Part VI

Appendices

Appendix A

Elements of Probability and Statistics

You've already studied some probability and statistics, but chances are that you could use a bit of review, so we supply it here, with emphasis on ideas that we will use repeatedly. Be warned, however: this section is no substitute for a full introduction to probability and statistics, which you should have had already.

A.1 Populations: Random Variables, Distributions and Moments

A.1.1 Univariate

Consider an experiment with a set O of possible outcomes. A random variable Y is simply a mapping from O to the real numbers. For example, the experiment might be flipping a coin twice, in which case $O = \{(Heads, Heads), (Tails, Tails), (Heads, Tails), (Tails, Heads)\}$. We might define a random variable Y to be the number of heads observed in the two flips, in which case Y could assume three values, $y = 0$, $y = 1$ or $y = 2$.[1]

Discrete random variables, that is, random variables with **discrete probability distributions,** can assume only a countable number of values

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[1] Note that, in principle, we use capitals for random variables (Y) and small letters for their realizations (y). We will often neglect this formalism, however, as the meaning will be clear from context.

y_i, $i = 1, 2, ...$, each with positive probability p_i such that $\sum_i p_i = 1$. The probability distribution $f(y)$ assigns a probability p_i to each such value y_i . In the example at hand, Y is a discrete random variable, and $f(y) = 0.25$ for $y = 0$, $f(y) = 0.50$ for $y = 1$, $f(y) = 0.25$ for $y = 2$, and $f(y) = 0$ otherwise.

In contrast, **continuous random variables** can assume a continuous range of values, and the **probability density function** $f(y)$ is a non-negative continuous function such that the area under $f(y)$ between any points a and b is the probability that Y assumes a value between a and b.[2]

In what follows we will simply speak of a "distribution," $f(y)$. It will be clear from context whether we are in fact speaking of a discrete random variable with probability distribution $f(y)$ or a continuous random variable with probability density $f(y)$.

Moments provide important summaries of various aspects of distributions. Roughly speaking, moments are simply expectations of powers of random variables, and expectations of different powers convey different sorts of information. You are already familiar with two crucially important moments, the mean and variance. In what follows we'll consider the first four moments: mean, variance, skewness and kurtosis.[3]

The **mean**, or **expected value**, of a discrete random variable is a probability-weighted average of the values it can assume,[4]

$$E(y) = \sum_i p_i y_i.$$

Often we use the Greek letter μ to denote the mean, which measures the **location**, or **central tendency**, of y.

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[2] In addition, the total area under $f(y)$ must be 1.

[3] In principle, we could of course consider moments beyond the fourth, but in practice only the first four are typically examined.

[4] A similar formula holds in the continuous case.

The **variance** of y is its expected squared deviation from its mean,

$$var(y) = E(y - \mu)^2.$$

We use σ^2 to denote the variance, which measures the **dispersion, or scale**, of y around its mean.

Often we assess dispersion using the square root of the variance, which is called the **standard deviation**,

$$\sigma = std(y) = \sqrt{E(y - \mu)^2}.$$

The standard deviation is more easily interpreted than the variance, because it has the same units of measurement as y. That is, if y is measured in dollars (say), then so too is $std(y)$. $Var(y)$, in contrast, would be measured in rather hard-to-grasp units of "dollars squared".

The **skewness** of y is its expected cubed deviation from its mean (scaled by σ^3 for technical reasons),

$$S = \frac{E(y - \mu)^3}{\sigma^3}.$$

Skewness measures the amount of **asymmetry** in a distribution. The larger the absolute size of the skewness, the more asymmetric is the distribution. A large positive value indicates a long right tail, and a large negative value indicates a long left tail. A zero value indicates symmetry around the mean.

The **kurtosis** of y is the expected fourth power of the deviation of y from its mean (scaled by σ^4, again for technical reasons),

$$K = \frac{E(y - \mu)^4}{\sigma^4}.$$

Kurtosis measures the thickness of the tails of a distribution. A kurtosis above three indicates "fat tails" or **leptokurtosis**, relative to the **normal, or Gaussian distribution** that you studied earlier. Hence a kurtosis above three indicates that extreme events ("tail events") are more likely to occur than would be the case under normality.

A.1.2 Multivariate

Suppose now that instead of a single random variable Y, we have two random variables Y and X.[5] We can examine the distributions of Y or X in isolation, which are called **marginal distributions**. This is effectively what we've already studied. But now there's more: Y and X may be related and therefore move together in various ways, characterization of which requires a **joint distribution**. In the discrete case the joint distribution $f(y, x)$ gives the probability associated with each possible pair of y and x values, and in the continuous case the joint density $f(y, x)$ is such that the area in any region under it gives the probability of (y, x) falling in that region.

We can examine the moments of y or x in isolation, such as mean, variance, skewness and kurtosis. But again, now there's more: to help assess the dependence between y and x, we often examine a key moment of relevance in multivariate environments, the **covariance**. The covariance between y and x is simply the expected product of the deviations of y and x from their respective means,

$$cov(y, x) = E[(y_t - \mu_y)(x_t - \mu_x)].$$

A positive covariance means that y and x are positively related; that is, when y is above its mean x tends to be above its mean, and when y is below its mean x tends to be below its mean. Conversely, a negative covariance means that y and x are inversely related; that is, when y is below its mean x tends to be above its mean, and vice versa. The covariance can take any value in the real numbers.

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[5] We could of course consider more than two variables, but for pedagogical reasons we presently limit ourselves to two.

Frequently we convert the covariance to a **correlation** by standardizing by the product of σ_y and σ_x,

$$corr(y, x) = \frac{cov(y, x)}{\sigma_y \sigma_x}.$$

The correlation takes values in [-1, 1]. Note that covariance depends on units of measurement (e.g., dollars, cents, billions of dollars), but correlation does not. Hence correlation is more immediately interpretable, which is the reason for its popularity.

Note also that covariance and correlation measure only *linear* dependence; in particular, a zero covariance or correlation between y and x does not necessarily imply that y and x are independent. That is, they may be *non-linearly* related. If, however, two random variables are jointly *normally* distributed with zero covariance, then they are independent.

Our multivariate discussion has focused on the joint distribution $f(y, x)$. In various chapters we will also make heavy use of the **conditional distribution** $f(y|x)$, that is, the distribution of the random variable Y *conditional* upon $X = x$. **Conditional moments** are similarly important. In particular, the **conditional mean** and **conditional variance** play key roles in econometrics, in which attention often centers on the mean or variance of a series conditional upon the past.

A.2 Samples: Sample Moments

A.2.1 Univariate

Thus far we've reviewed aspects of known distributions of random variables, in **population**. Often, however, we have a **sample** of data drawn from an unknown population distribution f,

$$\{y_i\}_{i=1}^{N} \sim f(y),$$

and we want to learn from the sample about various aspects of f, such as its moments. To do so we use various **estimators**.[6] We can obtain estimators by replacing population expectations with sample averages, because the arithmetic average is the sample analog of the population expectation. Such "analog estimators" turn out to have good properties quite generally. The **sample mean** is simply the arithmetic average,

$$\bar{y} = \frac{1}{N}\sum_{i=1}^{N} y_i.$$

It provides an empirical measure of the location of y.

The **sample variance** is the average squared deviation from the sample mean,

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^{N}(y_i - \bar{y})^2}{N}.$$

It provides an empirical measure of the dispersion of y around its mean.

We commonly use a slightly different version of $\hat{\sigma}^2$, which corrects for the one degree of freedom used in the estimation of $\bar{y}$, thereby producing an unbiased estimator of σ^2,

$$s^2 = \frac{\sum_{i=1}^{N}(y_i - \bar{y})^2}{N-1}.$$

Similarly, the **sample standard deviation** is defined either as

$$\hat{\sigma} = \sqrt{\hat{\sigma}^2} = \sqrt{\frac{\sum_{i=1}^{N}(y_i - \bar{y})^2}{N}}$$

or

$$s = \sqrt{s^2} = \sqrt{\frac{\sum_{i=1}^{N}(y_i - \bar{y})^2}{N-1}}.$$

It provides an empirical measure of dispersion in the same units as y.

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[6] An estimator is an example of a **statistic**, or **sample statistic**, which is simply a function of the sample observations.

The **sample skewness** is

$$\hat{S} = \frac{\frac{1}{N}\sum_{i=1}^{N}(y_i - \bar{y})^3}{\hat{\sigma}^3}.$$

It provides an empirical measure of the amount of asymmetry in the distribution of y.

The **sample kurtosis** is

$$\hat{K} = \frac{\frac{1}{N}\sum_{i=1}^{N}(y_i - \bar{y})^4}{\hat{\sigma}^4}.$$

It provides an empirical measure of the fatness of the tails of the distribution of y relative to a normal distribution.

Many of the most famous and important statistical sampling distributions arise in the context of sample moments, and the normal distribution is the father of them all. In particular, the celebrated central limit theorem establishes that under quite general conditions the sample mean $\bar{y}$ will have a normal distribution as the sample size gets large. The χ^2 **distribution** arises from squared normal random variables, the t **distribution** arises from ratios of normal and χ^2 variables, and the F **distribution** arises from ratios of χ^2 variables. Because of the fundamental nature of the normal distribution as established by the central limit theorem, it has been studied intensively, a great deal is known about it, and a variety of powerful tools have been developed for use in conjunction with it.

Because of the fundamental nature of the normal distribution as established by the central limit theorem, it has been studied intensively, a great deal is known about it, and a variety of powerful tools have been developed for use in conjunction with it. Hence it is often of interest to assess whether the normal distribution governs a given sample of data. A simple strategy is to check various implications of normality, such as $S = 0$ and $K = 3$, via informal examination of $\hat{S}$ and $\hat{K}$. Alternatively and more formally, the

Jarque-Bera test (JB) effectively aggregates the information in the data about both skewness and kurtosis to produce an overall test of the hypothesis that $S = 0$ and $K = 3$, based upon $\hat{S}$ and $\hat{K}$. The test statistic is

$$JB = \frac{T}{6}\left(\hat{S}^2 + \frac{1}{4}(\hat{K} - 3)^2\right),$$

where T is the number of observations. Under the null hypothesis of *iid* Gaussian observations, the Jarque-Bera statistic is distributed in large samples as a χ^2 random variable with two degrees of freedom.[7]

A.2.2 Multivariate

We also have sample versions of moments of multivariate distributions. In particular, the **sample covariance** is

$$\widehat{cov}(y, x) = \frac{1}{N}\sum_{i=1}^{N}[(y_i - \bar{y})(x_i - \bar{x})],$$

and the **sample correlation** is

$$\widehat{corr}(y, x) = \frac{\widehat{cov}(y, x)}{\hat{\sigma}_y \hat{\sigma}_x}.$$

A.3 Finite-Sample and Asymptotic Sampling Distributions of the Sample Mean

Here we refresh your memory on the sampling distribution of the most important sample moment, the sample mean.

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[7] Other tests of conformity to the normal distribution exist and may of course be used, such as the Kolmogorov-Smirnov test. The Jarque-Bera test, however, has the convenient and intuitive decomposition into skewness and kurtosis components.

A.3.1 Exact Finite-Sample Results

In your earlier studies you learned about *statistical inference*, such as how to form confidence intervals for the population mean based on the sample mean, how to test hypotheses about the population mean, and so on. Here we partially refresh your memory.

Consider the benchmark case of Gaussian **simple random sampling**,

$$y_i \sim iid\, N(\mu, \sigma^2), i = 1, ..., N,$$

which corresponds to a special case of what we will later call the "full ideal conditions" for regression modeling. The sample mean $\bar{y}$ is the natural estimator of the population mean μ. In this case, as you learned earlier, $\bar{y}$ is unbiased, consistent, normally distributed with variance σ^2/N, and indeed the minimum variance unbiased (MVUE) estimator. We write

$$\bar{y} \sim N\left(\mu, \frac{\sigma^2}{N}\right),$$

or equivalently

$$\sqrt{N}(\bar{y} - \mu) \sim N(0, \sigma^2).$$

We construct exact finite-sample confidence intervals for μ as

$$\mu \in \left[\bar{y} \pm t_{1-\frac{\alpha}{2}}(N-1)\frac{s}{\sqrt{N}}\right] \; w.p. \; \alpha,$$

where $t_{1-\frac{\alpha}{2}}(N-1)$ is the $1 - \frac{\alpha}{2}$ percentile of a t distribution with $N-1$ degrees of freedom. Similarly, we construct exact finite-sample (likelihood ratio) hypothesis tests of $H_0 : \; \mu = \mu_0$ against the two-sided alternative $H_0 : \; \mu \neq \mu_0$ using

$$\frac{\bar{y} - \mu_0}{\frac{s}{\sqrt{N}}} \sim t_{1-\frac{\alpha}{2}}(N-1).$$

A.3.2 Approximate Asymptotic Results (Under Weaker Assumptions)

Much of statistical inference is linked to large-sample considerations, such as the law of large numbers and the central limit theorem, which you also studied earlier. Here we again refresh your memory.

Consider again a simple random sample, but without the normality assumption,

$$y_i \sim iid(\mu, \sigma^2), i = 1, ..., N.$$

Despite our dropping the normality assumption we still have that $\bar{y}$ is unbiased, consistent, **asymptotically** normally distributed with variance σ^2/N, and best linear unbiased (BLUE). We write,

$$\bar{y} \overset{a}{\sim} N\left(\mu, \frac{\sigma^2}{N}\right).$$

More precisely, as $T \to \infty$,

$$\sqrt{N}(\bar{y} - \mu) \to_d N(0, \sigma^2).$$

This result forms the basis for asymptotic inference. It is a Gaussian central limit theorem, and it also has a law of large numbers ($\bar{y} \to_p \mu$) imbedded within it.

We construct asymptotically-valid confidence intervals for μ as

$$\mu \in \left[\bar{y} \pm z_{1-\frac{\alpha}{2}} \frac{\hat{\sigma}}{\sqrt{N}}\right] \ w.p. \ \alpha,$$

where $z_{1-\frac{\alpha}{2}}$ is the $1 - \frac{\alpha}{2}$ percentile of a $N(0, 1)$ distribution. Similarly, we construct asymptotically-valid hypothesis tests of $H_0: \ \mu = \mu_0$ against the two-sided alternative $H_0 : \ \mu \neq \mu_0$ using

$$\frac{\bar{y} - \mu_0}{\frac{\hat{\sigma}}{\sqrt{N}}} \sim N(0, 1).$$

A.4 Exercises, Problems and Complements

1. (Interpreting distributions and densities)

   The Sharpe Pencil Company has a strict quality control monitoring program. As part of that program, it has determined that the distribution of the amount of graphite in each batch of one hundred pencil leads produced is continuous and uniform between one and two grams. That is, $f(y) = 1$ for y in [1, 2], and zero otherwise, where y is the graphite content per batch of one hundred leads.

   a. Is y a discrete or continuous random variable?

   b. Is $f(y)$ a probability distribution or a density?

   c. What is the probability that y is between 1 and 2? Between 1 and 1.3? Exactly equal to 1.67?

   d. For high-quality pencils, the desired graphite content per batch is 1.8 grams, with low variation across batches. With that in mind, discuss the nature of the density $f(y)$.

2. (Covariance and correlation)

   Suppose that the annual revenues of world's two top oil producers have a covariance of 1,735,492.

   a. Based on the covariance, the claim is made that the revenues are "very strongly positively related." Evaluate the claim.

   b. Suppose instead that, again based on the covariance, the claim is made that the revenues are "positively related." Evaluate the claim.

c. Suppose you learn that the revenues have a *correlation* of 0.93. In light of that new information, re-evaluate the claims in parts a and b above.

3. (Simulation)

   You will often need to simulate data from various models. The simplest model is the $iidN(\mu, \sigma^2)$ (Gaussian simple random sampling) model.

   a. Using a random number generator, simulate a sample of size 30 for y, where $y \sim iidN(0, 1)$.
   b. What is the sample mean? Sample standard deviation? Sample skewness? Sample kurtosis? Discuss.
   c. Form an appropriate 95 percent confidence interval for $E(y)$.
   d. Perform a t test of the hypothesis that $E(y) = 0$.
   e. Perform a t test of the hypothesis that $E(y) = 1$.

4. (Sample moments of the wage data)

   Use the 1995 wage dataset.

   a. Calculate the sample mean wage and test the hypothesis that it equals \$9/hour.
   b. Calculate sample skewness.
   c. Calculate and discuss the sample correlation between wage and years of education.

5. Notation.

   We have used standard cross-section notation: $i = 1, ..., N$. The standard time-series notation is $t = 1, ..., T$. Much of our discussion will be valid in *both* cross-section and time-series environments, but still we have to pick a notation. Without loss of generality, henceforth we will typically use $t = 1, ..., T$.

A.5 Notes

Numerous good introductory probability and statistics books exist. Wonnacott and Wonnacott (1990) remains a time-honored classic, which you may wish to consult to refresh your memory on statistical distributions, estimation and hypothesis testing. Anderson et al. (2008) is a well-written recent text.

Appendix B

Elements of Nonparametrics

B.1 Density Estimation

B.1.1 The Basic Problem

$$\{x_i\}_{i=1}^N \overset{iid}{\sim} f(x)$$

f smooth in $[x_0 - h, x_0 + h]$

Goal: Estimate $f(x)$ at arbitrary point $x = x_0$

By the mean-value theorem,

$$f(x_0) \approx \frac{1}{2h} \int_{x_0-h}^{x_0+h} f(u)du = \frac{1}{2h} P(x \in [x_0 - h, x_0 + h])$$

Estimate $P(x \in [x_0 - h,\ x_0 + h])$ by $\frac{\#x_i \in [x_0-h,\ x_0+h]}{N}$

$$\hat{f}_h(x_0) = \frac{1}{2h}\ \frac{\#x_i \in [x_0 - h, x_0 + h]}{N}$$

$$= \frac{1}{Nh}\ \sum_{i=1}^{N} \frac{1}{2}\ I\left(\left|\frac{x_0 - x_i}{h}\right| \leq 1\right)$$

"Rosenblatt estimator"

Kernel density estimator with

kernel: $K(u) = \frac{1}{2}I(|u| \leq 1)$

bandwidth: h

B.1.2 Kernel Density Estimation

Issues with uniform kernels:

1. Why weight distant observations as heavily as nearby ones?

2. Why use a discontinuous kernel if we think that f is smooth?

Obvious solution: Choose *smooth* kernel

Standard conditions:

$\int K(u)du = 1$

$K(u) = K(-u)$

Common Kernel Choices

Standard normal: $K(u) = \frac{1}{\sqrt{2\pi}}e^{-\frac{u}{2}^2}$

Triangular $K(u) = (1 - |u|)I(|u| \leq 1)$

Epinechnikov: $K(u) = \frac{3}{4}(1 - u^2)I(|u| \leq 1)$

General Form of the Kernel Density Estimator

$$\hat{f}_h(x_0) = \frac{1}{Nh}\sum_{i=1}^{N} K\left(\frac{x_0 - x_i}{h}\right)$$

"Rosenblatt-Parzen estimator"

Figure B.1: Bandwidth Choice – from Silverman (1986)

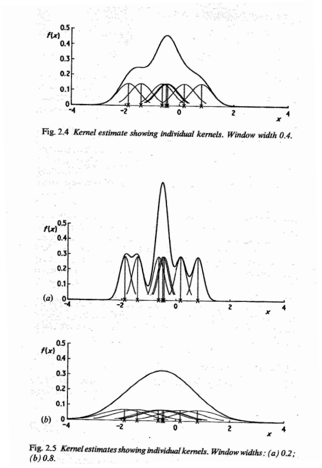

Fig. 2.4 Kernel estimate showing individual kernels. Window width 0.4.

Fig. 2.5 Kernel estimates showing individual kernels. Window widths: (a) 0.2; (b) 0.8.

B.1.3 Bias-Variance Tradeoffs

Inescapable Bias-Variance Tradeoff (in Practice, Fixed N)

Escapable Bias-Variance Tradeoff (in Theory, $N \to \infty$)

$$E(\hat{f}_h(x_0)) \approx f(x_0) + \frac{h^2}{2} \cdot O_p(1)$$

$$(So\ h \to 0 \implies bias \to 0)$$

$$var\ (\hat{f}_h(x_0)) \approx \frac{1}{Nh} \cdot O_p(1)$$

$$(So\ Nh \to \infty \implies var \to 0)$$

Thus,

$$\left.\begin{array}{c} h \to 0 \\ Nh \to \infty \end{array}\right\} \implies \hat{f}_h(x_0) \overset{p}{\to} f(x_0)$$

Convergence Rate

$$\sqrt{Nh}(\hat{f}_h(x_0) - f(x_0)) \overset{d}{\to} D$$

Effects of K minor; effects of h major.

B.1.4 Optimal Bandwidth Choice

$MSE\left(\hat{f}_h(x_0)\right) = E\Big(\hat{f}_h(x_0) - f(x_o)\Big)^2$

$IMSE \;= \int MSE\left(\hat{f}_h(x_0)\right) f(x)\, dx$

Choose bandwidth to minimize IMSE:

$h^* = \gamma^* N^{-1/5}$

Corresponding Optimal Convergence Rate

Recall:

$$\sqrt{Nh}\left(\hat{f}_h(x_0) - f(x_0)\right) \overset{d}{\to} D$$

$$h^* \propto N^{-1/5}$$

Substituting yields the best obtainable rate:

$$\sqrt{N^{4/5}}\left(\hat{f}_h(x_0) - f(x_0)\right) \overset{d}{\to} D$$

"Stone optimal rate"

Silverman's Rule

For the Gaussian case,

$h^* = 1.06\sigma N^{-1/5}$

So use:

$\hat{h}^* = 1.06\hat{\sigma} N^{-1/5}$

Better to err on the side of too little smoothing:

$\hat{h}^* = \hat{\sigma} N^{-1/5}$

B.2 Multivariate

Earlier univariate kernel density estimator:

$$\hat{f}_h(x_0) = \frac{1}{Nh}\sum_{i=1}^{N} K\left(\frac{x_0 - x_i}{h}\right)$$

Can be written as:

$$\hat{f}_h(x_0) = \frac{1}{N}\sum_{i=1}^{N} K_h(x_0 - x_i)$$

where $K_h(\cdot) = \frac{1}{h}K(\frac{\cdot}{h})$

or $K_h(\cdot) = h^{-1}K(h^{-1}\cdot)$
Multivariate Version (d-Dimensional)
Precisely follows equation (B.2):

$$\hat{f}_H(x_0) = \frac{1}{N}\sum_{i=1}^{N} K_H(x_0 - x_i),$$

where $K_H(\cdot) = |H|^{-1}K(H^{-1}\cdot),$ and H $(d \times d)$ is psd.

Common choice: $K(u) = N(0, I),\ H = hI$

$$\implies K_H(\cdot) = \frac{1}{h^d}K\left(\frac{1}{h}\cdot\right) = \frac{1}{h^d}K\left(\frac{x_0 - x_i}{h}\right)$$

$$\implies \hat{f}_h(x_0) = \frac{1}{Nh^d}\sum_{i=1}^{N} K\left(\frac{x_0 - x_i}{h}\right)$$

Bias-Variance Tradeoff, Convergence Rate, Optimal Bandwidth, Correpsonding Optimal Convergence Rate

$$\left.\begin{array}{c} h \to 0 \\ Nh^d \to \infty \end{array}\right\} \implies \hat{f}_h(x_0) \overset{p}{\to} f(x_0)$$

$$\sqrt{Nh^d}\left(\hat{f}_h(x_0) - f(x_0)\right) \overset{d}{\to} D$$

$$h^* \propto N^{-\frac{1}{d+4}}$$

$$\sqrt{N^{1-\frac{d}{d+4}}}\left(\hat{f}_h(x_0) - f(x_0)\right) \overset{d}{\to} D$$

Stone-optimal rate drops with d

"Curse of dimensionality"
Silverman's Rule

$$\hat{h}^* = \left(\frac{4}{d+2}\right)^{\frac{1}{d+4}} \hat{\sigma} N^{-\frac{1}{d+4}}$$

where

$$\hat{\sigma}^2 = \frac{1}{d}\sum_{i=1}^{d} \hat{\sigma}_i^2$$

(average sample variance)

B.3 Functional Estimation

Conditional Mean (Regression)

$$E(y|x) = M(x) = \int y \frac{f(y,x)}{f(x)} dy$$

Regression Slope

$$\beta(x) = \frac{\partial M(x)}{\partial x_j} = \lim_{h \to 0} \frac{(M(x+\frac{h}{2}) - M(x-\frac{h}{2}))}{h}$$

Regression Disturbance Density

$$f(u),\ u = y - M(x)$$

Conditional Variance

$$var(y|x) = V(x) = \int y^2 \frac{f(y,x)}{f(x)} dy - M(x)^2$$

Hazard Function

$$\lambda(t) = \frac{f(t)}{1 - F(t)}$$

Curvature (Higher-Order Derivative Estimation)

$$C(x) = \frac{\partial}{\partial\ x_j}\ \beta(x) = (\frac{\partial^2}{\partial\ {x_j}^2})M(x) = \begin{matrix} \lim \\ h \to 0 \end{matrix}\ \frac{\beta(x+\frac{h}{2}) -\ \beta(x-\frac{h}{2})}{h}$$

The curse of dimensionality is much worse for curvature...

d-vector: $r = (r_1, ..., r_d),\ |r| =\ \Sigma_{i=1}^{d}\ r_i$

Define $M^{(r)}(x) \equiv \partial^{\frac{|r|}{\partial^{r_1}x_1,\ldots,\partial^{r_d}x_d}}\ M(x)$

Then $\sqrt{Nh}^{2|r|+d}\ [\hat{M}^{(r)}(x_0) - M^{(r)}(x_0)] \to_d D$

B.4 Local Nonparametric Regression

B.4.1 Kernel Regression

$$M(x_0) = \int y f(y|x_0) dy = \int y \frac{f(x_0, y)}{f(x_0)} dy$$

Using multivariate kernel density estimates and manipulating gives the "Nadaraya-Watson" estimator:

$$\hat{M}_h(x_0) = \sum_{i=1}^{N} \left[\frac{K\left(\frac{x_0 - x_i}{h}\right)}{\sum_{i=1}^{N} K\left(\frac{x_0 - x_i}{h}\right)} \right] y_i$$

$$h \to 0,\ Nh \to \infty \implies$$

$$\sqrt{Nh^d}\ (\hat{M}_h(x_0) - M(x_0)) \overset{d}{\to} N(0, V)$$

B.4.2 Nearest-Neighbor Regression

Basic Nearest-Neighbor Regression

$\hat{M}_k(x_0)\ =\ \frac{1}{k}\ \sum_{i \in n(x_0)}\ y_i$ (Locally Constant, uniform weighting)

$k \to \infty,\ \frac{k}{N} \to 0 \ \ \Rightarrow\ \ \hat{M}_k\ (x_0) \overset{P}{\to} M(x_0)$

$\sqrt{k}\ (\hat{M}_k(x_0) - M(x_0)) \overset{d}{\to} D$

Equivalent to Nadaraya-Watson kernel regression with:

$K(u)\ =\ \frac{1}{2}\ I(|u| \leq 1)$ (uniform) and $h = R(k)$ (distance from x_0 to k^{th} nearest neighbor)

$\Rightarrow$ Variable bandwidth!

Locally-Weighted Nearest-Neighbor Regression (Locally Polynomial, Non-Uniform Weighting)

$$y_t \;=\; g(x_t) \;+\; \varepsilon_t$$

Computation of $\hat{g}(x^*)$:

$$0 \;<\; \xi \;\leq\; 1$$

$$k_T \;=\; int(\xi \cdot T)$$

Find K_T nearest neighbors using norm:

$$\lambda(x^*,\; x^*_{k_T}) \;=\; [\Sigma_{j=1}^{P}(x^*_{k_T\;j} - x^*_j)^2]^{\frac{1}{2}}$$

Neighborhood weight function:

$$v_t(x_t,\; x^*,\; x^*_{k_T}) \;=\; C\left(\frac{\lambda(x_t,\; x^*)}{\lambda(x^*,\; x^*_{k_T})}\right)$$

$$C(u) \;=\; \begin{cases} (1-u^3)^3 & for\ u<1 \\ 0 & otherwise \end{cases}$$

B.5 Global Nonparametric Regression

B.5.1 Series (Sieve, Projection, ...)

$$M(x_0) \;=\; \Sigma_{j=0}^{\infty}\, \beta_j\; \phi_j\; (x_0)$$

(the ϕ_j are orthogonal basis functions)

$$\hat{M}_J(x_0) \;=\; \Sigma_{j=0}^{J}\, \hat{\beta}_j\; \phi_j(x_0)$$

$$J \to \infty,\; \tfrac{J}{N} \to 0 \;\;\Rightarrow\;\; \hat{M}_J(x_0) \overset{P}{\to} M(x_0)$$

Stone-optimal convergence rate, for suitable choice of J.

B.5.2 Neural Networks

Run linear combinations of inputs through "squashing functions" $i = 1, ..., R$ inputs, $j =$ $1, ..., S$ neurons

$h_{jt} = \Psi(\gamma_{jo} + \Sigma_{i=1}^{R} \gamma_{ij} x_{it}), \; j = 1, ..., S \; (Neuron \; j)$
e.g. $\Psi(\cdot)$ can be logistic (regression), 0-1 (classification)

$O_t = \Phi(\beta_0 + \Sigma_{j=1}^{S} \beta_j h_{jt})$
e.g. $\Phi(\cdot)$ can be the identity function

Compactly: $O_t = \Phi(\beta_0 + \Sigma_{j=1}^{S}\beta_j\Psi(\gamma_{jo} + \Sigma_{i=1}^{R}\gamma_{ij}x_{it}) \equiv f(x_t;\theta)$

Universal Approximator: $S \to \infty, \; \frac{S}{N} \to 0 \; \Rightarrow \; \hat{O}(x_0) \to_p O(x_0)$

Same as other nonparametric methods.

B.5.3 More

Ace, projection pursuit, regression splines, smoothing splines, CART,

B.6 Time Series Aspects

1. Many results go through under mixing or Markov conditions.

2. Recursive kernel regression.

   Use recursive kernel estimator:

   $$\hat{f}_N(x_0) = (\tfrac{N-1}{N})f_{N-1}(x_0) + \tfrac{1}{Nh^d}K(\tfrac{x_0-x_N}{h})$$

   to get:

   $$\hat{M}_N(x_0) = \frac{(N-1)h^d\hat{f}_{N-1}(x_0)\hat{M}_{N-1}(x_0) + Y_N K(\frac{x_0-x_N}{h})}{(N-1)h^d\hat{f}_{N-1}(x_0) + K(\frac{x_0-x_N}{h})}$$

3. Bandwidth selection via recursive prediction.

4. Nonparametric nonlinear autoregression.

   $y_t \;=\; g(y_{t-1}, ..., y_{t-p}) \;+\; \varepsilon_t$
   $E(y_{t+1} \mid y_t, ..., y_{t-p+1}) = \; \int y_{t+1} \; f(y_{t+1} \mid y_t, ..., y_{t-p+1}) \; dy$
   $= \; \int y_{t+1} \frac{f(y_{t+1}, ..., y_{t-p+1})}{f(y_t, ..., y_{t-p+1})} \; dy$

   Implementation: Kernel, Series, NN, LWR

5. Recurrent neural nets.

   $h_{jt} \;=\; \Psi(\gamma_{jo} \;+\; \Sigma_{i=1}^{R} \gamma_{ij} x_{it} \;+\; \Sigma_{l=1}^{S} \delta_{jl} h_{l,\; t-1}), \; j \;=\; 1, \;..., \; S$

   $O_t \;=\; \Phi(\beta_0 \;+\; \Sigma_{j=1}^{S} \beta_j h_{jt})$

   Compactly: $O_t \;=\; \Phi(\beta_0 \;+\; \Sigma_{j=1}^{S} \beta_j \Psi \, (\gamma_{j0} \;+\; \Sigma_{i=1}^{R} \gamma_{ij} x_i t \;+\; \Sigma_{l=1}^{S} \delta_{jl} h_{l,\; t-1})$

   Back substitution:
   $O_t \;=\; g(x_t, x_{t-1}, ..., x_1; \theta)$

B.7 Exercises, Problems and Complements

1. Tightly parametric models are often best for time-series prediction.

   Generality isn't so great; restrictions often help!

2. Semiparametric and related approaches.

   $\sqrt{N}$ consistent estimation. Adaptive estimation.

B.8 Notes

Appendix C

“Problems and Complements” Data

Here we provide data for the in-chapter examples as well as end-of-chapter EPC’s. The data are also available on the web.

C.1 Liquor Sales

480 467 514 505 534 546 539 541 551 537 584 854 522 506 558 538 605 583 607 624 570 609 675 861 605 537 575 588 656 623 661 668 603 639 669 915 643 563 616 645 703 684 731 722 678 713 725 989 687 629 687 706 754 774 825 755 751 783 804 1139 711 693 790 754 799 824 854 810 798 807 832 1142 740 713 791 768 846 884 886 878 813 840 884 1245 796 750 834 838 902 895 962 990 882 936 997 1305 866 805 905 873 1024 985 1049 1034 951 1010 1016 1378 915 854 922 965 1014 1040 1137 1026 992 1052 1056 1469 916 934 987 1018 1048 1086 1144 1077 1036 1076 1114 1595 949 930 1045 1015 1091 1142 1182 1161 1145 1119 1189 1662 1048 1019 1129 1092 1176 1297 1322 1330 1263 1250 1341 1927 1271 1238 1283 1283 1413 1371 1425 1453 1311 1387 1454 1993 1328 1250 1308 1350 1455 1442 1530 1505 1421 1485 1465 2163 1361 1284 1392 1442 1504 1488 1606 1488 1442 1495 1509 2135 1369 1320 1448 1495 1522 1575 1666 1617 1567 1551 1624 2367 1377 1294 1401 1362 1466 1559 1569 1575 1456 1487 1549 2178 1423 1312 1465 1488 1577 1591 1669 1697 1659 1597 1728 2326 1529 1395 1567 1536 1682 1675 1758 1708

1561 1643 1635 2240 1485 1376 1459 1526 1659 1623 1731 1662 1589 1683
1672 2361 1480 1385 1505 1576 1649 1684 1748 1642 1571 1567 1637 2397
1483 1390 1562 1573 1718 1752 1809 1759 1698 1643 1718 2399 1551 1497
1697 1672 1805 1903 1928 1963 1807 1843 1950 2736 1798 1700 1901 1820
1982 1957 2076 2107 1799 1854 1968 2364 1662 1681 1725 1796 1938 1871
2001 1934 1825 1930 1867 2553 1624 1533 1676 1706 1781 1772 1922 1743
1669 1713 1733 2369 1491 1445 1643 1683 1751 1774 1893 1776 1743 1728
1769 2431

C.2 Housing Starts and Completions

"OBS" "STARTS" "COMPS"
"1968M01" 1.38 1.257
"1968M02" 1.52 1.174
"1968M03" 1.466 1.323
"1968M04" 1.554 1.328
"1968M05" 1.408 1.367
"1968M06" 1.405 1.184
"1968M07" 1.512 1.37
"1968M08" 1.495 1.279
"1968M09" 1.556 1.397
"1968M10" 1.569 1.348
"1968M11" 1.63 1.367
"1968M12" 1.548 1.39
"1969M01" 1.769 1.257
"1969M02" 1.705 1.414
"1969M03" 1.561 1.558
"1969M04" 1.524 1.318
"1969M05" 1.583 1.43

”1969M06” 1.528 1.455

”1969M07” 1.368 1.432

”1969M08” 1.358 1.393

”1969M09” 1.507 1.367

”1969M10” 1.381 1.406

”1969M11” 1.229 1.404

”1969M12” 1.327 1.402

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”1970M02” 1.305 1.43

”1970M03” 1.319 1.317

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”1970M09” 1.534 1.383

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”1970M11” 1.647 1.457

”1970M12” 1.893 1.437

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”1971M06” 2.026 1.637

”1971M07” 2.083 1.699

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”1972M03” 2.334 1.981
”1972M04” 2.249 1.97
”1972M05” 2.221 1.896
”1972M06” 2.254 1.936
”1972M07” 2.252 1.93
”1972M08” 2.382 2.102
”1972M09” 2.481 2.053
”1972M10” 2.485 1.995
”1972M11” 2.421 1.985
”1972M12” 2.366 2.121
”1973M01” 2.481 2.162
”1973M02” 2.289 2.124
”1973M03” 2.365 2.196
”1973M04” 2.084 2.195
”1973M05” 2.266 2.299
”1973M06” 2.067 2.258
”1973M07” 2.123 2.066
”1973M08” 2.051 2.056
”1973M09” 1.874 2.061
”1973M10” 1.677 2.052
”1973M11” 1.724 1.925
”1973M12” 1.526 1.869
”1974M01” 1.451 1.932
”1974M02” 1.752 1.938
”1974M03” 1.555 1.806

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”1974M06” 1.513 1.897

”1974M07” 1.316 1.695

”1974M08” 1.142 1.634

”1974M09” 1.15 1.651

”1974M10” 1.07 1.63

”1974M11” 1.026 1.59

”1974M12” 0.975 1.54

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”1975M02” 0.904 1.346

”1975M03” 0.993 1.293

”1975M04” 1.005 1.278

”1975M05” 1.121 1.349

”1975M06” 1.087 1.234

”1975M07” 1.226 1.276

”1975M08” 1.26 1.29

”1975M09” 1.264 1.333

”1975M10” 1.344 1.134

”1975M11” 1.36 1.383

”1975M12” 1.321 1.306

”1976M01” 1.367 1.258

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”1976M03” 1.421 1.347

”1976M04” 1.395 1.332

”1976M05” 1.459 1.44

”1976M06” 1.495 1.39

”1976M07” 1.401 1.322

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”1976M09” 1.72 1.371

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”1976M11” 1.641 1.428

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”1977M01” 1.527 1.457

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”1977M05” 1.971 1.555

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”1978M07” 2.092 1.957

”1978M08” 1.996 1.976

”1978M09” 1.97 1.944

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”1980M08” 1.436 1.44

”1980M09” 1.471 1.267

”1980M10” 1.523 1.272

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”1981M04” 1.36 1.469

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”1981M07” 1.041 1.337

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”1981M09” 0.911 1.221

”1981M10” 0.873 1.206

”1981M11” 0.837 1.074

”1981M12” 0.91 1.129

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”1982M06” 0.902 0.93

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”1983M06” 1.733 1.344

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”1983M10” 1.715 1.586

”1983M11” 1.785 1.462

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”1984M05” 1.774 1.732

”1984M06” 1.843 1.714

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”1984M08” 1.586 1.685

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”1985M09” 1.676 1.806

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”1985M11” 1.698 1.749

”1985M12” 1.942 1.732

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”1986M07” 1.782 1.722

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”1986M09” 1.687 1.732

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”1987M09” 1.695 1.587

”1987M10” 1.515 1.577

”1987M11” 1.656 1.578

”1987M12” 1.4 1.632

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”1988M02” 1.473 1.45

”1988M03” 1.532 1.6

”1988M04” 1.573 1.615

”1988M05” 1.421 1.483

”1988M06” 1.478 1.512

”1988M07” 1.467 1.527

”1988M08” 1.493 1.551

”1988M09” 1.492 1.531

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”1988M11” 1.569 1.407
”1988M12” 1.563 1.547
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”1989M03” 1.422 1.442
”1989M04” 1.339 1.542
”1989M05” 1.331 1.449
”1989M06” 1.397 1.346
”1989M07” 1.427 1.386
”1989M08” 1.332 1.429
”1989M09” 1.279 1.338
”1989M10” 1.41 1.333
”1989M11” 1.351 1.475
”1989M12” 1.251 1.304
”1990M01” 1.551 1.508
”1990M02” 1.437 1.352
”1990M03” 1.289 1.345
”1990M04” 1.248 1.332
”1990M05” 1.212 1.351
”1990M06” 1.177 1.263
”1990M07” 1.171 1.295
”1990M08” 1.115 1.307
”1990M09” 1.11 1.312
”1990M10” 1.014 1.282
”1990M11” 1.145 1.248
”1990M12” 0.969 1.173
”1991M01” 0.798 1.149
”1991M02” 0.965 1.09
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”1991M03” 0.921 1.176
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”1991M05” 0.996 1.07
”1991M06” 1.036 1.093
”1991M07” 1.063 1.076
”1991M08” 1.049 1.05
”1991M09” 1.015 1.216
”1991M10” 1.079 1.076
”1991M11” 1.103 1.013
”1991M12” 1.079 1.002
”1992M01” 1.176 1.061
”1992M02” 1.25 1.098
”1992M03” 1.297 1.128
”1992M04” 1.099 1.083
”1992M05” 1.214 1.187
”1992M06” 1.145 1.189
”1992M07” 1.139 1.251
”1992M08” 1.226 1.14
”1992M09” 1.186 1.123
”1992M10” 1.244 1.139
”1992M11” 1.214 1.224
”1992M12” 1.227 1.199
”1993M01” 1.21 1.135
”1993M02” 1.21 1.236
”1993M03” 1.083 1.105
”1993M04” 1.258 1.216
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”1993M09” 1.343 1.172

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”1995M04” 1.278 1.331

”1995M05” 1.3 1.324

”1995M06” 1.301 1.256

”1995M07” 1.45 1.332

”1995M08” 1.401 1.247

”1995M09” 1.401 1.267

”1995M10” 1.351 1.32

”1995M11” 1.458 1.36

”1995M12” 1.425 1.225

"1996M01" 1.453 1.403
"1996M02" 1.514 1.328
"1996M03" 1.439 1.391
"1996M04" 1.511 1.35
"1996M05" 1.478 1.392
"1996M06" 1.474 1.398

C.3 Shipping Volume

"VOL" "VOLJ" "VOLQ"
19.2717057789 17.459748181 18.7609251809
19.5739427053 17.0051823285 18.971430836
20.2496352454 20.0632864458 21.5160397659
18.7581267693 19.0300416396 22.511024212
18.9623879164 19.27406249 23.6257746082
18.7082264913 17.7225435923 18.9527794473
17.5583325839 16.3996071649 16.3155079075
16.200570162 16.0688532171 16.4200268795
17.5672224715 15.9365700733 16.7922075809
18.3506389645 15.174656225 17.2723634089
19.6108588322 17.399682921 18.8658616044
19.0548224273 18.3899433918 17.5524349924
17.8562732579 17.3099553279 17.59936768
17.3026348251 15.7391009507 17.3483112881
16.992232973 16.2263804308 16.1946474378
16.6783874199 15.0494683232 15.8035069624
17.440059836 14.8752473335 16.715966412
16.6618026428 17.0961955995 17.5161819485
16.384313619 16.7257725533 17.0938652092

16.050331812 13.739513488 15.0166120316

15.7184746166 13.4520789836 14.2574702548

15.849494067 15.2452098373 16.1171312944

15.2144285697 13.7662941367 14.7161769243

15.5820599759 13.2800857116 13.5803587289

16.504876926 14.5873238183 15.4941943822

16.3726266283 14.752659297 15.6499708269

17.441672857 16.3451653899 17.4300040172

18.1500104973 17.4648816444 18.1790757434

18.874365366 18.1946450784 17.9539480087

18.1098021573 16.5289387474 17.0578268847

18.6816660898 16.9546621479 17.4720840136

18.246280095 16.8771274629 17.5725869706

18.0012782954 17.1811808451 17.7155012812

19.587794813 17.494746818 20.0703911986

19.5981770221 17.217759479 21.6789128429

19.2223298359 17.8218538169 19.8239398072

20.0634140058 18.5751902922 18.3789688332

20.0809239368 20.6290468968 20.7880586038

19.1786299632 19.0280437604 21.0085727009

19.1588286054 17.0601921473 19.933106815

19.3928968784 18.122574268 19.2421396264

18.9646978349 17.3292945255 18.511792914

20.435792902 19.6739965193 20.1347179524

20.4202833337 20.4439466979 20.4893669879

20.9052188136 19.7398566084 20.1878793077

19.6652673577 18.6546627952 18.249475265

20.0951191985 19.4590133306 20.6924024332

20.7800095041 19.0475471902 20.5414206421

21.2965069366 18.9291269898 20.0406867042

20.8192028548 20.769493656 20.7077992512

21.0614028758 20.9171131275 19.9587141065

20.9765403357 18.7027400646 19.2220417591

19.1698867079 17.6112329857 17.6748160012

18.9439652669 17.8949795526 18.0725569547

19.8093280201 19.4476632819 19.1307345563

19.5802661175 20.7283882343 19.7635124256

20.188836761 22.031446338 21.3858459329

18.1792990372 19.8472107672 19.1444759791

17.7548507547 18.72925784 18.6182439677

17.2289318147 16.8165314785 17.7292985173

16.4121068243 15.5654085445 15.9577922676

16.2045884936 16.3836619494 15.7260238369

15.0130253194 15.3710289573 16.7921314762

13.8266097099 13.052907295 14.9949652982

13.1843688204 11.6840841891 12.0034276402

12.870213406 11.6770775998 12.9172510431

13.2334442709 11.5479612775 14.9634674363

12.9173329865 11.5577104018 13.6367828064

12.7117169428 11.3733253536 11.787848607

14.1808084522 13.3665634393 12.8296343543

12.9484589444 12.878562356 12.4906568078

13.6214661551 14.483489407 16.3731037573

14.7098312316 15.4407091256 17.3526214658

14.4560809397 13.8277648613 14.4358762589

14.8227523736 13.6335269982 12.5340732824

16.5007559885 15.7312209525 15.0489415034

17.5106649474 17.840872251 17.7325640302

17.6557029729 18.2473296622 16.7282353463
17.8485823627 17.6195057968 16.7266919875
19.67633947 18.3353228515 19.0679266201
20.277734492 19.3934423648 20.0492502848
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C.4 Hungarian Exchange Rate

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4.0659293 3.0434249 2.0164477 2.8522073 2.8140498 2.1848722 1.5950817 2.2429898
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4.1155835 2.6670804 2.4475717 2.205739 2.4292855 2.0911023 2.0898105 3.043442
3.6113511 3.7893799 3.2121155 3.1678467 3.2550351 2.9450505 2.7632934 2.9777748
3.7541152 2.3789054 1.5524019 2.4166115 2.8760458 2.6712716 2.9638433 2.3101149
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2.6977919 2.340162 2.2215253 2.5238235 1.9671895 2.1577204 2.7455625 2.8270665
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4.7105699 5.4896383 7.1584 6.805053 8.6144752 5.9383055 7.7796817 8.7711985
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21.102066 22.330443 23.778787 22.421849 24.318354 23.276223 22.928615 23.516817
23.192031 24.439776 25.646361 24.013173 22.800817 21.930868 22.396108 21.948636
22.63152 22.843553 21.965477 22.086954 21.484588 20.992311 22.040551 20.708479
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19.324093 19.563313 20.169274 21.064913 21.383854 21.115485 21.041537 20.447841
19.167268 19.282245 19.669213 19.842788 19.645848 19.393213 20.050257 20.285704
21.206103 21.741322 22.684601 21.714409 21.517625 22.221479 23.531033 22.094984
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22.746644 22.104567 23.903777 23.660177 23.246803 23.582306 23.02254 23.442928
23.342003 23.239028 22.862249 22.922168 22.296407 22.855469 22.513571 22.223672
21.680807 21.434534 22.321791 22.649076 21.389639 20.758869 20.44828 19.735225
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18.781568 18.886111 19.397855 19.525678 19.275408 19.967104 18.91759 20.965193
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17.070625 17.208007 16.974044 16.865031 16.961967 17.3944 17.143105 16.718039
16.753252 16.844759 16.818118 17.125553 18.288319 19.027563 18.176753 17.891172
16.673228 15.974902 16.879096 18.164183 20.231471 19.295101 20.248109 19.621391
21.030168 22.29068 21.865809 19.379118 19.267991 17.739786 16.340927 15.767129
13.917159 16.121872 15.741244 14.635771 14.533094 12.590452 14.162835 13.254253
13.759789 13.531156 11.736316 10.540666 10.701331 11.714493 12.102604 12.890286
11.972669 13.447575 15.760853 17.270281 15.322816

C.5 Eurostar

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102.800003052 93.0999984741 94.1999969482 81.4000015259 57.4000015259
52.5 59.0999984741 73.8000030518 99.6999969482 97.6999969482 103.400001526
103.5 94.6999969482 86.5 101.800003052 75.5999984741 65.5999984741

C.6 BankWire Transfers

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13.3202588 14.9058295 13.3445574 13.60132 13.9392483 13.8055779 14.7512005
15.7884112 14.425972 12.1438859 12.1084447 11.6292785 9.7112687 8.8009283
8.5336967 7.4968967 6.1815601 6.3582354 5.0254212 5.8837991 7.6623125 8.086742
8.2718261 7.6887475 6.556665 6.8305189 8.3272832 10.3902244 11.1315264
11.8735433 14.2927949 17.5727407 18.2033083 20.0942024 20.7989315 19.7259136
19.7014543 19.6978237 20.601377 21.5619129 20.0131328 15.9583137 14.9364171
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12.8937596 13.3888684 12.6286618 12.6561732 13.979903 12.8926098 11.0750817
10.5342009 11.1250268 10.6263291 9.7332819 10.2946824 9.5062875 10.1611082
11.7902245 11.7399657 12.2417721 14.0038044 15.0152511 16.0313511 16.53824
16.5422309

C.7 Nile.com Hits

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15791 11032 11552 15616 10698 13013 11990 18108 13341 19639 14734 10308
20065 15601 12745 14778 14227 16321 11750 12596 11046 8203 21149 15019
13109 15456 17693 16824 13117 11156 15489 18109 17760 20384 11889 12650
18174 13942 16485 16015 15010 11684 16182 9811 18900 16397 20547 21057
14467 9365 19399 19388 14776 12164 10494 16762 12231 17009 16362 23383
17742 18326 16453 15082 13735 13893 11698 13851 15218 14424 17427 15253

15230 20236 14149 18682 18458 20022 15808 20427 19109 14244 17348 19860
17013 16165 11351 16602 17804 19386 14606 15158 20604 15041 21182 14643
21980 15930 13342 18783 18262 20398 16426 18919 16314 15636 11820 38742
55050 45774 22393 16737 21300 13452 15563 17914 22325 19595 20574 18968
23808 23364 21628 18773 16804 15599 18642 20220 22484 18273 14450 23979
18250 21754 18832 19441 18701 21359 18468 22955 21661 19033 18164 22093
19848 20138 18353 20090 16290 18583 25099 21317 20996 20529 19307 19044
20879 17008 23621 15661 23771 24859 17587 14257 13307 21755 26337 11135
11589 14550 23208 19635 19707 22167 21662 16799 16689 21876 17366 22486
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26188 21825 22849 25099 19081 19485 24610 24377 24091 23478 23282 24906
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C.8 Thompson Energy Investors

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-2.07931297888 -2.82223513021 -2.63656651073 -1.37640273576
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-0.081508050013 -1.6685618537 -1.36252335414 -1.36854847243
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Appendix D

Some Pop and "Cross-Over" Books and Sites Worth Examining

Lewis (2003) [Michael Lewis, *Moneyball*]. Appearances may lie, but the numbers don't, so pay attention to the numbers.

Silver (2012) [Nate Silver, *The Signal and the Noise*]. Entertaining general investigation of forecasting's successes and failures in a variety of disciplines (including in baseball, speaking of *Moneyball*), with an eye toward extracting general principles for what makes a good forecaster.

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Appendix E

Construction of the Wage Datasets

We construct our datasets from randomly sampling the much-larger Current Population Survey (CPS) datasets.[1]

We extract the data from the March CPS for 1995, 2004 and 2012 respectively, using the National Bureau of Economic Research (NBER) front end (`http://www.nber.org/data/cps.html`) and NBER SAS, SPSS, and Stata data definition file statements (`http://www.nber.org/data/cps_progs.html`). We use both personal and family records.

We summarize certain of our selection criteria in Table ??. As indicated, the variable names change slightly in 2004 and 2012 relative to 1995. We focus our discussion on 1995.

CPS Personal Data Selection Criteria

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[1] See `http://aspe.hhs.gov/hsp/06/catalog-ai-an-na/cps.htm` for a brief and clear introduction to the CPS datasets.

Variable	Name (95)	Name (04,12)	Selection Criteria
Age	PEAGE	A_AGE	18-65
Labor force status	A_LFSR		1 working (we exclude armed forces)
Class of worker	A_CLSWKR		1,2,3,4 (we exclude self-employed and pro bono)

There are many CPS observations for which earnings data are completely missing. We drop those observations, as well as those that are not in the universe for the eligible CPS earning items (A_ERNEL=0), leaving 14363 observations. From those, we draw a random unweighted subsample with ten percent selection probability. This weighting combined with the selection criteria described above results in 1348 observations.

As summarized in the Table ??, we keep seven CPS variables. From the CPS data, we create additional variables AGE (age), FEMALE (1 if female, 0 otherwise), NONWHITE (1 if nonwhite, 0 otherwise), UNION (1 if union member, 0 otherwise). We also create EDUC (years of schooling) based on CPS variable PEEDUCA (educational attainment), as described in Table ??. Because the CPS does not ask about years of experience, we construct the variable EXPER (potential working experience) as AGE (age) minus EDUC (year of schooling) minus 6.

Variable List

The variable WAGE equals PRERNHLY (earnings per hour) in dollars for those paid hourly. For those not paid hourly (PRERNHLY=0), we use PRERNWA (gross earnings last week) divided by PEHRUSL1 (usual working hours per week). That sometimes produces missing values, which we treat as missing earnings and drop from the sample. The final dataset contains 1323 observations with AGE, FEMALE, NONWHITE, UNION, EDUC, EXPER and WAGE.

Variable	Description
PEAGE (A_AGE)	Age
A_LFSR	Labor force status
A_CLSWKR	Class of worker
PEEDUCA (A_HGA)	Educational attainment
PERACE (PRDTRACE)	RACE
PESEX (A_SEX)	SEX
PEERNLAB (A_UNMEM)	UNION
PRERNWA (A_GRSWK)	Usual earnings per week
PEHRUSL1 (A_USLHRS)	Usual hours worked weekly
PEHRACTT (A_HRS1)	Hours worked last week
PRERNHLY (A_HRSPAY)	Earnings per hour
AGE	Equals PEAGE
FEMALE	Equals 1 if PESEX=2, 0 otherwise
NONWHITE	Equals 0 if PERACE=1, 0 otherwise
UNION	Equals 1 if PEERNLAB=1, 0 otherwise
EDUC	Refers to the Table
EXPER	Equals AGE-EDUC-6
WAGE	Equals PRERNHLY or PRERNWA/ PEHRUSL1
NOTE: Variable names in parentheses are for 2004 and 2012.	

Definition of EDUC

mn3—l—**Definition of EDUC**		
EDUC	PEEDUCA (A_HGA)	Description
0	31	Less than first grade
1	32	Frist, second, third or four grade
5	33	Fifth or sixth grade
7	34	Seventh or eighth grade
9	35	Ninth grade
10	36	Tenth grade
11	37	Eleventh grade
12	38	Twelfth grade no diploma
12	39	High school graduate
12	40	Some college but no degree
14	41	Associate degree-occupational/vocational
14	42	Associate degree-academic program
16	43	Bachelor' degree (B.A., A.B., B.S.)
18	44	Master' degree (M.A., M.S., M.Eng., M.Ed., M.S.W. M.B.A.)
20	45	Professional school degree (M.D., D.D.S., D.V.M. L.L.B., J.D.)
20	46	Doctorate degree (Ph.D., Ed.D.)

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