.

Real Activity Forecasts From the Term Premium

Unexpectedly tight monetary policy now, by raising short rates, produces an inverted (negatively-sloped) yield curve now, and recession often follows (both theoretically and empirically). Conversely, loose monetary policy now produces an upward-sloping yield curve now, and a boom later. This suggests that the shape of the yield curve now, and in particular a long-short term spread, has predictive content for real activity (real GDP and its components, and more generally "the business cycle.")

Real Activity Forecasts From the Default Premium

A simple direct argument regarding market-perceived recession risk suggests that we compare the prevailing yield on an N-year ("risk-free") government bond to an index of N-year (risky, defaultable) corporate bond yields.[4] The larger the spread, the larger the market-perceived corporate bond default probability, presumably driven by an increase in market-perceived recession probability.

[4]The corporate bond yield index can moreover be broken down by grade.

Long-Run Equity Return Forecasts from the Dividend Yield

13.2 "Prediction Markets"

13.2.1 Arrow-Debreu Contingent Claims

The name "prediction markets" is a misnomer, as predictions are not traded in those markets; rather, Arrow-Debreu contracts are traded. Hence there's really nothing new relative to digital options. As with digital options, we interpret the prices in markets for Arrow-Debreu securities as market-based combined forecasts of event probabilities.

But prediction markets are run purely for the purpose of inferring predictions, so we don't simply have to take markets "as is," as with financial markets. Instead, we *design* the markets to provide exactly what we want.

Prediction markets are proving useful in many forecasting situations, and they may be unusually useful in the very hardest forecasting situations, such as assessing the probability of events like "an earthquake hits Tehran before December 31, 2050 and kills 100,000 or more people."

13.2.2 Parimutual Betting Markets

Parimutual is like Arrow-Debreu but without the ability to resell a security once bought. So it suffers in terms of dynamic tracking of market-based probabilities. A way to fix it would be to have a secondary market in pari "receipts".

13.3 Issues with Market-Based Forecasts

There are many interesting issues yet to be thoroughly explored.

13.3.1 Market Inefficiencies and No-Arbitrage Conditions

Issues may arise with composite bets (e.g., Duke wins it all in March Madness). In particular if the market is arbitrage-free, then the price of the composite bet should equal that of a replicating portfolio of simple bets. Another no-arbitrage issue is that the the bid price on one exchange should never be higher than the ask price on another.

A related issue is apparent mis-pricing of extreme events, such as overpricing of far out-of-the-money puts. This is often called "favorite / longshot bias," in reference to parimutual betting markets.

13.3.2 Moral Hazard and Market Manipulation

Moral hazard – the temptation to be less vigilant against risks that are insured – is always an issue in "insurance" markets such as those under discussion. Related, incentives arise to manipulate markets so as to increase the likelihood of payoffs.

13.3.3 True Moral Issues

The public sometimes finds trading on extreme events immoral (e.g., "we shouldn't let someone profit from an earthquake that kills 100,000 people"). But the profiting hedgers are precisely those that need help – earthquake victims who bought earthquake contracts!

13.3.4 Risk Neutrality

The main issue is that market-assessed probabilities are risk neutral, so they may not be reliable guides to the physical world. Perhaps market can behave as risk neutral even if individual agents are not. Many of the bets are for small entertainment purposes, so risk neutrality may not be unreasonable. At any

rate, as an empiricall matter, event probabilities extracted from prediction markets are often highly accurate.

13.3.5 Beyond Risk Neutrality

Charles Manski argues that there are problems with market-assessed probabilities even if traders are risk-neutral, as long as they have heterogeneous beliefs. In particular, he argues that

> ... the price of a contract in a prediction market reveals nothing about the dispersion of traders' beliefs and only partially identifies the central tendency of beliefs. Most persons have beliefs higher than price when price is above 0.5, and most have beliefs lower than price when price is below 0.5. The mean belief of traders lies in an interval whose midpoint is the equilibrium price.

The first part of Manksi's critique (that price reveals nothing about the dispersion of traders' beliefs) seems disingenuous. Why would anyone think that price *would* reveal anything about dispersion of beliefs?

The second part of Manski's critique (that price only partially identifies the central tendency of beliefs) seems relevant, if not particularly trenchant. Indeed he shows that the mean belief of traders nevertheless "lies in an interval whose midpoint is the equilibrium price." And as an empirical matter, market-based probability assessments are typically highly accurate, for whatever reason. For a broad overview of these and related issues, see Snowberg et al. (2013).

13.3.6 A Bit More on Market Efficiency

We have seen that groupthink may wreak havoc in certain survey environments (Delphi, focus-group), but we hasten to add that it can similarly pollute market-based probability assessments, as price bubbles and the like

("panics, manias and crashes," in the colorful language of Kindleberger and Aliber (2011)) may seriously compromise the independence of the opinions being aggregated.

In addition, if surveys may suffer from the fact that "no money is on the line," markets may suffer from selection bias – markets aggregate only the views of those who choose to trade, and traders may be a very special group with special characteristics, whereas randomized surveys cover entire populations.

13.4 Exercises, Problems and Complements

1. "Parimutual" market-based forecasts inside Intel.

   Read Gillen, Plott and Shum (2014) (GPS), "A Parimutual-Like Mechanism for Information Aggregation: A Field Test Inside Intel".

   (a) GPS aggregate information (combine forecasts) using a "parimutual betting market" as opposed to an Arrow-Debreu (AD) securities market. Discuss the similarities and differences, pros and cons, etc. Clearly the GPS parimutual market-based information aggregation mechanism is *different* from the AD mechanism, but is it necessarily *better*? Why or why not?

   (b) Why do GPS reveal the bet distribution in real time? Might that not promote groupthink? Discuss.

   (c) GPS admirably try try to provide new insight into why parimutual prediction markets work. But isn't it basically the usual story, namely that people in prediction markets behave in approximately risk-neutral fashion, for whatever reason, allowing us to infer market-assessed conditional event probabilities?

   (d) Recall Manski's critique of AD markets: Even under risk neutral-

ity AD markets identify only a range for the conditional probability (centered at the true conditional probability). Perhaps the parimutual mechanism has provably better properties under risk neutrality, nailing the conditional expectation as opposed to a range? Discuss.

(e) Density forecasts are often evaluated using the sequence of probability integral transforms (PIT's). What is the PIT sequence, what two properties should it have, and why?

(f) Do GPS check the two PIT sequence conditions for their parimutual density forecast? One? None? Discuss in detail.

(g) Density forecasts are often compared using predictive likelihood (PL). What is the PL, and how do such comparisons proceed? Do GPS do a PL comparison of their parimutual forecast to the official Intel forecast? Why or why not?

(h) GPS compare parimutual forecasts to official Intel forecasts, but they neglect a key comparison parimutual vs. AD. How would you do it?

(i) In a forecast combination exercise (GPS vs. official Intel), Intel receives a *negative* combining weight. Discuss.

2. Comparing parimutual and AD information aggregation mechanisms.

   http://authors.library.caltech.edu/44358/1/wp1131.pdf

3. Are combined prediction markets likely valuable?

   PredictWise aggregates prices from alternative prediction markets. But prediction market forecasts are effectively combined forecasts, so averages of different prediction markets are effectively combined combined forecasts. Are such averages likely to be superior to any single prediction market? And isn't the existence of different prices for the same contract in different prediction markets a violation of the law of one price, and

hence of market efficiency, enabling arbitrage? If the answer is that the contracts in different markets aren't identical, then does it really make sense to average their prices?

4. A diary of experiences trading in prediction markets (among other things).

   See the material under "STATISTICS/Predicting and Prediction Markets" at gwern.net. Much other material on the site is interesting and also worth a look.

5. Interesting people working on prediction markets.

   Miro Dudik, Dan Goldstein, Jake Hofman, Patrick Hummel, Adam Isen, Neil Malhotra, David Pennock, David Rothchild, Florian Teschner, Duncan Watts, Justin Wolfers, Eric Zitzewitz. For links to names not hyperlinked, see Rothchild's site.

6. Prediction markets encourage "foxy" behavior.

   Read Silver (2012), Tetlock and Gardner (2015) and Tetlock (2006) on forecasting "foxes" and "hedgehogs" (e.g., Silver, pp. 53-54). Note the key to the success of prediction markets – particularly as typically used for complicated event forecasting – may be their encouragement of foxy behavior, by virtue of their making non-foxy (hedgehog) behavior explicitly *costly*! Note also, however, that it's important that prediction-market wagers be set at levels not so small as to encourage strong risk-seeking, or so high as to encourage strong risk aversion. We need approximate risk neutrality to be able to credibly interpret prediction-market prices as probabilities.

13.5 Notes

Financial Markets

Macroeconomic derivatives. CME futures and options on the S&P Case-Shiller house price index.

VIX implied volatility index

CBOE options exchange

CME futures exchange

Prediction Markets

ipredict. New Zealand. Wide range of contracts.

Tradesports. In trouble. Re-emerging with virtual currency?

The Journal of Prediction Markets

Lumenogic. "Collective intelligence solutions." Consultancy providing prediction markets to client firms. Uses virtual currency.

European Betting Markets

Microsoft Prediction Lab

PredictWise Prediction/Betting Market Aggregator. Combined prediction-market forecasts.

From their FAQ's (regarding the markets that they follow / combine):

Q:What are the Iowa Electronic Markets, http://tippie.uiowa.edu/iem/index.cfm?

A:The Iowa Electronic Markets (IEM) is an exchange of real-money prediction markets operated by the University of Iowa Tippie College of Business. The IEM is not-for-profit; the markets are run for educational and research purposes. Because of the small sums wagered and the academic focus, the IEM has received no-action relief from the United States government, meaning U.S.-based speculators can legally risk up to $500 on the exchange.

Q:What is Betfair, https://www.betfair.com/us?

A:Betfair, based in the United Kingdom, is the world's largest internet betting exchange. Rather than having a bookmaker create odds, the odds for every bet are determined by the market of bettors, working similarly to a stock market. Bettors can either "Back" (buy) or "Lay" (sell) a given bet at

certain odds, and the odds move as Backs and Lays are matched. Betfair is legal in the UK and other countries, but it is illegal to bet money on Betfair as a resident of the United States.

Q:What is Intrade, https://prev.intrade.com/v4/home/?

A:Intrade, based in Ireland, is a prediction market which allows individuals to trade contracts on whether future events will or will not occur. For any given contract, the value at expiration is either 100 (if the event happens) or 0 (if it does not). Contracts therefore trade between 0 and 100 at all times, with the price representing the market's prediction for the likelihood of that event. Intrade is legal in the Republic of Ireland and other countries, but it is illegal to bet money on Intrade as a resident of the United States.

Q:What is HuffPost Pollster, http://www.huffingtonpost.com/news/pollster/?

A:HuffPost Pollster, is a site that discusses and aggregates polling data. Polling data is subject to random fluctuations and Pollster's aggregation methods cleanly and transparently aggregate polls over time to provide a more meaningful snapshot of where the polls are at any given moment.

Q:What is PredictIt, https://www.predictit.org/?

A:PredictIt is an exchange of real-money prediction markets operated by the Victoria University and Aristotle. Because of the small sums wagered PredictIt has received no-action relief from the United States government, meaning U.S.-based speculators can legally risk upwards of $850 in any of the markets.

Q:What is BETDAQ, https://www.betdaq.com/Default.aspx?

A:BETDAQ, based in Ireland, is an internet betting exchange. Rather than having a bookmaker create odds, the odds for every bet are determined by the market of bettors, working similarly to a stock market. Bettors can either "Back" (buy) or "Lay" (sell) a given bet at certain odds, and the odds move as Backs and Lays are matched. BETDAQ is legal in Ireland and other

countries, but it is illegal to bet money on BETDAQ as a resident of the United States.

Q:What is the Hollywood Stock Exchange (HSX), http://www.hsx.com/?

A:The Hollywood Stock Exchange (HSX) is a play-money prediction market in which users can buy or sell shares in movies, actors, directors, and other Hollywood-related topics. For example, users can buy or sell shares of an upcoming film as a means predicting how well that film will do at the box office in its first four weekends of wide release, and then be ranked based on the accuracy of their predictions. Because HSX involves only simulated money, it is legal for all participants.

Q:What is Smarkets, https://smarkets.com/?

A:Smarkets, based in the United Kingdom, is an internet betting exchange. Rather than having a bookmaker create odds, the odds for every bet are determined by the market of bettors, working similarly to a stock market. Bettors can either bet "For" (buy) or "Against" (sell) a given bet at certain odds, and the odds move as Fors and Againsts are matched. Smarkets is legal in the UK and other countries, but it is illegal to bet money on Smarkets as a resident of the United States.